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RISK AVERSION AND THE UTILITY OF ANNUITIES

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Risk aversion and the utility of annuities*

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Abstract

A well-known result in life-cycle models with uncertain lifetime is that, absent other sources of uncertainty, egoistic agents should annuitize all their wealth. The gain from access to an annuity market, as measured by the increase in non-annuitized wealth required to obtain the same utility level, has repeatedly been shown to be a positive function of risk aversion in expected-utility models. This paper extends the analysis by considering the recursive utility function introduced by Epstein and Zin. By disentangling risk aversion from the elasticity of intertemporal substitution it is shown that the utility value of annuitization is decreasing with both parameters. The classical Yaari result that access to a fair annuity market leads to the same consumption dynamics as in the certainty scenario is also shown to obtain only in the expected-utility case.

1 Introduction

Economists have long acknowledged the apparent discrepancy between life-cycle theory, which predicts that selfish agents with uncertain lifetimes should hold all their wealth in the form of life annuities, and empirical evidence showing the extreme rarity of private individual annuity contracts[1]. Within this broader "annuity puzzle", it is interesting to note that economic theory and conventional wisdom seem to be at odds with respect to the relation of annuity demand to risk aversion.

In fact, recent findings from consumer focus groups conducted by the American Council of Life Insurance suggest that annuities are widely perceived as a source of increased risk, and even equated with gambling on one's life with odds favouring the insurance company[2]. This opinion is not only common but quite ancient, since the famous XVIII-century encyclopedists Diderot and D'Alembert

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[1] This observation is found e.g. in Modigliani (1986).

[2] Cfr. Brown and Warshawsky (2000).

(1765) concluded that "*les rentes viagères, de quelque manière qu'elles soient faites, sont des jeux ou loteries où l'on parie à qui vivra le plus.*"[3]

On the other hand, insurers and economists alike take it for granted that annuities are meant to insure against an existing risk. This is usually identified with the so-called "longevity risk", namely the risk of outliving one's assets, but one can refer to the risk of consuming too conservatively, rather than too aggressively, thereby leaving unintended bequests upon one's death[4].

The insurance interpretation is apparently supported by simulations of the utility gain deriving from the access to an annuity market: the increase in wealth required to obtain the same utility level without annuitization is invariably shown to increase with the coefficient of relative risk aversion.

This paper attempts to shed new light on the issue by analyzing different formulations of the utility function: section 2 discusses the role of risk aversion and the elasticity of intertemporal subtitution in a general case; section 3 considers the recursive Epstein-Zin utility function, which allows a disentagling of the two parameters, which are constrained to be reciprocals of one another in the conventional case of an additive and homogeneous von Neumann-Morgenstern intertemporal utility function; section 4 concludes.

2 Risk aversion, the elasticity of intertemporal substitution and the utility of annuities

Yaari (1965) was the first to demonstrate that in a life-cycle model with uncertain lifetimes and no other sources of risk, retirement consumption will be entirely financed with the purchase of actuarially fair annuities[5]. Accordingly, a consumer with no bequest motives will annuitize all her wealth; this result has subsequently been shown to be valid in a rather general setting.

Annuities can be construed as a sequence of actuarial notes, which dominate traditional financial assets because their rate of return is equal to the market interest rate plus a mortality premium. Such a premium can be paid because an actuarial note is cancelled when the holder dies, and thus there is a strictly positive probability that it does not have to be repaid at maturity.

While actual insurance-pricing strategies do not necessarily imply positive mortality premia for every actuarial note embedded in an annuity, this assumption is obviously much weaker than actuarial fairness: a convenient - albeit slightly stronger than necessary - way to represent it is to denote the price of

[3]Life annuities, however constructed, are gambles or lotteries where people bet on who shall live the longest.

