THE BEHAVIOUR OF UK ANNUITY PRICES FROM 1972 TO THE PRESENT

Edmund Cannon and Ian Tonks

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Introduction

Although pensions are not usually considered the most interesting of topics, there has been considerable discussion in the British press (e.g., The Sun, 5 March 2002, The Observer, February 17, 2002) about supposedly low rates of return on annuities and the legislative compulsion to devote 75 per cent of one's pension fund to the purchase of such an annuity by the age of 75. The government has become so concerned about the functioning of the annuities market in the UK that it issued a consultation paper\(^1\) in February 2002. In this paper we consider a variety of potential criticisms of both the legal requirements and the way that annuities operate.

We start first by noting that at least part of this discussion seems to be based upon a mis-apprehension. This is that the money paid towards an annuity by annuitants who then die relatively young their wealth is received by the life insurer because any implicit capital available at the point of death is not paid to the deceased annuitant's estate. In fact, however, this money is used to subsidise those people who live for a relatively long time and who receive considerably more money from their annuity than they actually paid in. This is a key component of the idea of an annuity: it insures people against uncertain lifetimes by redistributing from those who die relatively young to those who die relatively old. The risk insured is the possibility that one might live too long, or, if that seems an odd thing to avoid, the possibility that one might live longer than one has assets to finance. Criticism of annuity markets on this basis is clearly faulty and it is a matter of educating the public (or perhaps the press) as to the true state of the matter. It is noteworthy that analyses of the UK annuity market by Murthi, Orszag and Orszag (1999) and Finkelstein & Poterba (2002) suggest that the market is approximately efficient and that annuities are not actuarially mis-priced.

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\(^1\) Department for Work and Pensions and Inland revenue "Modernising Annuities: A Consultative Document", February 2002
An alternative interpretation of the critique that annuities fail to pay back any capital is that some individuals do not wish to use the pension fund to finance their retirement income at all, but wish to preserve the wealth (which has been accumulated tax-free) to pass on to the heirs. For this reason, London & Colonial have issued a form of annuity which does involve a component of wealth being paid to the estate of an annuitant at the point of death. There are two problems with allowing annuity markets to develop in this way. The first is definitional. If pensioners are to be allowed to preserve their capital indefinitely, then this means that they are effectively being allowed to buy infinite-dated financial instruments, or “Consols”, in other words an asset which is not an annuity. While one might accept that financial intermediaries and Humpty Dumpty may choose what words mean, using the word “annuity” to describe a “Consol” is likely to lead to more confusion than less. (For a discussion see Quinton, 2001). The second problem is that allowing financial instruments such as that of L&C has significant tax implications, both for revenue and distribution. Most individuals in the UK have a pension fund of less than £50,000, corresponding to a pension income of at most £4,250 per year at current Annuity rates compared with an annual income of £2,300 which could be earned from Consols. Given the current level of the UK state pension, such pensioners will need to annuitise their whole pension fund to obtain a satisfactory income. Those individuals who will most wish to avoid annuitising their pension fund will be rich people using their pension plan as a way to avoid tax.

A more promising criticism of the current annuity rules is that annuity rates are unusually low. It is certainly true that annuity rates have fallen from about 15 per cent to about 8 per cent over the last ten years. Part of the explanation for this is that longevity has increased. As people live longer, a given sum of money paid for an annuity has to finance a longer stream of income and so income per year has had to fall. This reduction in annuity rates is unavoidable. However, the fall in annuity rates over the last ten years is clearly far too large to be due to changes in longevity alone.

Another important contributory factor is that all interest rates have fallen as can be seen in Figure 1, which illustrates the time series behaviour of the annuity rate alongside that of the consol rate. The question that this paper addresses is whether the fall in annuity rates is larger than justified by the fundamental changes in interest rates.
and longevity. However, even if low annuity rates are in some sense “justified” it is still reasonable to ask whether pensioners should be forced to purchase annuities at such low rates. To do so appears to make pensioners particularly susceptible to annuity rate risk. Apart from the effects this has on people retiring now, it may also influence potential saving behaviour of people who will retire in the future. Blundell and Stoker (1999) have shown that the timing of risk may be significant in determining agents’ savings decisions and that even quite small future risks may influence current savings behaviour.

