Learning, Ambiguity and Life-Cycle Portfolio Allocation*

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Preliminary and Incomplete — Comments Welcome

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Abstract

The empirical evidence shows two notable features of household financial portfolios: the allocation to stocks for equity market participants is approximately constant in age and increasing in wealth. The standard life-cycle model with uninsurable idiosyncratic earnings instead predicts conditional stock shares that are declining in wealth and age, the latter especially early in life. In the present paper I develop a life-cycle portfolio choice model where agents perceive stock returns to be ambiguous and are ambiguity averse. As in Epstein and Schneider (2005) part of the ambiguity vanishes over time as a consequence of learning over observed returns. When calibrated to match average participation rates and conditional portfolio shares the model is able to generate patterns of conditional stock allocations across age groups that are roughly constant as in the data. It also shows promise in reproducing the increasing pattern of conditional shares with wealth.

Keywords: Portfolio choice, life-cycle, ambiguity, learning

JEL codes: G11, D91, H55

1 Introduction

Survey studies from different developed countries show two important features of household allocation to stocks. First the fraction of financial assets invested in stocks for market participants is constant or mildly increasing in wealth; second the conditional stock share is also largely insensitive to age. [1].

The basic life-cycle model with power utility and non-tradeable labor income predicts that the portfolio share should decline with both financial wealth and age. The intuition, well explained for example in Bodie, Merton and Samuelson (1992) is that the present value of future labor income acts as a non-tradeable risk-free bond in the agent's portfolio. At any age, that is, fixing residual human capital, optimal portfolio allocation would require that the household initially allocates all its financial wealth to the stock and then diversify towards bonds only as financial wealth increases. Similarly at any given financial wealth level, as the agent ages, its human capital ends up making a lower portion of his total wealth hence the desire to reduce the exposition to risky stocks. Once this general portfolio rules are interacted with life-cycle optimal savings the resulting pattern of conditional stock shares are decreasing in both wealth and age, with the decline in age especially pronounced early in life. [2]

The goal of the present paper is to provide a unified theory to address both asset allocation issues. The key assumption that characterize the present paper is that stock market returns are assumed to be ambiguous and that agents are

[1] The empirical evidence is summarized in Curcuru et al. (2004), the book edited by Guiso et al. (2001) and in Ameriks and Zeldes (2004)

[2] Notable papers that have simulated life-cycle portfolio choice models with realistic life-cycle profiles and risk of earnings are Campbell et al. (1999), Cocco, Gomes and Michaelides (2005) and Gomes and Michaelides (2005).

averse to ambiguity. The extent of ambiguity can be reduced over time by observing the sequence of realized returns, however ambiguity never vanishes altogether. This framework, developed theoretically in Epstein and Schneider (2005), offers a natural explanation to the two facts mentioned above. In a first version of the model, I assume that information about stock market returns is free so that all households get it as they age. In this way the ambiguity faced by older agents is less than the one faced by younger agents. This force counteracts the desire to diversify towards bonds as the fraction of total wealth represented by safer human capital declines, thus increasing the allocation to stocks of older agents compared to younger agents and flattening the life-cycle profile. In a second version of the model I assume that a fixed cost needs to be paid to observe the return process. Under this assumption agents who receive better earnings shocks will be more willing to pay the cost and become more confident about the stock market. Endogenously wealthier agents ends up facing less ambiguity hance they will allocate larger share of their portfolio to the stock.

In the quantitative exercise I use average participation rates and conditional shares to calibrate the parameters governing the extent of ambiguity and the speed at which this is reduced through learning. I then show that under that parametrization the model is able to reproduce the observed flat profile of stock shares with both age and wealth. [3]

The present study joins an active line of research that tries to explain the observed pattern of household choice concerning the allocation of their financial resources under risky labor income. This line of research was originally

[3]Indeed so far only results concerning the relationship between age and stock shares are available as those regarding the wealth-share relationship are still under construction.

pursued by Heaton and Lucas (1997 and 2000) and Haliassos and Michaelides (2003) in an infinite horizon framework and by Campbell et al. (1999), Cocco, Gomes and Meanhout (2005) and Gomes and Michaelides (2005) in life-cycle models. The common finding of these papers is that the share of stocks conditional on participation is declining in financial resources except for implausibly large and positive correlations between stock market returns and labor earnings. The models cast in a finite horizon framework also find that the share of stock for participants declines with age during working life and then increases afterwards. These counterfactual implications of the basic model have stimulated the search for alternative specifications. Benzoni, Collin-Dufresne And Goldstein (2007) and Lynch and Tan (2006) focused their attention on alternative specification of the labor process in particular with reference to their correlation with the stock market returns that allow to reduce the stock market exposition of younger agents. Wachter and Yogo (2007) abandoned the assumption of homothetic utility by introducing luxury goods in the model. They show that their model can generate stock portfolio shares that are increasing in wealth beside explaining a number of features about consumption.

