Opting Out of Social Security over the Life-Cycle

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Abstract

We augment a life-cycle model of consumption with given undiversifiable labor income risk, to include opting out of social security. Agents can move mandatory contributions on labor income to an individual account earning a risky rate of return. The cost for a dollar opted out is a dollar debt to the social security earning a safe and lower rate of return. This paper uses a calibrated partial-equilibrium model of optimal opting out over the life-cycle. We find that the typical pattern is actually a out/in strategy: young workers contribute to the private pillar and, as they age, switch to public social security. The length of periods out and in is responsive to behavioral and economic assumptions, but the pattern is maintained.

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1 Introduction

Giving workers the choice to put mandatory contributions out of labor income either in a private individual account or in a PAYG scheme is a widely adopted pension reform strategy -usually called opting out- to move towards funding. Countries such as Great Britain, Argentina, Colombia, Peru, Uruguay have reformed their pension schemes in a similar vein and, in almost every country where social security privatization took place, this kind of choice has been given to transition cohorts.

But what reaction should we expect from workers when the Government gives this kind of choice?

It proves useful to answer this question before examining the macroeconomic effect of an opting out reform strategy: microfoundations for opting out of social security are needed.

This paper examines a break-even opting out mechanism, in that a dollar opted out from the public pillar into an individual account -which earns a higher and risky rate of return- is run in a debt with a lower constant yield. At the time of retirement accumulated debt is rolled into a negative annuity which is subtracted from public pension benefits.

We can think that this latter rate of return is the yield on public debt, so that opting out do not worsen the present discounted value budget constraint of the social security system.

We consider three different public pillars: notional defined contribution, average wage and flat rate schemes. From the microeconomic point of view, this model is always a joint portfolio/consumption problem under uncertainty over the life-cycle, where portfolio choice regards mandatory contributions out of labor income to be invested either in an individual account or in the public pillar.

Our framework and methodology is very similar to the life-cycle portfolio theory (Cocco, Gomes and Maenhout (1998) and Campbell, Cocco, Gomes and Maenhout (1999)), but in the fact that our focus is on pension wealth rather than discretionary wealth allocation and therefore the horizon is different.

We consider a life-cycle model with exogenous labor supply, nontradable earnings subject to both transitory and permanent shocks, and mandatory contributions to pension system that are neither available for consumption nor tradable before retirement. Since these problems are far from being analytically solvable -when using CRRA utility function-, we adopt the simulation approach with parameters calibrated on the United States economy: first we numerically derive policy functions, then we keep record of optimal behavior for households receiving different draws of random variables.

As our results are dependent on the parameter set, we also consider different behavioral and economic calibrations for the model in order to understand sensitivity of benchmark results. In the first group we examine optimal patterns for both more risk averse and less patient households. Concerning the second group, we consider positive correlation between permanent income innovation and stock return, lower -and safer- yield on funded wealth and different earnings profiles. While different behavioral calibrations give hints on the importance of heterogeneity in determining optimal opting out strategies, changing economic parameters is useful to understand how much our results rely on U.S. market features or how heavily some of our simplifying assumptions affect findings.

This portfolio approach is novel in opting out literature, that has usually examined optimal individual behavior assuming a safe rate of return on individual accounts. To make the problem interesting it has been considered a money-loser opting out, in that the cost of a dollar opted out depends on the accrual mechanism of public pension benefits. Therefore the cost for a dollar opted out is the present value of pension benefits for a dollar contributed to the public pillar and the optimal strategy is to stay out as long as the cost for opting out is lower than one.

Disney, Palacios and Whitehouse (1999) consider actuarially unfair pension schemes -such as average wage or last wage models- where agents opt out or in to game the system. Since in such pension schemes the present value of public pension benefits for a dollar contributed is lower the younger the worker is, the optimal strategy is monothonic in age: young workers stay in and then shift to the public pillar at an age depending on the rate of return on funded wealth. Under typical calibrations this age is very close to retirement. On a similar vein is the opting out model by Gustman and Steinmeier (1998).

Samwick (1997) models a buy out problem in a three periods setting, where the trade off between the two pillars is higher safe return versus lower contribution rate, since every dollar bought out has to be matched by more than a dollar of funded retirement contributions. Because of borrowing constraints, he obtains an optimal strategy where the older the worker is, the higher is buying out.

Dutta, Kapur and Orzag (1999) assume uncertainty on funded wealth, but they look for the optimal portfolio only at the time of retirement and use a mean variance utility function, which has the undesirable feature of a constant degree of absolute risk aversion.