[4]While the literature does not usually stress this point, the latter is the more relevant risk for a rational risk-averse agent in conventional models. The usual assumption of infinite marginal utility in the origin implies that the optimal consumption plan shall never contemplate the complete exhaustion of wealth before an age associated with an infinitesimal probability of survival.

[5]Other sources of risks may either be absent, or perfectly insured thorugh complete and fair insurance markets in the Arrow-Debreu sense.

an actuarial note of maturity t as $B_t l_t$, where B_t is the price of a riskless zero-coupon bond of identical maturity and l_t the probability of survival for t years derived from any mortality table, with an arbitrary degree of selection[6].

In other words, a life annuity is a means of attaining a more favourable intertemporal budget constraint by sacrificing the opportunity to bequeath unconsumed wealth. Thus, if the representative agent derives utility only from her own consumption and is able to plan it optimally in a single initial period, the optimal plan undoubtedly involves full and immediate annuitization, irrespective of the precise specification of the utility function. This is the case in a conventional model with no bequest motive, no uncertainty which is not perfectly and completely insurable, and intertemporally consistent preferences.

Having established that the fundamental effect of annuitization is that of shifting the budget constraint, we can analyze the utility gain resulting from this sfhit with the standard tools of microeconomic theory. To begin with, let us consider the compensating variation *à la* Slutsky $CV_S = \sum_{t=0}^{T} c_t B_t - c_t B_t l_t = \sum_{t=0}^{T} c_t B_t (1 - l_t)$, where c_t indicates optimal consumption in period t before annuitization is allowed, and T is the maximum possible lifetime.

Given two consumers i and j who are offered annuities at the same price, the comparison between their respective utility gains is thus

$$CV_{S,i} - CV_{S,j} = \sum_{t=0}^{T} (c_{t,i} - c_{t,j}) B_t - \sum_{t=0}^{T} (c_{t,i} - c_{t,j}) B_t l_t$$

The first sum is the difference in wealth between the two agents, and as such is set to zero for a meaningful comparison. Consequently, the expression reduces to

$$CV_{S,i} - CV_{S,j} = \sum_{t=0}^{T} (c_{t,j} - c_{t,i}) B_t l_t$$

By definition, the successions $\{B_t\}$ and $\{l_t\}$ are both monotonically non-increasing and bound in the interval $(0, 1]$; therefore, if $\{c_{t,j} - c_{t,i}\}$ is also monotonically non-increasing, it follows from $\sum_{t=0}^{T} (c_{t,j} - c_{t,i}) B_t = 0$ that $CV_{S,i} - CV_{S,j} > 0$. In other words, the benefit of annutization, as measured by the compensating variation *à la* Slutsky, is greater for consumer i than for consumer j if the former's consumption grows at a higher rate, or decreases at a lower rate, than the latter's when no annuities are available.

Of course, the Slustsky measure is only a linear approximation of the utility gain, but bearing in mind that it coincides with the variation in consumer surplus for infinitesimal price movements, we can conclude that the utility gain resulting from annuitization is greater for consumer i than for consumer j if the former's consumption growth rate is not lower than the latter's for every price vector belonging to the segment $(1 - \alpha + \alpha \mathbf{l}) \mathbf{B}$.

Thus annuitization is more attractive for longer-lived people[7], which is pretty

[6] The multiplicity of asset classes and the uncertainty of future returns can both be disregarded without loss of generality: for every existing financial asset an equivalent actuarial note could always be devised.

[7] Analytically, it is required that agent i's survival probability dominates agent j's.

obvious, but also for those whose intertemporal discount rate is lower and, most important in this context, for those whose elasticity of intertemporal substitution is lower, provided the optimal consumption path is never increasing in time. The last hypothesis is both conventional and eminently plausible, since it is equivalent to assuming that the individual discount rate is not so greater than the market interest rate as to offset the difference between actual and actuarially fair annuity prices.