The present model adds to that literature by providing a unified theory for both the age and wealth dependence of households' allocation to stocks conditional on participation. It does so by retaining the homotheticity assumption and by relaxing the required ability to process information and understand the distribution of stock returns and its correlation with labor earnings risk, rather than increasing it as in Benzoni et al. (2007) or Lynch and Tan (2006).

The rest of the paper is organized as follows. In Section 2 I describe the model, in Section 3 the choice of parameters, in Section 4 the results and finally in Section 5 I briefly make some provisional conclusions and point to directions currently under development in the present paper.

2 The Model

2.1 Demography and Preferences

Time is discrete and the model period is assumed to be 1 year. Adult age is denoted with the letter t and can range from 1 to $T = 81$ years. Agents are assumed to enter the model at age 20 so that real life age is equal to $t + 20$. Each agent faces an age changing conditional probability of surviving to the next period which will be denoted with π_t. Surviving agents work the first $S = 45$ years and retire afterwards.

Agents do not value leisure, hence they derive utility from the stream of consumption they enjoy during their life-time only. Utility over consumption is defined by a period utility index $u(c_t)$ which will be assumed to be of the standard iso-elastic form.

In the economy there are two independent sources of uncertainty. The first one is determined by the stochastic process for labor earnings and it is standard in that I assume that agents know its distribution. This process will be described in a later subsection. The second one is the process for stock returns. Following Epstein and Schneider (2005) it is assumed that this process is i.i.d. and that agents perceive it as ambiguous. In other words they assume that stock returns are draws from a family of distributions and even if they can learn from past observations of realized returns, they can never shrink the set of distributions to a singleton.

In every period an element $x_t \in X$ is observed: this pair consists of a realization of the stock return $s_t \in S$ and a realization of the labor efficiency unit shock $z_t \in Z$. At time t then the agent's information set consists of the history $x^t = (x_1, x_2, ..., x_t)$. Given that the horizon is finite the full state space will be X^T. The agent ranks consumption plans $c = \{c_t\}$ where consumption c_t depends on the history x^t. At any date $t = 1, 2, ..., T$ and given history x^t, the agent's ordering is represented by a conditional utility function U_t defined recursively by:

$$U_t(c; x^t) = \min_{p \in \mathcal{P}_t(s^t)} E^p[u(c_t) + \beta E^{z_{t+1}} U_{t+1}(c; x^{t+1})] \tag{1}$$

where β and u are defined above. The set of probability measures $\mathcal{P}_t(s^t)$ models beliefs about the next observation s_{t+1} given history up to s_t. When this set is a non-singleton such beliefs reflect ambiguity and the minimization over p reflects ambiguity aversion.[4] The set of probability measures $\{\mathcal{P}\}$ is called *process of conditional one-step ahead beliefs* and together with $u(\cdot)$ and β constitute the primitives of the functional form.

2.2 Learning

The investor knows the distribution of future labor earnings, however he perceives stock returns as ambiguous. In particular he thinks that they are generated by the same memoryless mechanism in each period and that even if there are features of the data generating process that can be learned others are not.[5] Mathematically learning is represented by a tuple $(\Theta, \mathcal{M}_0, \mathcal{L}, \alpha)$ where Θ is a

[4] The minimization is taken with respect to p only since the process for labor earnings is independent and is not ambiguous.

[5] In this subsection I present a minimal exposition of the subject which is entirely based on Epstein and Schneider, (2005) to which the reader is referred.

parameter space whose elements θ represent features of the data generating process that the agents think are learnable. The set $\mathcal{M}_0$ is the set of priors on Θ and $\mathcal{L}$ is a set of likelihood functions whose multiplicity reflects the existence of poorly understood factors driving the returns. Finally α is a parameter that governs the reevaluation process through which posteriors are constructed based on the past observed returns. The set of posteriors is constructed based on a likelihood ratio test and will be defined as:

$$\mathcal{M}_t^{\alpha}(s^t) = \{\mu_t(s^t; \mu_0 \in \mathcal{M}_0, \ell^t \in \mathcal{L}^t), \int \prod_{j=1}^{t} \ell_j(s_j|\theta) d\mu_0(\theta) \geq \alpha \max_{\tilde{\mu}_0 \in \mathcal{M}_0, \tilde{\ell}^t \in \mathcal{L}^t} \int \prod_{j=1}^{t} \tilde{\ell}_j(s_j|\theta) d\tilde{\mu}_0(\theta)\}. \quad (2)$$