The rest of the paper is organized as follows. Section 2 examines all of the assumption we made, the structure of the problem and technicalities about its solution method. Section 3 lays out the calibration we adopt and the results we obtain, with a special stress on opting out policy function. Section 4 presents sensitivity analyses. Section 5 briefly concludes.

2 The Model

2.1 Assumptions

The following set of assumptions is maintained over the different parametrizations of the model and most of them are common within the life-cycle asset allocation literature: see among the others Cocco, Gomes and Maenhout (1998) or Campbell, Cocco, Gomes and Maenhout (1999).

Assumption 1: life-cycle structure The representative agent we consider is an adult who lives T periods: she works during the first $k-1$ periods, then she retires and dies in $T+1$. The length of the work period and of the entire life is certain and exogenous. Assumption 1 is strong since it can potentially overshadow the insurance properties of endogenous labor offer, thus entailing an overstatement in both precautionary saving and riskless share in portfolio. Moreover, this hypotesis eliminates the need for the agents to hedge life-span risk, thus allowing us to disregard the modelling of the annuity market[1].

Assumption 2: labor income Agent's age t labor income is exogenously given by three multiplicative components: a deterministic function of age $f(t)$ that captures the hump-shape of earnings, an idiosyncratic temporary shock $\tilde{\varepsilon}_t$ describing bonuses or temporary unemployment, and a permanent random variable $\tilde{z}_t$ that represents career shocks. During retirement labor income is set to zero. This characterization of labor income process is consistent with the microeconomic evidence on individual shocks (Carroll (1992)).

More precisely we have:

$$
\begin{aligned}
\tilde{y}_t &= f(t)\tilde{z}_t\tilde{\varepsilon}_t \qquad \text{if } t < k; \\
y_t &= p \qquad\qquad\quad \text{if } t \geq k;
\end{aligned}
\tag{1}
$$

where $\log \tilde{z}_t$ follows an $Ar(1)$ process with unit root and normal innovation

$$\tilde{z}_t = z_{t-1}\tilde{u}_t; \tag{2}$$

$$\tilde{u}_t \sim LN\left(0; \sigma_u^2\right);$$

the temporary shock is lognormally distributed

$$\tilde{\varepsilon}_t \sim LN\left(0; \sigma_\varepsilon^2\right);$$

and the two random variables are stochastically independent

$$\sigma_{u,\varepsilon} = 0.$$

As usual in life-cycle theory, labor earnings are nontradable: that is the investor cannot write claims against her future human capital. Nor borrowing is allowed against retirement wealth. Thus discretionary wealth is constrained to be non-negative:

$$w_t \geq 0, \; w_1 = 0. \tag{3}$$

Assumption 3: investment opportunity set We consider a single tradable financial asset with uncertain gross real return normally distributed with constant mean and variance:

$$\tilde{R} \sim N(R; \sigma_R^2).$$

In some simulations we also allow for covariance σ_{Ru} between R and the innovation in the permanent labor shock u. Since we consider transitory shocks ε completely idiosyncratic -i.e. independent of the business cycle- they are stochastically independent of $\tilde{R}$.

Assumption 4: pension scheme The scheme we examine is based on mandatory contributions out of labor income at the constant rate τ. The agent is allowed to opt out from public social security and invest her contribution in the saving market where it earns the risky rate of return $\tilde{R}$; the dollar opted out is a debt to the social security with a safe rate of return G, and this debt is rolled into a negative annuity ate the time of retirement. To make opting out break-even from the social security point of view, we can think about G as the rate of return on public debt.

Three different pension schemes are considered:

(i) Notional Defined Contribution (NDC): this is a intragenerational non-redistributive mechanism, where worker contributions are recorded on an individual account earning GDP growth rate, that, for the sake of simplicity, we assume is still G^2. As the agent retires, virtually accrued retirement wealth is converted into a positive annuity.

(ii) Average Wage (AW): here the pension paid to the retirees is a fraction of the average wage earned during work life. Since contributions have the same effect on pension entitlements at any age, this scheme entails a redistribution from young to old workers.

(iii) Flat Rate (FR): the pension paid is independent of income earned and therefore to contributions. This scheme is redistributive from young to old workers and obviously from high to low income people.

Retirement wealth is not disposable to finance consumption before the agent exits work force -to maintain the commitment properties of social security-, nor switching accrued wealth between pillars is allowed. Therefore if we denote α the share of contributions put outside the public pillar, w^p and x respectively funded retirement wealth and public pension entitlements, the agent faces the following constraints for $t + 1 < k$:

$$w^p_{t+1} = R_{t+1}\left(w^p_t + \alpha_{t+1}\tau y_t\right),\ w^p_1 = 0 \tag{4}$$

$$0 \leq \alpha_{t+1} \leq 1 \tag{5}$$

that hold in any pension scheme.