This proves that the positive relation between the parameter of an intertemporally additive power utility function and the benefit of annuitization can be unequivocally interpreted in terms of the elasticity of intertemporal substitution. The alternative interpretation in terms of relative risk aversion, while more common, seems to be less straightforward. In fact, annuitization entails a reallocation of resources from nearer to remoter periods, leading to an increase in the variance as well as the expected value of total utility: this suggests that annuities should, if anything, be less attractive to more risk-averse consumers.

3 The case of the Epstein-Zin utility function

While traditional models involving time-additive expected utility maximization cannot distiunguish between risk aversion and the elasticity of intertemporal substitution, the separation of the two parameters is a crucial feature of the Epstein-Zin recursive intertemporal utility function. The two key assumptions underlying its specification are "that the agent forms a *certainty equivalent* of random future utility using his risk preferences", and "that to obtain current-period lifetime utility, this certainty equivalent is combined with deterministic current consumption via an *aggregator* function."[8] Thus lifetime utility in period t is given by

$$U_t = W\left(c_t, \mu\left[\tilde{U}_{t+1}|I_t\right]\right)$$

where $\mu\left[\tilde{U}_{t+1}|I_t\right]$ is the certainty equivalent of stochastic future utility $\tilde{U}_{t+1}$ given the information available to the agent in the planning period I_t, and $W(.)$ is the aggregator function.

Let the latter have the form

$$W(c, z) = \left(c^{\phi} + \beta z^{\phi}\right)^{\frac{1}{\phi}}$$

with $c, z \geq 0$ and $0 < \beta < 1$; $0 \neq \phi < 1$ reflects the elasticity of intertemporal substitution $\eta = (1-\phi)^{-1}$. Define furthermore the certainty equivalent as

$$\mu[\tilde{x}] = (E\tilde{x}^{\alpha})^{\frac{1}{\alpha}}$$

where $0 \neq \alpha < 1$ may be interpreted as an inverse measure of relative risk aversion. This leads to the recursive structure for intertemporal utility

$$U_t = \left[c_t^{\phi} + \beta\left(E_t\tilde{U}_{t+1}^{\alpha}\right)^{\frac{\phi}{\alpha}}\right]^{\frac{1}{\phi}}$$

[8] Epstein and Zin (1991), p. 265.

which for $\alpha = \phi$ coincides with the traditional expected utility

$$U_t = \left(\sum_{j=0}^{\infty} \beta^j E_t \tilde{c}_{t+j}^{\alpha} \right)^{\frac{1}{\alpha}}$$

With uncertain lifetime and no other sources of uncertainty, it follows that

$$\begin{aligned} U_t &= \left[c_t^{\phi} + \beta \left(1 - q_t\right)^{\frac{\phi}{\alpha}} U_{t+1}^{\phi} \right]^{\frac{1}{\phi}} = \\ &= \left(\sum_{j=0}^{T} \beta^j l_{t+j|t}^{\frac{\phi}{\alpha}} c_{t+j}^{\phi} \right)^{\frac{1}{\phi}} \end{aligned}$$

where q_t is the probability of dying between t and t+1, and $l_{t+j|t} = \prod_{i=0}^{j-1} \left(1 - q_{t+i}\right)$ that of living from t to $t + j$. It is necessary to restrict the parameters α and ϕ to strictly positive values: otherwise, since there is a risk of death in every period $\alpha < 0 \Rightarrow \mu \left[\tilde{U}_{t+1} | I_t \right] = 0 \; \forall t$, and analogously, since death before time T is certain $\phi < 0 \Rightarrow U_t = 0 \; \forall t$[9].