In this specific context Θ is assumed to be a one-dimensional set with elements $\theta \in [\bar{\lambda}, 1-\bar{\lambda}]$ where $\bar{\lambda} < \frac{1}{2}$. The set of likelihoods is defined by $\ell(1|\theta) = \theta + \lambda$ for some $\lambda \in [-\bar{\lambda}, \bar{\lambda}]$ and $\ell(1|\theta)$ is the probability of observing a high stock return given the value of θ. The set of priors $\mathcal{M}_0$ is given by all the Dirac measures on Θ. Finally α is a constant that determines how the set of posteriors responds to new information: were it equal to zero the set of posteriors $\mathcal{M}_t$ would be equal to $\mathcal{M}_0$ for all t and no updating would occur. A value of $\bar{\lambda} > 0$ is needed for returns to be ambiguous signals.

It can be proved that under the simple specification used here the set of posteriors depends on the sample only through the fraction of high stock returns ϕ_t observed before t. More specifically it will obey the following law:

$$\mathcal{M}_t^{\alpha}(s^t) = \left\{\theta \in \Theta : g(\theta; \phi_t) \geq \max_{\tilde{\theta} \in \Theta} g(\tilde{\theta}; \phi_t) + \frac{log(\alpha)}{t}\right\} \quad (3)$$

where $g(\theta; \phi_t) = \phi_t \log(\theta + \bar{\lambda}) + (1 - \phi_t) \log(1 - \theta + \bar{\lambda})$.

2.3 Labor Income and Pensions

I use the indexed letter Y_t to denote income. During working life income is determined by an uncertain stream of labor earnings. Earnings can be expressed as the product of two components:

$$Y_t = G(t)z_t \tag{4}$$

where the function $G(t)$ is a deterministic function of age meant to capture the hump in life-cycle earnings that is observed in the data. The second term, z_t, is a stochastic component that follows an AR(1) process in logarithms:

$$ln(z_t) = \rho ln(z_{t-1}) + \varepsilon_t \tag{5}$$

where ε_t is an i.i.d. normal random variable.

In the retirement years agents receive a fixed pension benefit, so that

$$Y_t = Y_{ss}. \tag{6}$$

The social security payment is independent of past earnings history so that it is equal for all agents.

2.4 Financial Assets

Agents can use two different assets to carry out their investment plans. First there is a one period risk free bond with price q and return $R^f = \frac{1}{q}$. The second asset is a risky stock whose return R^s_{t+1} follows an i.i.d. process and that can take two values: $\mu \pm \delta$ with equal probability. Consequently $\mu - \frac{1}{q}$ is the expected equity premium and δ is the standard deviation of the equity return. This simple formulation of equity returns is somewhat non-standard since in the related quantitative literature the usual assumption is of normal stock returns.

It is adopted in this context because it is convenient to the model of learning about stock returns used here.

Trade in the two assets is subject to two frictions. First households are prevented both from borrowing and from selling short stock. Denoting bond and stock-holdings with B_t and S_t respectively this implies:

$$B_t \geq 0 \tag{7}$$

$$S_t \geq 0. \tag{8}$$

Second it is assumed that participation in the stock market requires payment of a fixed cost F_p in each period.

2.5 The Optimization Problem

With the description of the model given above it is now possible to state the household's optimization problem in dynamic programming form. [6]

The agent maximizes:

$$V_t(X_t, z_t, \phi_t) = \max_{c_t, B_{t+1}, S_{t+1}} \min_{p_t \in \mathcal{P}_t(s^t)} \Big\{ u(c_t) + \beta \pi_{t+1} E V_{t+1}(X_{t+1}, z_{t+1}, \phi_{t+1}) \Big\}$$

$$= \max_{c_t, B_{t+1}, S_{t+1}} \min_{\lambda_t \in [-\bar{\lambda}, \bar{\lambda}], \theta \in \mathcal{M}_t^{\alpha}} \Big\{ u(c_t) + \beta \pi_{t+1} [(\theta + \lambda_t) V_{t+1}(X_{t+1}(1), z_{t+1}, \phi_{t+1}(1)) + (1 - \theta - \lambda_t) V_{t+1}(X_{t+1}(0), z_{t+1}, \phi_{t+1}(0))] \Big\} \tag{9}$$

subject to the budget constraint

$$c_t + qB_{t+1} + S_{t+1} \leq X_t + Y_t - I_p F_p \tag{10}$$

the transition equation for financial resources

$$X_{t+1} = B_{t+1} + R(s_{t+1}) S_{t+1} \tag{11}$$

[6] The model described below refers to the case where update of beliefs is free and occurs in each period of life for which results are already available.