On the contrary, pension entitlements constraints vary according to the pension scheme.

In NDC we have:

$$x_{t+1} = x_t + A(1 - \alpha_{t+1})\tau y_t G^{k-t},\ x_1 = 0,\ \ p = x_k \tag{6}$$

where A is the annuity value $\left(\frac{G-1}{G}\right)\left(1 - G^{k-T-1}\right)^{-1}$.

Within the AW, we have:

$$x_{t+1} = x_t + \frac{\psi y_t}{k-1} - A\alpha_{t+1}\tau y_t G^{k-t},\ x_1 = 0,\ x_{t+1} \geq 0,\ \ p = x_k \tag{7}$$

where ψ is the substitution rate of wage at retirement.

And in FR:

$$x_{t+1} = x_t - A\alpha_{t+1}\tau y_t G^{k-t},\ x_1 = \bar{p},\ x_{t+1} \geq 0,\ \ p = x_k \tag{8}$$

where $\bar{p}$ is the flat pension.

As the individual retires, funded retirement wealth becomes fully disposable. This different treatment from public pillar retirement wealth mirrors the typical choice given to workers contributing to an individual account to get all of their retirement wealth as capital. Therefore discretionary wealth is:

$$\begin{aligned} w_{t+1} &= R_t w_t + (1-\tau)\, y_t - c_t && \text{if } t < k; \\ w_{t+1} &= R_t w_t + w_t^p + p - c_t && \text{if } t = k; \\ w_{t+1} &= R_t w_t + p - c_t && \text{if } t > k. \end{aligned} \tag{9}$$

Taken at its face value, our model is a life-cycle asset allocation problem, where portfolio choice regards retirement wealth. To keep things manageable, we assume that there is a single and risky asset in which invest wealth on the saving market. What will happen considering also public debt in the investment opportunity set? Is it likely that people opt out -trading public debt for risky asset- and still have public debt in their discretionary wealth? The answer is positive because retirement wealth is not a perfect substitute for discretionary wealth due to the borrowing constraints on the former. Moreover they are typically poor substitutes during the first half of work life when agents voluntarily hold wealth not as life-cyclers in the classic sense but to insulate consumption from labor income shocks. As agent ages, retirement and discretionary wealth become better substitutes and therefore people holding a mixed discretionary portfolio and opting out would be rather puzzling. However we will show that the typical strategy in work life is to opt-in.

Assumption 5: maximand The agent solves a dynamic program of optimal consumption over the life-cycle with preferences described by a time-separable utility function

$$\sum_{t=1}^{T} u(c_t)\beta^t;$$

specifically we consider an isoelastic form

$$u(c) = \frac{c^{1-\rho}}{1-\rho},$$

where ρ is the degree of relative risk aversion, $1/\rho$ is the elasticity of intertemporal substitution and β^{-1} is the one-period psychological discount rate on future consumption utility.

2.2 Setup of the Problem

During her work life the agent has to take two choices in every period: how much to consume out of cash-on-hand and how to split her mandatory contributions to the social security system between the private and the public pillar. Thus control variables are c and α. During retirement the control variable is only the optimal level of consumption out of cash-on-hand and annuity.

Even with all of the simplifying assumptions made, the dynamic program has five state variables: gross return on non-retirement wealth Rw, labor income y, the permanent shock z, private retirement wealth w^p and public pension entitlements x. Usually the first two collapse in a single state variable defined cash-on-hand, but this is not the case since the level of labor income affects contributions to the pension scheme.

But, as usual when assuming isoelastic utility function, the problem is scale independent, in that the value function is homogeneous of degree $1-\rho$, the saving function is homogeneous of degree 1 and the opting out function is homogeneous of degree 0. We can exploit this property by reducing the number of state variables to four, specifically we can eliminate the permanent income z.