To answer this question we need to consider the effects of two additional effects on people's pensions. The first is the interaction of government legislation with the national debt and the second is the behaviour of the stock market.

Because annuities are relatively long-lived financial liabilities from the point of view of an insurance company, it has been common practice to match these with long-lived assets such as long-dated government debt. A series of government regulations following on from the Maxwell pension scandal has led many pension funds also to hold significant quantities of such assets. Unfortunately, this increased demand for long-dated government debt coincides with a reduction in the size of the national debt through a series of budget surpluses or very small deficits, as well as the large transfer to the government of the money raised in the 3G telecommunications auction. This has led to very high prices of long-dated government debt and relatively low yields, to the point where the yield curve is now perversely shaped, ie downward sloping, at the long end. Thus, although low annuity rates merely reflect low long-term interest rates, this may be because long term interest rates themselves have been artificially distorted. This phenomenon has led to calls for pension funds to be able to hold a wider variety of assets and this alone might allow annuity rates to rise. According to this line of argument (which relies upon a “preferred-habitat” view of the term structure), there is a major distortion of all long-term interest rates which has a corresponding effect on annuity prices.

On the other hand, there is good reason to believe that low annuity rates in themselves are irrelevant. If the annuity rate is $A$ and the value of a pension fund at the point of retirement is $V$, then the value of a pension that can be purchased is $AV$. For a pension fund invested predominantly in the stock market, $V$ effectively represents the stock
market index, which tends to be negatively correlated with interest rates. Thus it is quite possible for the annuity rate, A, to be relatively low while AV is close to its long-run value. For this reason, no discussion of annuity rates can be divorced from a wider discussion of financial markets.

The rest of this paper is organised as follows: in the next section we present an annuity rate for the last thirty years for the UK and discuss its relationship with the long term interest rates. We then calculate the net present value of such an annuity over this period, a figure sometimes referred to as the “money's worth”. To relate the decumulation phase of a pension to the accumulation phase we then conduct a simple analysis of the relationship of annuity rates and the stock market before concluding.

Annuity Rates and Long Term Interest Rates

There are a paucity of time series data on annuity rates in the United Kingdom, contrasting with the historical analysis of the USA by Warshawsky (1988). Our data are taken from bi-monthly or monthly figures published in Pensions World from September 1972 to November 1977 and monthly data from April 1980 to May 1998. During these periods Pensions World published a consistent series of data of non-escalating purchase annuities guaranteed for five years, for both men and women of different ages from a variety of different annuity providers. The gaps in the series are filled with data from Money Management and Money Facts. More details can be found in Appendix A. One concern with using non-escalating annuities is that these are the forms of annuity most at risk from inflation, apart from the fact that they fail to keep pace with secular increases in average earnings. However, from the point of view of analysing the UK annuity markets they are appropriate, since most annuities which are purchased are of this form. An additional consideration is that data for index-linked or escalating annuities are not available for much of the period.

Our data are illustrated in Figure 1: lest there be any confusion, the annuity prices are usually quoted in the form of an annuity of £600 per £10,000 purchased, which we would represent as 6 per cent on our graph. For comparison with other interest rates we also plot the consol rate. Theoretically these two series should be linked because
they are both long-term assets: if longevity were constant and the short term rate were not particularly variable then the two would differ on average only by a constant.

Descriptive statistics are presented in the table below. As can be seen from the graph, the series are highly correlated (the correlation between the two series is 0.906). Given a 5 per cent critical value of 3.59, it is impossible to reject the null hypotheses that the they each have unit roots, but it is possible to reject the null of a unit root in the difference between the two variables, suggesting that the series are cointegrated.

<table>
<thead>
<tr>
<th></th>
<th>Annuity Rate</th>
<th>Consol Rate</th>
<th>Annuity Rate - Consol Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>13.06 per cent</td>
<td>9.85 per cent</td>
<td>3.21 per cent</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>2.64 per cent</td>
<td>2.76 per cent</td>
<td>1.18 per cent</td>
</tr>
<tr>
<td>ADF test for Unit Root 5 per cent critical value</td>
<td>3.31</td>
<td>2.87</td>
<td>3.89*</td>
</tr>
</tbody>
</table>

Multivariate analysis of the two series is hazardous with so few observations and so further analysis of the cointegration properties of the two variables is suggestive rather than conclusive. In a bivariate model of the two series whose residuals seem well-behaved it is possible to reject the null of two unit roots with a trace statistic of 26.2 (p-value 0.43), but it is possible to reject the null hypothesis that the cointegrating vector is simply the difference of the annuity and consol rates but for a linear trend (the test for the homogeneity restriction is $\chi^2 = 8.12$).