the transition equation for the fraction of time a high return was observed

$$\phi_{t+1}(s_{t+1}) = \frac{t\phi_t + s_{t+1}}{t+1} \tag{12}$$

, the non negativity constraints (7) and (8) and equations (4), (5) and (6) that define the nonfinancial income available to the agents from labor earnings or pensions. In the above equation the variable I_p is an index that takes a value of one if the agent decides to participate in the stock market — that is, if $S_t > 0$ — and a value of zero if he decides to set $S_t = 0$, that is, if he does not participate. Finally the arguments 1 and 0 that appear on both X_{t+1} and ϕ_{t+1} stand for high and low stock return respectively.

3 Parameter Calibration

3.1 Preferences Parameters

Preferences are defined by the functional form of the period utility index and by the subjective discount factor. The utility index is chosen to be of the standard iso-elastic form: $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ and a value of 6 is chosen for σ, the coefficient of relative risk aversion. This value falls in the middle of the range of values used in the life-cycle portfolio choice literature. The subjective discount factor β is set equal to 0.96 a value commonly used in the macro and finance literature. The effective discount rate is determined also by the conditional survival probabilities which are taken from male survival probabilities available at the "Berkeley Mortality Database ".

3.2 Learning Parameters

The process for learning is defined by two parameters. The first one, denoted above with $\bar{\lambda}$, determines the extent of the component of ambiguity that does not vanish even in the long run. I fix the value of this parameter to 0.01. Following Epstein and Schneider (2005) this value implies that in the long run, that is, after a large set of data on the stock return has been observed, the set of posteriors of the probability of high stock returns shrinks to $[0.49, 0.51]$ which implies a range of equity premia of 64 basis points. This number seems sufficiently small to leave substantial scope for learning in the model. The second parameter is α which determines the speed at which learning occurs. Since there is no reference on how to fix this parameter we choose a baseline value of 0.4 and perform some sensitivity analysis. The value itself though is chosen so that once the other parameters are fixed the average stock market participation rate is reasonably close to its empirical counterpart.

3.3 Labor Income and Pensions

The specification of the labor earnings process during working life requires fixing two sets of parameters. The first one refers to the function $G(t)$ which defines the deterministic hump-shaped component of earnings. This function is assumed to be a 3-rd degree polynomial in age and the coefficients are taken form the estimates by Cocco, Gomes and Maenhout (2005) for high school graduates. These estimates when aggregated over five year groups are also consistent with the ones of Hansen (1993) based on the whole population. The second one is the idiosyncratic component z_t which is assumed to follow an $AR(1)$ process with autocorrelation coefficient $\rho = 0.95$ and a standard deviation of the innovation

$\sigma_\varepsilon = 0.158$, both values taken from Hubbard; Skinner and Zeldes (1994).

During retirement it is assumed that agents receive a fixed pension benefit equal to 40 percent of average working life income, a number somewhat below but not far from the figure for the US social security system.

3.4 Asset Returns and Fixed costs

The bond price is set at 0.99 which implies a risk free return of 1 percent annually corresponding to the value used by Mehra and Prescott (1985). The stock return process is modeled as a two point i.i.d. process with the expected value μ set at 6 percent annually and a standard deviation δ of 16 percent. These values imply an equity premium of 5 percent, somewhat lower than the historical equity premium targeted in the asset pricing literature but in line with the one used in the literature on household portfolio choice and indeed somewhat larger than that.

Empirical work that tried to measure the magnitude of fixed stock market participation costs found values in the range of 50 to 200 dollars.[7] The cost in the model is then set so that when compared to model wages it is consistent with values in the middle of the range reported above.

4 Results

In this section I report the results of the simulation of the model. The section is organized in four parts. In the first one I describe in details the conditions under which the model simulation is performed and the evolution of ambiguity over the life-cycle that results from them. In the second one I report the results

[7] See Paiella (2001) and Vissing-Jørgensen (2002)

from a baseline simulation. In the third one I perform a sensitivity analysis on the values of α, that is, the parameter that regulates the speed of learning and on the risk aversion coefficient. Finally in the last subsection I report the relationship between wealth and both participation rates and conditional stock shares in the model.