The problem

$$\max_{\{c_t\}} U = \left(\sum_{t=0}^{T} \beta^t l_t^{\frac{\phi}{\alpha}} c_t^{\phi} \right)^{\frac{1}{\phi}}$$

subject to the usual constraint in the absence of annuities and with initial wealth x

$$\sum_{t=0}^{T} c_t B^t = x$$

has f.o.c.

$$c_t = \left[\left(\frac{\beta}{B} \right)^t l_t^{\frac{\phi}{\alpha}} \right]^{\frac{1}{1-\phi}} c_0 \; \forall t$$

equivalent to the Euler equation

$$c_{t+1} = \left[\left(\frac{\beta}{B} \right) \left(1 - q_t\right)^{\frac{\phi}{\alpha}} \right]^{\frac{1}{1-\phi}} c_t \quad \forall t$$

both of which naturally simplify to the conventional ones for $\alpha = \phi$.

Access to an actuarially fair annuity market leads to the budget constraint

$$\sum_{t=0}^{T} c_t B^t l_t = x$$

[9] The restriction is rather unfortunate from an empirical point of view, as most econometric studies, including Epstein and Zin (1991), estimate negative values for α and ϕ.

and thus to the f.o.c

$$c_t = \left[\left(\frac{\beta}{B} \right)^t l_t^{\frac{\phi}{\alpha}-1} \right]^{\frac{1}{1-\phi}} c_0 \ \forall t$$

i.e. to the Euler equation

$$c_{t+1} = \left[\left(\frac{\beta}{B} \right) (1-q_t)^{\frac{\phi}{\alpha}-1} \right]^{\frac{1}{1-\phi}} c_t \quad \forall t$$

This immediately shows that the well-known Yaari (1965) result that the Euler equation is identical without uncertainty and in the presence of actuarially fair annuities is only valid in the special case where $\alpha = \phi$. Otherwise, relatively risk-averse agents (those with $\alpha < \phi$) have decreasing consumption profiles even when $\beta = B$ and all their wealth is converted into actuarially fair annuities.

Moreover, returning to the role of risk aversion, the benefit of annuitization may be computed in the standard form of the equivalent wealth $\hat{x}$ such that

$$\left[\sum_{t=0}^{T} B^t \left(\frac{\beta}{B} \right)^{\frac{t}{1-\phi}} l_t^{\frac{\phi}{\alpha(1-\phi)}} \right]^{\frac{1-\phi}{\phi}} \hat{x} = \left[\sum_{t=0}^{T} B^t \left(\frac{\beta}{B} \right)^{\frac{t}{1-\phi}} l_t^{\frac{(1-\alpha)\phi}{\alpha(1-\phi)}} \right]^{\frac{1-\phi}{\phi}} x$$

$$\frac{\hat{x}}{x} = \left[\frac{\sum_{t=0}^{T} B^t \left(\frac{\beta}{B} \right)^{\frac{t}{1-\phi}} l_t^{\frac{(1-\alpha)\phi}{\alpha(1-\phi)}}}{\sum_{t=0}^{T} B^t \left(\frac{\beta}{B} \right)^{\frac{t}{1-\phi}} l_t^{\frac{\phi}{\alpha(1-\phi)}}} \right]^{\frac{1-\phi}{\phi}}$$

The proof that that $\frac{\partial \frac{\hat{x}}{x}}{\partial \alpha} > 0$ is provided in the Appendix: it follows that the utility gain resulting from the opening of an actuarially fair annuity market is indeed a negative function of risk aversion.

4 Conclusions and directions for future research

This paper has analyzed the role of risk aversion in determining the welfare benefit of complete annuitization for an egoistic life-cycle agent. Provided the optimal consumption path is never increasing in time, this benefit is a negative function of the elasticity of intertemporal substitution, which in conventional expected-utility models is bound to be the reciprocal of the relative risk-aversion index. Separating the two parameters by means of an Epstein-Zin utility function, it can be demonstrated that the latter has a negative effect on the utility of annuitization; furthermore, outside the expected-utility framework, an uncertain lifetime has an influence on consumption dynamics which cannot be elminated through access to fair insurance markets. An intuitive explanation for both results is that, whereas a conventional insurance contract can offer full compensation for a loss, thereby making the insured party indifferent to the eventual resolution of uncertainty, an annuity can never remove the risk of dying, but merely offers the valuable opportunity to transfer resources across states of nature: since these remain antithetic, the effect of uncertainty is not removed.