Thus, during work life, the Bellman equation for this problem is given by:

$$\begin{aligned} v_t(R_t w_t; y_t; w_t^p; x_t; z_t) &= \\ & \max_{w_{t+1}} u(R_t w_t + (1-\tau) y_t - w_{t+1}) + \\ & \max_{\alpha_{t+1}} \beta E_t[v_{t+1}(R_{t+1} w_{t+1}; y_{t+1}; w_{t+1}^p; x_{t+1}; z_{t+1})]; \end{aligned} \tag{10}$$

$$\text{sub } (1), (2), (3), (4), (5) \text{ and either } (6), (7) \text{ or } (8).$$

In period k the agent retires and the problem becomes:

$$v_t(R_t w_t;p) = \max_{w_{t+1} \geq 0} u(R_t w_t + p - w_{t+1}) + \beta E_t[v_{t+1}(R_{t+1}w_{t+1};p)]; \tag{11}$$

without bequest motive the value function in T is simply the uniperiodal utility of overall wealth:

$$v_T(R_T w_T;p) = u(R_T w_T + p): \tag{12}$$

2.3 Solution

As usual the problem is solved backwards.

Starting with the last but one period, where with $w_{T+1}(R_T w_T;p) = 0$, we search for the value w^*_{t+1} that satisfies the first order condition

$$\begin{aligned} \left(u'(R_t w_t + p - w^*_{t+1}) - \beta E_t[R_{t+1}u'(R_{t+1}w^*_{t+1} + p) - w_{t+2}(R_{t+1}w^*_{t+1};p)]\right) w^*_T &= 0; \\ u'(R_t w_t + p - w^*_{t+1}) - \beta E_t[R_{t+1}u'(R_{t+1}w^*_{t+1} + p) - w_{t+2}(R_{t+1}w^*_{t+1};p)] &\geq 0; \end{aligned} \tag{13}$$

that returns the saving function $w^*_{t+1}(R_t w_t;p)$ and the value function v_t which depends on the same arguments. Moving one period backwards we face the very same problem as long as $t \quad k$.

During work life the problem involves two choices.

Within NDC, the first order condition for α_{t+1} is

$$\begin{aligned} E_t\left[R_{t+1}\frac{\partial v_{t+1}}{\partial w^p_{t+1}} - AG^{k-t}\frac{\partial v_{t+1}}{\partial x_{t+1}}\right]\alpha^*_{t+1}\left(1 - \alpha^*_{t+1}\right) &= 0; \\ E_t\left[R_{t+1}\frac{\partial v_{t+1}}{\partial w^p_{t+1}} - AG^{k-t}\frac{\partial v_{t+1}}{\partial x_{t+1}}\right] &\geq 0 \quad \text{if } \alpha^*_{t+1} = 1; \\ E_t\left[R_{t+1}\frac{\partial v_{t+1}}{\partial w^p_{t+1}} - AG^{k-t}\frac{\partial v_{t+1}}{\partial x_{t+1}}\right] &\; 0 \quad \text{if } \alpha^*_{t+1} = 0: \end{aligned} \tag{14}$$

From this we obtain a policy function $\alpha^*_{t+1}(w_{t+1};y_t;w_t;x_t;z_t)$. For other pension schemes the first order condition on α_{t+1} is straightforwardly derived from (7) or (8). It should be noted that in AW and FR the upper constraint on α_{t+1} has to incorporate the lower bound on x_{t+1} and it may be therefore lower than 1.

By using $\alpha^*_{t+1}(w_{t+1};y_t;w_t;x_t;z_t)$, we can define λ^*_{t+1} as

$$\lambda^*_{t+1}(w_{t+1}; y_t; w^p_t; x_t; z_t) = E_t\left[R_{t+1}u'(c^*_{t+1})\right];$$

and the first order condition for w_{t+1} is therefore:

$$\begin{aligned}\left[u'(R_t w_t + (1-\tau)y_t - w^*_{t+1}) - \beta\lambda^*_{t+1}(w^*_{t+1}; y_t; w^p_t; x_t; z_t)\right] w^*_{t+1} &= 0; \\ u'(R_t w_t + (1-\tau)y_t - w^*_{t+1}) - \beta\lambda^*_{t+1}(w^*_{t+1}; y_t; w^p_t; x_t; z_t) &\geq 0.\end{aligned} \tag{15}$$

Thus we can define a saving function $w^*_{t+1}(R_t w_t; y_t; w^p_t; x_t; z_t)$, and the value function v_t. The loop is complete and we move one period backwards.

This problem cannot be solved analytically and thus we search numerical solution for a given set of parameters[3]. This has become a standard approach in precautionary saving literature since the pioneering work of Zeldes (1989): actually this is the approach followed -among many others- by Carroll (1992, 1997), Hubbard, Skinner and Zeldes (1994).