Thus we conclude that there is sufficient statistical evidence to confirm what is visible by eye, namely that the annuity and consol rates have behaved similarly over the last thirty years, but insufficient statistical evidence to specify the relationship closely. For this reason we turn to a more theoretically based analysis of the annuity rate.

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2 Data is 1973-2000. The unit root tests are based on a regression involving a constant and a trend, since both series are trending downwards over the period, and includes two lags. Alternative specifications yielded very similar results.

3 Our cointegration analysis was based upon one lag of the variables concerned with a linear trend restricted to be within the cointegration space. Apart from the fact that it was statistically significant, the trend was included as a crude proxy for any possible convergence between the two rates over the period due to increased longevity (as life expectancy rises, the length of time for which an annuity pays out increases and hence it becomes “more similar” to a consol). However, since the annuity and consol rates are themselves trending down, the inclusion of a trend is itself problematic. Its exclusion makes it impossible to reject the null hypothesis of no cointegration between the two series.
Net Present Value Calculations

There are two ways to compare the value of annuities with other assets. The simplest way to calculate the value of an annuity is to use the measure called the “money’s worth”, which is simply the ratio of the expected present value of the flow of payments made by an annuity to the money paid for an annuity. This procedure has been used by Mitchell et al (1999) to analyse the annuities market in the USA and by Murthi, Orszag and Orszag (1999) and Finkelstein and Poterba (2002) to analyse the UK annuity market. These three papers all analyse a cross-section of annuity rates offered by different firms and also consider how the money’s worth depends upon the type of annuity purchased: the evidence suggests that annuities purchased by older annuitants yield a lower money’s worth. A detailed description of the method can be found in the references cited: for a general discussion see the Introduction to the collection of papers in Brown et al (2001).

The alternative way to compare the value of annuities with other assets is to consider the internal rate of return implied by an annuity rate. The advantage of the latter approach is that it is necessary only to project life expectancies, whereas the money’s worth approach requires assumptions about expectations of future interest rates as well. However, since the results we obtain the two approaches are consistent, we relegate our results from the internal rate of return approach to the Appendix C.

Returning to the money’s worth approach: define the annuity rate $A_t$ as the payment made per year of an annuity which cost £1 to buy in year $t$. The money’s worth for a level 5-year guaranteed annuity can be written as

\[
A_t \left\{ \sum_{k=66}^{70} \prod_{j=1}^{k-65} (1 + r_{t+j})^{-1} + \sum_{k=71}^{\infty} \pi_{t,k,65} \prod_{j=1}^{k-65} (1 + r_{t+j})^{-1} \right\}
\]

where $r_{t+j}$ is the interest rate in period $t + j$ and $\pi_{t,65,k}$ is the probability of someone born in year $t - 65$ surviving to age $k$ having reached age 65.$^4$

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$^4$ This assumes that no (fraudulent) payments are made to the annuitant or his or her estate after the point of death (disregarding payments made in the guaranteed five years period). Evidence from the Audit Commission (2002) suggests that such fraud may not be
We estimate expectations of future interest rates from the term structure of interest rates. This means that the 1988 interest rate used to value an annuity sold in 1980 is the implicit rate in 1980 yield curve. We choose to use this interest rate rather than the ex post interest rate because is provides a consistent series of interest rates for us to use over the whole period. Apart from consistency across time, the approach has the advantage that it can be compared directly with Mitchell et al (1999), Murthi, Orszag and Orszag (1991) and Finkelstein and Poterba (2002), as well as giving an idea of the profits anticipated by life assurers at the time of selling the annuity, rather than the ex post realisation.

It is, of course, well known that the term structure is rather a mediocre predictor of future interest rates. According to theories where there is a liquidity or term premium or according to preferred habitat theories, it may be a biased predictor. Most discussion assumes that longer rates are higher because of a term premium, in which case the interest rates we infer from the yield curve will be biased upwards and the money’s worth biased downwards. Since we shall be arguing that the money’s worth is quite high, this bias will be against our argument and in fact strengthen our conclusions.