4.1 The Experiment

The results reported in the next two subsections are obtained under the following two conditions. First we have to fix the number of observations prior to entry in the workforce upon which the first posterior is formed. The number chosen is 10 as this seems to plausibly reflect the number of years a newly entered workers has been exposed to some information about the stock market through family members, teachers or friends before entering the labor force. Second we simulate the model under a sequence of stock market returns in which each high return is followed by a low return. This choice is made to highlight the impact of the evolution of ambiguity on the life-cycle portfolio allocation without blurring it with the idiosyncracies of actual random realizations that might indeed depart substantially from the historical experience.[8] Figure 1 reports the past fraction of high returns under the sequence of stock realizations used in the following experiments and the expected stock return that minimizes the investor utility with respect to the distributions in the posterior set. In addition, for comparison purposes the dashed line reports the risk-free return. Given the chosen draw of stock returns the fraction of past high returns shows a saw-tooth path that never departs substantially from 0.5. In response to this pattern the set of posteriors

[8]Notice though that the agents dynamic programming problem is solved under the assumption that the stock return process is random.

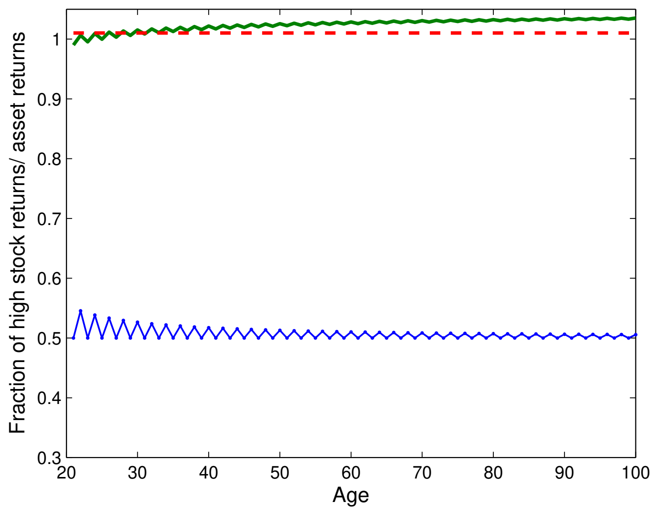

Figure 1: Simulated returns

shrinks so that the expected stock return using the minimizing probabilities smoothly increases towards its long run value. Consequently the expected equity premium effectively relevant for agents' decisions moves from slightly below the risk free rate in the first decade of life to a few percentage points late in life.

4.2 Life-Cycle Profiles: A Benchmark Case

In this section I report results for the benchmark case. The benchmark case is defined by a value of the coefficient of relative risk aversion of 6 and a value of the parameter α that determines the speed of learning of 0.4. In Figure 2 I report the average life-cycle profiles of income, consumption and wealth. These are quite standard: income is given by the exogenous, hump-shaped earnings process during working life and then is constant and equal to the pension benefit after retirement. Consumption is also hump-shaped: it is equal to earnings

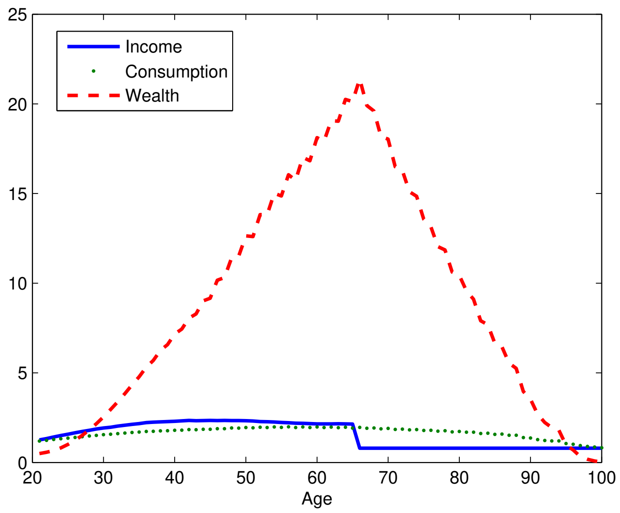

Figure 2: Life-cycle profiles of income, consumption and wealth.

early in life when agents face the steepest part of their income profile and are at the borrowing constraint and then it becomes lower than earnings as households start to save for precautionary reasons and for retirement. Mirroring this pattern, wealth starts close to zero and then picks up quickly until the retirement date after which it decreases as agents deplete their savings to supplement income to finance consumption.