Further research may concentrate on the relevance of these findings for an explanation of the limited size of annuity markets around the world. In particular, it would be interesting to examine whether selfish but risk-averse consumers facing many imperfectly insurable sources of uncertainty, chiefly relating to health and disability status, should rationally abstain from converting their entire wealth into life-annuities.

A Appendix

Given

$$\frac{\hat{x}}{x} = \left[\frac{\sum_{t=0}^{T} B^t \left(\frac{\beta}{B}\right)^{\frac{t}{1-\phi}} l_t^{\frac{(1-\alpha)\phi}{\alpha(1-\phi)}}}{\sum_{t=0}^{T} B^t \left(\frac{\beta}{B}\right)^{\frac{t}{1-\phi}} l_t^{\frac{\phi}{\alpha(1-\phi)}}} \right]^{\frac{1-\phi}{\phi}}$$

the logarithmic derivative with respect to α is

$$\frac{\partial \ln \frac{\hat{x}}{x}}{\partial \alpha} = \frac{1}{\alpha^2} \left[\frac{\sum_{t=0}^{T} B^t \left(\frac{\beta}{B}\right)^{\frac{t}{1-\phi}} l_t^{\frac{\phi}{\alpha(1-\phi)}} \ln l_t}{\sum_{t=0}^{T} B^t \left(\frac{\beta}{B}\right)^{\frac{t}{1-\phi}} l_t^{\frac{\phi}{\alpha(1-\phi)}}} - \frac{\sum_{t=0}^{T} B^t \left(\frac{\beta}{B}\right)^{\frac{t}{1-\phi}} l_t^{\frac{(1-\alpha)\phi}{\alpha(1-\phi)}} \ln l_t}{\sum_{t=0}^{T} B^t \left(\frac{\beta}{B}\right)^{\frac{t}{1-\phi}} l_t^{\frac{(1-\alpha)\phi}{\alpha(1-\phi)}}} \right]$$

which has the sign of

$$\sum_{t=0}^{T} B^t \left(\frac{\beta}{B}\right)^{\frac{t}{1-\phi}} l_t^{\frac{\phi}{\alpha(1-\phi)}} \ln l_t \sum_{t=0}^{T} B^t \left(\frac{\beta}{B}\right)^{\frac{t}{1-\phi}} l_t^{\frac{(1-\alpha)\phi}{\alpha(1-\phi)}} +$$
$$- \sum_{t=0}^{T} B^t \left(\frac{\beta}{B}\right)^{\frac{t}{1-\phi}} l_t^{\frac{(1-\alpha)\phi}{\alpha(1-\phi)}} \ln l_t \sum_{t=0}^{T} B^t \left(\frac{\beta}{B}\right)^{\frac{t}{1-\phi}} l_t^{\frac{\phi}{\alpha(1-\phi)}}$$

This can be simplified by substituting $\gamma_t = B^t \left(\frac{\beta}{B}\right)^{\frac{t}{1-\phi}} l_t^{\frac{\phi}{\alpha(1-\phi)}} > 0$ and $\delta_t = B^t \left(\frac{\beta}{B}\right)^{\frac{t}{1-\phi}} l_t^{\frac{(1-\alpha)\phi}{\alpha(1-\phi)}} > \gamma_t \ \forall t > 0$, bearing in mind that $0 < \alpha, \phi < 1$:

$$\sum_{t=0}^{T} \delta_t \ln l_t^{-1} \sum_{t=0}^{T} \gamma_t - \sum_{t=0}^{T} \gamma_t \ln l_t^{-1} \sum_{t=0}^{T} \delta_t > \sum_{t=0}^{T} (\delta_t - \gamma_t) \ln l_t^{-1} \sum_{t=0}^{T} \gamma_t > 0$$

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