As mentioned in the Introduction, however, there may be some reason to believe that long term interest rates are currently biased downwards, in which case we would be over-estimating the money’s worth. We shall partially address this problem in the following section where we discuss the value of pensions.

To implement equation (1) we also need to infer expected survival probabilities. Ideally we should use a panel of \textit{ex ante} survival probabilities. However, because of data constraints we have used the \textit{ex post} realised probabilities where they are available, which can be justified on the basis that these move relatively slowly and are less prone to severe mis-prediction. Where we do need to project survival probabilities into the future we do so using a simple model quadratic in time and age,
described more fully in Appendix B. We use two estimates: one is relatively conservative and assumes longevity will only increase slowly, whereas the other assumes faster increases in longevity. In the Appendix we discuss the possibility that our estimates of survival probabilities based on mortality rates as a whole will provide a further downward bias to our estimates of the money’s worth.

The money's worth under these two assumptions is illustrated in Figure 2. The observations for 2001 and 2002 are not perfectly comparable with previous years, since not all of the relevant mortality data is available, and not all of the yield curve data is yet published: the 2002 observation is based on the first quarter only. The money’s worth in 2000 was between 89 pence and £1.01, falling to a range of 85-96 pence for the first few months of 2002.

Figure 2 illustrates that the money’s worth is not particularly poor at the moment. Certainly there have been times when it was worse, notably the 1970s. Since a fixed nominal payment was worth less in the 1970s because it was more quickly eroded by the high inflation of that period, something which will be inadequately accounted for by discounting with the yield curve because it is well known that interest rates failed to fully incorporate changes in inflation at that time.

Furthermore the money’s worth only appears poor if one assumes that future life expectancy will be relatively low: using the higher estimate of life expectancy leads to money’s worth estimates that still appear high by comparison with the last thirty years.

These results are very similar to the cross-sectional analysis of Finkelstein and Poterba (2002) and Murthi Orszag and Orszag (1999). The former found the money’s worth to be 90 pence and the latter 93.2 pence in 1998, comparing with our figure of 93 pence based on a smaller increase in life expectancy. Murthi, Orszag and Orszag (1999) also provide estimates of 99.6 pence in 1990 and 92.1 pence in 1994: our analogous figures are 99 pence and 84 pence.

This section has illustrated that using the money’s worth criterion, annuity rates appear competitive compared with the last thirty years. This result depends to a
considerable extent on the yield curve being a valid source of information for the calculation of the money’s worth. As we have noted, however, this may not be valid if, as would be the case in preferred habitat theories, the long-term interest rate is itself artificially low: the fact that there is efficient arbitrage between annuity markets and bond markets is of little comfort if all financial markets are distorted. We address this question indirectly in the next section by asking how much pension an annuity can buy at the present time.

**Value of Pension Funds**

The previous discussion centres on the actuarial fairness of annuity contracts. Perhaps more important to the actual annuitant is the pension that he receives, regardless of how long he lives. This means that we need to consider both the accumulation of pension fund and the annuity rate together. The value of the pension received by a pensioner is $AV$, where $A$ is the annuity rate and $V$ is the value of the pension fund at the point of retirement (assuming that all of the pension fund is annuitised). It is a well-known empirical regularity that the value of assets is negatively correlated with interest rates. This is theoretically unsurprising, since the formula for the value (strictly speaking net present value) of an asset is negatively related to the discount rate. Thus there is very good reason to believe that the variables $A$ and $V$ will be negatively correlated. If this is so then any discussion of pensions needs to address the relationship between these two variables.

To do this we need to determine how the value of individuals' pension funds have changed over time. The earliest discussion of this issue of which we know is the discussion in Diamond (1977). Since we have no data on this variable, we construct the pension funds of a series of hypothetical individuals who save according to a well-specified rule.

Diamond introduces the concept of a “replacement ratio”, defined as the ratio of the pension income to labour income (net of pension contributions) in the final year of employment. If the savings rate is 10 per cent and pension income is 60 per cent of labour income, then the replacement ratio is $60/90 = 2/3$ and Diamond suggests that
this replacement ratio might be appropriate. Empirically such replacement ratios are common in UK company pension schemes where employees have completed their full set of contributions.