Next I move to the description of participation rates and conditional stock shares. The average participation in the simulation is 52 percent and the average conditional share is 44 percent two figures that are in line with the empirical evidence. The average life-cycle profiles of participation rates and the share of stocks for market participants are reported in Figures 3 and 4 respectively. It can be seen from Figure 3 that the model generates a hump-shaped profile of participation rates, qualitatively consistent with what is often found in the

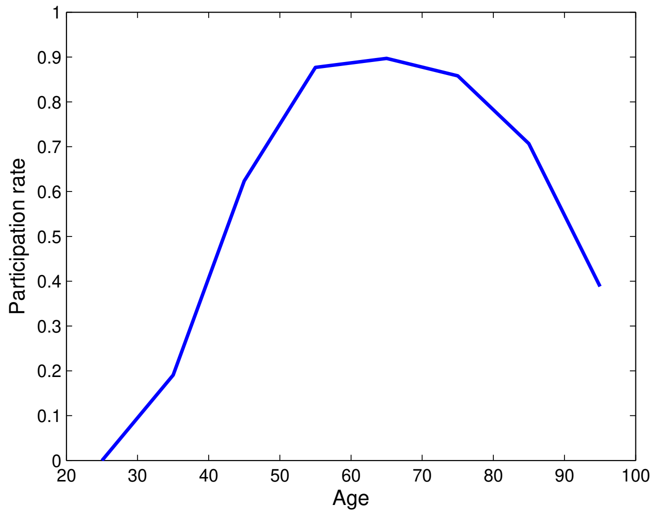

Figure 3: Life-cycle profile of participation rates.

data.[9] The hump that is observe in the model is too pronounced though, especially early in life. The reason can be understood by looking once more at Figure 1. What can be seen there is that in the first decade of working life the extent of ambiguity the agents perceive to be in the data is still large enough that the minimizing expected stock return is below the return on the risk free security, hence agents won't participate in the stock market. To a lesser degree the problem persists in the second decade of working life. In this case the minimizing expected stock return is large enough to generate a positive premium, however this is small so that only very wealthy individuals will find it optimal to pay the fixed cost and participation will still be only at 20 percent. Finally

[9]The hump-shaped profile of stock market participation rates emerges in plot of cross-sectional data from different years and industrialized countries. To be precise though the profile that is found indeed depends on the assumption made in the estimation. See Ameriks and Zeldes (2004) for the US on this point.

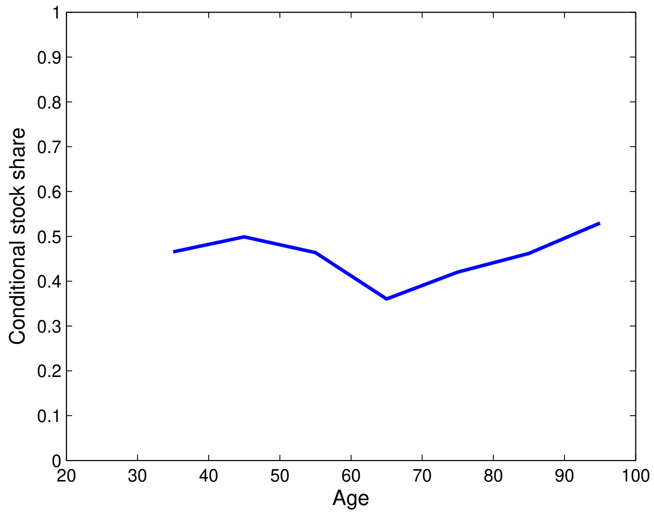

Figure 4: Life cycle profiles of conditional shares.

the large drop in participation rates late in life — which is new compared to the existing literature — is partly determined directly by agents completely running out of wealth and partly by the interaction of low late life wealth and the fixed per period participation cost.