The optimal value of the replacement ratio is unclear. In a simple utility maximisation framework where agents only wish to smooth consumption flows, the optimal ratio would be one. However, this result does not follow if agents also obtain utility from leisure and utility is not additively separable in consumption and leisure: because leisure discontinuously increases at the point of retirement we should also expect consumption to discontinuously fall. We might note that there are at least two reasons for consumers’ expenditure to change upon retirement: the elimination of work-related expenditure (commuting etc) and variation in expenditure on leisure activities.

Consider the pension fund of someone retiring at time $t$, who has contributed a proportion $s$ of their income $y_{t-i}$ in year $t-i$ to a fund for the last $R$ years. Each year $t-j$ the entire value of the fund (including previous years’ returns which are re-invested) earns a rate of return $r_{t-j}$. Then the value of the fund at retirement is

$\sum_{i=0}^{R-1} y_{t-j} \prod_{j=0}^{i} (1 + r_{t-j})$.

(We assume that interest is earned on the last year's contribution.)

Note that this rule underlies the so-called “60:40:20:10:5:2 rule”. With a constant rate of return $r$ and with constant income growth so that $y_t = y_0(1 + g)^t$, the formula for the value of the fund at retirement simplifies to

$s \sum_{i=0}^{R-1} y_0 (1 + g)^{t-i} (1 + r)^{i+1}$.

This fund can then be converted into an annuity over the expected retirement life of the individual. Manipulation of the formula leads to the surprisingly tidy result that: if one wants one's pension to be 60 per cent of one's final salary and one works for 40
years followed by 20 years of retirement, then it is sufficient to save just 10 per cent of one's income every year, so long as the real rate of return (on both investments and annuities) is 5 per cent per year and real wages are growing at 2 per cent per year. Again, this rule implies a replacement ratio of 2/3.

Rather than assume a constant rate of return and labour income growth, we use actual data to calculate possible pension fund values. We consider a series of hypothetical individuals whose labour income is proportional to the UK average earnings index in each year of their life. From the age of 26 to 65, they save 10 per cent of their labour income and invest it in the stock market: all dividends are re-invested. To account for charges, we assume that there is a 5 per cent charge for purchasing shares, so that the effective savings rate is 9.5 per cent instead of 10 per cent, that there is a 2 per cent charge every year on the capital invested, and that the spread is zero. These charges are consistent with the estimates of charges found by Chapman (1999). At 65 the agents purchase an annuity at the prevailing annuity rate.

Figure 3 shows the replacement ratio under these assumptions. This replacement ratio may appear very high, averaging a little under one, whereas the typical defined benefit company pension yields a replacement ratio closer to 2/3 (often consisting of an annual income of half final salary and a lump sum). However, defined benefit company pensions are not level pensions but usually index linked and they also usually contain a reduced pension for any spouse who survives the principle pensioner. Direct comparison is thus impossible. We are currently attempting to gain additional data on indexed pensions and joint pensions to see how these replacement ratios compare, but back-of-envelope calculations suggest that if appropriate reductions were made then the resulting replacement ratio would be lower at about 2/3 as we should expect.

5 These assumptions about the accumulation of pension funds are not fully realistic, since agents would hold some of their wealth in bonds and might change the proportions of bonds and equities over their life. Consideration of this issue is the subject of on-going research.

6 In January 2002 Legal and General is quoted in Pensions World as being the 5th best provider of annuities for Male aged 65 level no guarantee (being only 0.2 per cent behind the best provider, Friends Provident) and as the best provider of joint annuities for Male aged 65 Wife aged 63, escalating 3 per cent no guarantee. The annuity rates are respectively 8.32 per cent and 5.13 per cent, so the escalating annuity with income for spouse is just over 3/5 the level annuity with no income for spouse.
It is clear from the graph that the last two years have been less favourable times to retire than 1999, when the stock-market was at its peak. But even after the fall in stock market prices, the replacement ratio does not appear particularly bad, certainly not as bad as the mid-1970s. Even compared with the period 1982-2000, when the replacement ratio averaged just over unity, the replacement ratio does not appear disastrously low.

The reason for this is that there has been a strong negative correlation between stock markets and annuity rates since then. This is illustrated in Figure 4, which plots the annuity rate against the ratio of pension fund to final salary. Although the points 2001 and 2002 lie below the curve, they do not do so significantly.

It should be noted that the steady replacement ratio since 1980 is despite increases in longevity over the last 20 years, which is ignored in this section. Thus individuals retiring in 2002 on the same replacement ratio as people 20 years earlier are better off, since they live longer and have a similar annual income.
Conclusion

In this paper we have constructed a time series of annuity prices since 1972. This is considerably longer than the only other time series that we know to have been published in Murthi, Orszag & Orszag (1999) which included a graph of the best annuity rate for the period 1988-98.

Using this series we answer two questions: are annuity rates unfairly low and does it matter that they are low?

In answer to the first question we find no evidence that the average annuity rate is unfairly low. Depending on the assumptions we make about future longevity, the present value of an annuity is of the order of between 90 per cent and 100 per cent of the purchase price. Compared with the typical costs of buying financial services this figure looks suspiciously good: annuity providers must earn a profit and cover the real resource costs of annuity provision. Perhaps even more worrying is the fact that annuity providers appear to have no cushion if longevity rises as fast as our more liberal projections allow. The insurance provided by annuity markets is to smooth longevity risk between individuals, but there is an additional risk of mis-estimating average longevity, which is faced either by insurance companies or, if the companies fail, the annuitants.

A possible response to this conclusion is that current annuity rates may appear to provide a good money’s worth because the latter is calculated using interest rates which are themselves distorted.

In answer to our second question, we find no reason to suggest that individuals are worse off by annuity rates being low, since this has been off-set by increases in the value of pension funds over the last thirty years. Even apart from the fact that people retiring today expect to live longer, their pension income (compared to their final salary) looks as good as ever. This is prima facie evidence to suggest that government policy need not be mis-placed and that any distortions of the pensions or bond markets are unimportant.
Appendix A: Discussion of the Data

The majority of our data are taken from bi-monthly or monthly figures published in *Pensions World* from September 1972 to November 1977 and monthly data from April 1980 to May 1998. During these periods *Pensions World* published a consistent series of data of non-escalating purchase annuities guaranteed for five years, for both men and women of different ages from a variety of different annuity providers.

Because the *Pensions World* series is incomplete, we have also used data from *Money Management* and *Money Facts* to fill in the missing periods from 1978-1980 and 1998-2001 respectively. A consequence is that there are a total of three problems in aggregating these data: changing composition of annuity providers, stale prices and inconsistent series. We discuss how we overcome these problems in turn.

The first problem that we have is that the composition of annuity providers changes over time even within *Pensions World*. To overcome this we have analysed several different measures: the mean of all prices quoted, the median of all prices quoted, the best price quoted, the mean of prices from a relatively constant sub-set of companies and the median from the same subset.

The second problem is that some companies appear to keep their prices exactly constant for long periods of time, during which time they are often quite uncompetitive. This may be because the price in the data source is stale, being just rolled over from the previous month, or it may be because the company was not actively seeking to gain custom, in which case the annuity price may be unrepresentative of annuities which were actually purchased at that time. To overcome this situation we have also considered using only those prices of firms which have changed since the previous month. With the possible exception of 1974 this makes little difference: in that year there is some evidence that annuity rates of firms whose prices had changed were systematically 1 per cent better than all annuity rates, consistent with some stale prices during a period of rising prices. From the purposes of our annual analysis, the difference is so small (and only affects one observation), that this makes virtually no difference.
The resulting monthly series is illustrated in Figure 5. These are the raw data, with no adjustment made for changes in definition.

This brings us to our third problem of maintaining consistency between the data in *Pensions World* and the data in the other two sources. The data in *Money Management* are for annuities paid half yearly in arrears and not guaranteed, whereas the data in Pensions World are for annuities paid monthly in advance, guaranteed for five years.

To correct this definitional difference we perform some actuarial calculations to maintain rough consistency: fortunately the data in *Money Management* are being used merely to fill in a small gap in the otherwise consistent Pensions World series. To make the series compatible we performed a calculation in two steps. Annuities paid half yearly in arrears result in payments being paid on average a quarter of a year later, so we subtracted one-quarter of the then prevailing consol rate (actually 11.5 per cent) to effect the increased discount given to payments made further in the future.