Figure 4 reports the conditional stock share profile which is the main object of this study. As we can see this is fairly constant over the life-cycle: it starts at slightly less than 50 percent, then after a minor surge in the next decade of life it decreases by a few percentage points in mid-life and then returns to about the initial value of 50 percent towards the end of life. To support the intuition for the result obtained, Figure 5 reports results from two other experiments. In that figure the continuous line represents the average stock share in the standard case with no ambiguity. The dashed and the dotted lines report the stock share for market participants in the case when agents do perceive stock returns as ambiguous but feel that there is no part of this ambiguity that can be resolved through learning; the difference between the two lines is in the extent of ambiguity which is larger in the case of the dotted one. The continuous line shows the well known result that in the basic model agents would enter the stock market very early in life and invest in it 100 percent of their financial wealth. As they age and their financial wealth increases relative to their non-tradable and safer human wealth they would reduce their exposition to stock market risk but still keeping most of their assets in stocks. This gives rise to a counter-factually high and declining profile of equity investment for holders of the asset. The dashed line shows that with ambiguity averse agents a certain amount of ambiguity can reduce the average conditional stock share to the empirical one, more precisely the value in the experiment is 42 percent. The dotted line shows that by further increasing the extent of ambiguity it is also possible to match the empirical average share early in life. The reason is that if agents are ambiguity averse the relevant return probabilities differ from the true ones. The utility minimizing probability of high stock return is lower implying a lower expected equity premium: 1.15 percent in the experiment underlying the dashed line and 0.67 percent in the experiment underlying the dotted line. However ambiguity alone cannot improve the overall performance of the model. This is because with a constant amount of ambiguity perceived in the data, the minimizing expected equity premium is constant so that as agents age they would still want to reduce the share of equity in their financial portfolio to compensate the decline in relatively safe non-tradable income. Indeed the presence of ambiguity worsens this problem making conditional stock shares even more steeply declining over the life-cycle. The key element then is learning. When agents use the sequence

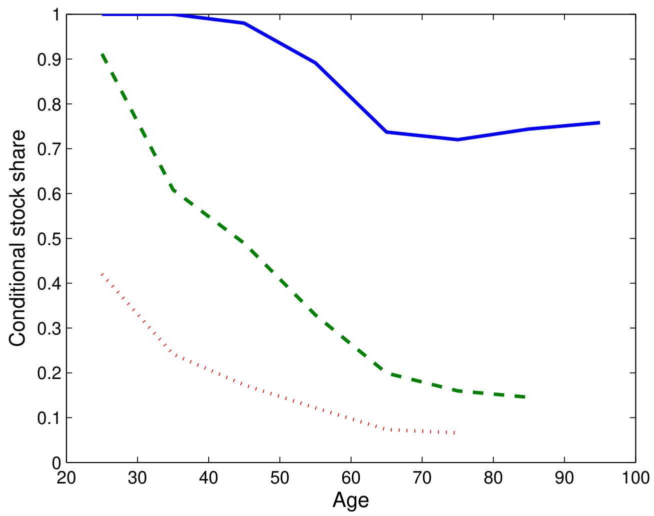

Figure 5: Life cycle profiles of conditional shares without learning.

of realized returns to learn away part of the ambiguity that is present in the data then the probability distribution of stock returns that minimizes their utility improves increasing the decision relevant expected equity premium. This creates a force that balances the desire to reduce the exposition of financial wealth to stock market risk as relatively safe human wealth declines. As Figure 4 shows at least for the proposed choice of parameters the two forces balance out allowing for a flat profile of stock shares for market participants over the whole life-cycle. In the next section I'll check if small perturbations in this choice of parameters preserve this result.

4.3 Life-Cycle Profiles: Sensitivity Analysis

In this section I report the life-cycle profiles of equity shares for stock market participants for some alternative choices of parameters. This is done in Figure

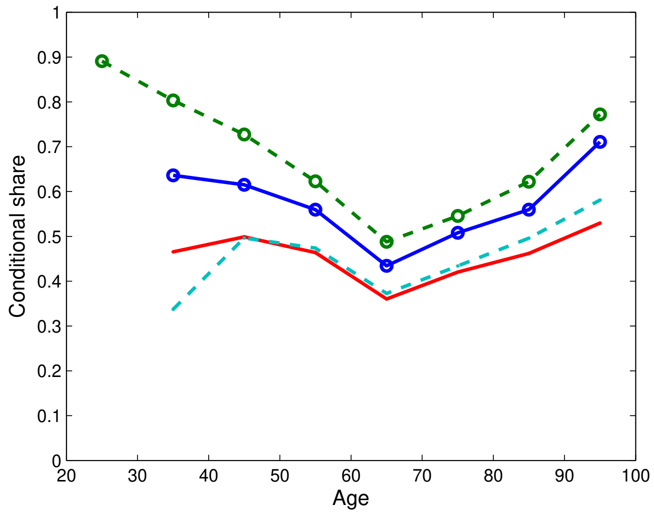

Figure 6: Life cycle profiles of conditional shares.