Data on monthly hazard rates are unavailable, so we determined the effect of an annuity being guaranteed for five years on annual data. The probabilities of dying between the age of 65 and the ages of 66-70, (i.e., $1 - \pi_{165,66}, 1 - \pi_{166,67},$ etc.) for the two years 1978 and 1979 are 0.03, 0.07, 0.11, 0.15, 0.19. Write the headline annuity rate (i.e., the payment per year as A per pounds 1 paid to the annuity provider).

Assuming a constant interest rate $r$, which we assume to be equal to the consol rate, the present value of an annuity guaranteed five years is

$$A_N \left\{ \sum_{k=66}^{70} \prod_{j=1}^{k-65} (1 + r_{1+j})^{-1} + \sum_{k=1}^{\infty} \pi_{1,k,65} \prod_{j=1}^{k-65} (1 + r_{1+j})^{-1} \right\},$$

whereas that not guaranteed five years is

$$A_N \left\{ \sum_{k=66}^{\infty} \prod_{j=1}^{k-65} (1 + r_{1+j})^{-1} \right\}.$$
Now suppose that the money’s worth of these two annuities were the same and thus they were equal. Then, assuming a constant interest rate,

\[
\frac{1}{A^N_t} - \frac{1}{A^t} = \sum_{k=65}^{199} \pi_{1,65} (1 + r)^{65-k} = 0.37
\]

and

\[
A_t = \frac{1}{1/A^N_t - 0.37}.
\]

It is this estimate of the annuity rate that we use for illustrative purposes in Figure 1. However, the analysis of Finkelstein and Poterba (2001) suggests that the money’s worth is not going to be the same for the two series because of adverse selection effects: agents who choose an annuity guaranteed five years are more likely to be those who have a relatively high probability of dying in the first few years of the annuity. Unfortunately we do not have sufficient data to calculate how much premium this is likely to involve to a non-guaranteed annuity. To partially overcome this problem, our analysis of money’s worth in the main section of the paper does not use the annuity rate constructed above, but uses the non-guaranteed annuity (with an appropriate change in the definition of the money’s worth). Our results in Figure 2 show such a variable money’s worth around these few years that it is impossible to discern whether this causes any economically significant bias.

The data in *Money Facts* are defined similarly to those in *Pensions World*, but are drawn from a different list of companies. More worrying, the annuity rates for the subset of companies in both sources are different. We have contacted the relevant annuity providers to seek an explanation for this and have been told that it arises due to different commission charges being included in the two quotes (e.g, one included a commission of 1 per cent and the other a commission of 1.3 per cent). To overcome this difference we have spliced the series together using a shift factor derived from the overlap period of four months (1998 January-April), which turns out to be 0.38 per cent, comfortably close to figure we might expect given the commission charges quoted.
Appendix B: Estimation of Longevity

To calculate the money’s worth or the internal rate of return it is necessary to have estimates of the probabilities of the annuitant living to a given age. Back-of-envelope calculations regarding the money’s worth can be conducted using life expectancy alone and assuming that annuity payments are made for the same period as the money’s worth: this is the simplification used in Diamond (1977) and in 60:40:20:10:5:2 rule. However, this is a very crude method because there is considerable variability of actual age of death around life-expectancy; it will also introduce biases over time since the patterns of mortality are changing (see e.g., Craig, 1998).

To discuss the issue of the pattern of mortality we introduce the concept of the hazard, or probability, of dying at a given age. Empirical values of male hazards are plotted in Figure 6 over a long time horizon. These are cross sectional hazards: the series of data labelled 1932 shows the probability of a 50-year old in 1932 (born in 1882) dying before age 51 and the probability of a 51-year old in 1932 (born in 1881) dying before age 52. In the demographic literature cross-sectional mortality rates such as these are referred to as a “secular” tabulation.

We can contrast these with cohort hazards, which determine the probability of dying at a given age for someone born in a given year, e.g., 50-year old in 1932 (born in 1882) dying before age 51 followed by a 51-year old in 1933 (born in 1882) dying before age 52. The demographic terminology here is that these data are “generational” mortality rates.

It is these mortality rates that are needed to determine the money’s worth of an annuity. However, we are interested not so much in the probability of dying as the probability of surviving (and hence receiving an annuity payment). Write the hazard rate for someone age \( i \) born in period \( t - i \) as \( h_t \). The probability of someone aged \( j \) at time \( t \) being alive at age \( k \) is then
\[ \pi_{t,j,k} = \prod_{i=j+1}^{k} (1 - h_{it}) \]

We now turn to the calculation of these survival probabilities in practice.

Mitchell et al (1999) note that purchasers of annuities tend to be longer lived than typical agents of the same age. This merely reflects the adverse selection problem faced by insurers, that agents who choose to buy annuities are self-selected to be those who will benefit most. Comparative data for annuitant and population mortality are illustrated on pp.18-19 of Brown et al (2001).

We do not currently have mortality data for UK annuitants and have used population mortality figures. Unless there have been systematic changes in the relative life expectancies of the two groups, using population mortality figures will make no difference to our comparison of money’s worth over time. Regarding the absolute size of error in our estimates of the money’s worth, it should be noted that the bias in our estimates of the money’s worth is downwards, since annuitants tend to live longer than the population as a whole. As we are arguing that the money’s worth is relatively high, the bias will lead us to understate our case that annuities are not under-priced.

Hazard rates are published in “life tables” based on Census and other data. Because there are sometimes relatively few people of a given age (especially for the very elderly), the raw data is usually smoothed to ensure that sampling error is minimised (see e.g., Haberman, 1997). However, English Life Tables are only published every ten years (no. 15 was published in 1997, based on the 1991 Census). Because we wanted annually available data we have constructed our own life tables using data for death rates in Population Trends for ten year groups (45-54; 65-64 etc.) and log-linearly interpolating to obtain an annual panel of hazards for every year of age.

Figure 7 illustrates the resulting empirical survival probabilities (one minus the hazard) for males aged 65 in years 1972 onwards: the longest (and lowest) curve is that for someone aged 65 in 1972, while the shortest is for someone aged 65 in 2000,
for whom the hazard is only available for one year. The clear impression is that survival probabilities have risen over the last thirty years as the curves have shifted up. There is also some evidence that each generation’s survival probability curve is getting flatter.

To project these survival probabilities into the future we fitted two possible relationships between survival probability, cohort effect and age:

\[
1 - h_{it} = \alpha_0 + \alpha_i i + \alpha_2 i^2 + \alpha_3 (t - i) + \alpha_4 (t - i)^2
\]

and

\[
1 - h_{it} = \alpha_0 + \alpha_i i + \alpha_2 i^2 + \alpha_3 (t - i) + \alpha_4 (t - i)^2 + \alpha_5 (t - i) i
\]

The first of these regressions restricts the “survival probability curve” to be the same shape and allows it to shift up over time. We use a quadratic to allow for the very slight curvature in the empirical curve. The second regression allows the curve to change shape in that it can become gradually flatter.

The projected survival probabilities from the second of the two equations are considerably larger and we thus refer to these estimates as the “Larger Life Expectancy”, whereas the former relationship is the “Smaller Life Expectancy” case. Although these projections are econometrically rather crude, they provide a useful interval of life expectancies. The historical record of life expectancy projections is that they tend to under-estimate increases in life expectancy (Cox, 1975), suggesting considerable room for error: this is another reason for considering a range of possibilities.
Appendix C: Internal Rates of Return

An alternative method of evaluating annuity rates is to look at the internal rate of return of an annuity instead of the money’s worth. The advantage of this approach is that it is not necessary to have expectations about future interest rates, although it is still necessary to have information on survival probabilities.

The internal rate of return for an annuity \( r \) for a Male aged 65 guaranteed five years is defined implicitly from the equation

\[
1 = A \left\{ \sum_{k=0}^{70} (1 + r)^{65-k} \right\} + \sum_{k=71}^{\infty} \pi_{r,65}(1 + r)^{65-k}.
\]

We calculate \( r \) numerically in Excel to obtain a series which is illustrated in Figure 8: we calculate the series using the two different projections of survival probabilities used in calculating the money’s worth and plot the consol rate for comparison. In the mid-1970s, and to a smaller extent in the mid-1990s, consol rates are considerably higher than the internal rate of return on annuities, whereas for the rest of the period consol rates move closely with the internal rate of return. As with the money’s worth calculations, the projection with the larger increase in life expectancy leads to firms being unprofitable: internal rates of return on annuities (ie a cost to the firm as these are payments) are higher than the long run interest rate.
References


Figure 1: Interest Rates and Annuity Rates
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- Smaller increases in Life Expectancy
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