6 where the dashed lines correspond to simulations where α, the parameter governing the speed of learning, is set to 0.5 while the continuous lines correspond to simulations where α is set to 0.4. Lines with dot markers represent results with risk aversion of 6 and lines with hollow circle markers represent results with risk aversion of 5. As it can be seen from the Figure the ability of the model to reproduce a flat life-cycle profiles is better with a risk aversion of 6. A larger risk aversion coefficient increases financial wealth holdings both at young and old ages because of precautionary reasons. This increases the ratio of financial to relatively safe human wealth, thus reducing conditional stock shares at the extremes of the life-cycle and flattening the profiles. When α is equal to 0.5 indeed there is even an increasing region in the profile between age groups 30 to 40 and 40 to 50. When risk aversion is set to 5 instead we still observe that the life-cycle profile of conditional stock shares is declining in the first part of life,

especially for the case of α equal to 0.5 — the one represented by the top line — when learning is faster hence the minimizing stock return distribution implies a larger expected equity premium starting from earlier in life. It has to be said though that under this parametrization both the average participation rate and the average conditional stock share are quite larger than in the data, especially the former which is 65 percent in the model while in the data it reaches at most 52 percent for the most recent editions of the SCF.

4.4 The Wealth-Conditional Stock Share Relationship

In Table 1 I report participation rates and equity shares for market participants by wealth levels. I partition the wealth distribution into 5 groups defined by the following thresholds: 3, 6, 10, and 16. To give an idea of the magnitude of these wealth levels observe that the average earnings in the model is 2.09, average wealth is 12.3 and, as it can be seen from Figure 2 average wealth at retirement age is somewhat above 20. As it can be seen from the first line of each panel of the table the participation rate is monotonically increasing across all wealth groups: it starts at around 20 percent for the poorest group and reaches more than 95 percent in the top wealth group. This results occurs for two reasons. One is the presence of the fixed per period participation cost. This effect is reinforced by the fact that an important part of poor agents are among the youngest groups whose set of posteriors is still large enough for the minimizing expected equity premium to be low, hence for the expected benefit of investing in the stock market to be small. While this result is consistent with the empirical evidence, the same cannot be said for the profile of conditional shares. These are somewhat increasing in the data, while as the table shows

Table 1: Participation rates and conditional stock shares by wealth groups

	0-3	3-6	6-10	10-16	16+
Participation $\sigma = 6$, $\alpha = 0.4$	17.1	63.2	69.2	86.6	95.9
Conditional share	93.2	64.4	43.4	33.4	23.0
Participation $\sigma = 6$, $\alpha = 0.5$	25.5	82.0	93.2	94.7	99.3
Conditional share	96.7	74.8	52.8	40.6	28.1
Participation $\sigma = 5$, $\alpha = 0.4$	20.0	70.2	82.3	90.4	97.1
Conditional share	98.2	74.1	52.6	40.7	28.2
Participation $\sigma = 5$, $\alpha = 0.5$	30.5	89.3	93.9	97.8	100.0
Conditional share	99.6	85.2	64.8	49.7	34.6

they are decreasing for all choices of parameters considered in the model. The second lines of each panel in the table show stock shares for market participants that are above 90 percent in the bottom wealth group and decrease to about 20 percent in the top group.

Motivated by this shortcoming of the current version of the model, in the next section I modify the setup and consider a version of the model where payment of the fixed cost is needed not only to participate in the stock market but also to acquire information about the stock return process.

The Model with Costly Information Acquisition

UNDER CONSTRUCTION: RESULTS WILL BE AVAILABLE SOON

5 Conclusions and Ongoing Research

In the present paper I have constructed a life-cycle asset allocation model with uninsurable earnings uncertainty and with borrowing and short sales constraint. I assumed that stock returns are generated by an i.i.d. process but that agents perceive that process as ambiguous and that they are averse to ambiguity. It was also assumed that part of the ambiguity can be learned away through the observation of the sequence of realized returns. I showed that when agents perceived stock returns as ambiguous average stock shares for market participants are substantially reduced but that their life-cycle profile is strongly monotonically decreasing. However when learning is taken into account the life-cycle profile of conditional equity share becomes roughly constant for some parameter choice, thus aligning the model prediction with the data. While this represents an improvement over the basic model, it was also shown that the relationship between asset holdings and conditional stock shares is still declining, a prediction that is at odd with the data. Motivated by this failure I am currently exploring the implications of assuming that the fixed participation cost is needed not only to buy stocks but also to observe the stock return and update the beliefs about the process that determines it. Intuitively in this way agents who get better earnings realizations should start paying the information cost earlier, thus at the same time face a higher minimizing expected equity premium and be wealthier. This mechanism is still under testing and results will be available soon.

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