As usual we solve the problem backwards. In each retirement period we derive a saving function $w^*_{t+1}(R_t w_t; p)$ by checking the first order condition for a grid of points and then interpolating them. With this in hand we derive the value function $v_t(R_t w_t; p)$ and move one period back. During working life we first find α^*_{t+1} -within a fine grid of points- that maximizes the expected value of v_{t+1}, then we interpolate to derive the policy $\alpha^*_{t+1}(w_{t+1}; y_t; w^p_t; x_t; z_t)$. As a second step we find $w^*_{t+1}(R_t w_t; y_t; w^p_t; x_t; z_t)$ using first order condition and interpolation. At this stage we can obtain the value function $v_t(R_t w_t; y_t; w^p_t; x_t; z_t)$.

For the sake of simplicity random variables are reduced to a binomial variable. To enhance interpolation quality, we follow the techniques suggested by Carroll (1999). Once we have obtained all the policy functions, we calculate the optimal paths for 10,000 agents receiving different draws of income and returns. We take means and plot them against age.

3 Benchmark Calibration

3.1 Parameters

Our calibration follows closely that of Campbell, Cocco, Gomes and Maenhout (1999) that is based on U.S. data.

Calibration 1: life-cycle structure Agents start working at 26, retire at 60 and die at 74.

Calibration 2: labor income We use the labor income process that Campbell, Cocco, Gomes and Maenhout estimate from the PSID for a male household with High School Degree. The deterministic age-related component of labor income is a third order polynomial, the temporary shock and the innovation in the random walk process have both mean zero and variance respectively .0738 and .0106. High School educational group has a human wealth between No High School and College Degree groups and the same is true for variance in transitory and permanent shocks to labor income.

Calibration 3: investment opportunity set The mean return on discretionary and funded retirement wealth is set to 1.05 with standard deviation .157, that is a calibration typical for U.S. stock market. This is below the historical average, but we chose it because stock prices have tended to increase in recent years relative to corporate earnings (as noted in Campbell, Cocco, Gomes and Maenhout).

According to Assumption 3 and Calibration 3, all funded wealth is invested in stocks. As already noted adding public debt in the investment opportunity set is not likely to modify heavily opting out strategies, but it would ‡atten the consumption pro…le by reducing mean return and risk -especially late in life, when individuals typically have a less aggressive portfolio, and for those highly risk averse. This unrealistic feature will be crudely addressed in one of the following calibrations by decreasing mean and variance on …nancial wealth to mimic a mixed portfolio[4].

Calibration 4: pension schemes Rate of return on public debt G is set to 1.015 and it is assumed to be equal to the GDP growth rate. In turn we can derive both Ã (.3453) and þ (.735) respectively in AW and FR pension schemes by using social security budget constraint.

Calibration 5: maximand The isoelastic utility function has a benchmark degree of risk aversion ½= 4 and the psychological discount rate ¯ is set to :96. As a counterpart of well known equity premium puzzle, the value for ½ is higher than in models where the asset is safe. For instance Carroll (1999) sets $\rho = 10$, Campbell, Cocco, Gomes and Maenhout (1998) set $\rho = 5$, Cocco, Gomes and Maenhout (1999) use $\rho = 10$.

3.2 Opting Out Policy Function

Before looking at simulated life-cycle patterns, we study the opting out policy function to highlight main forces at work. We disregard the saving policy since it has the standard shape in modern consumption theory. The number of state variables makes the graphical representation of portfolio allocation rules difficult, so we will study separately the effect of each argument of the policy function.

Age Effect First of all, we have to remark that human wealth is an implicit investment in an asset earning an uncertain return depending on labor income shocks. If this yield is not heavily correlated with the risky asset, then human wealth is a closer substitute for the safe investment[5]. This finding has been shown in recent literature -namely since Cocco, Gomes and Maenhout (1997)- for both two periods and multiperiod frameworks[6]. This is the clue to understand the age effect.

Now we consider what this phenomenon entails in our model of portfolio allocation for retirement. Consumption during retirement depends on human and financial wealth. Over the life-cycle, human wealth is hump shaped with the peak at age 30 and this pattern is mirrored in the opting out policy function by age. This finding is typical within life-cycle portfolio allocation theory, but still there is a noteworthy difference. Studying the optimal allocation of discretionary wealth, Cocco, Gomes and Maenhout (1998) find a much more stressed hump-shape for the age effect.

This discrepancy is due to the different portfolio choice we examine. When dealing with discretionary wealth allocation, the agents consider correlation between next-period marginal utility and R. Those very young have a consumption function which is highly responsive to wealth shocks -they are buffer-stock savers and they act as if they had a short horizon. Therefore their optimal portfolio is not fully made up of the risky asset. As they approach income peak, they become life-cyclers and their consumption policy turns out to be less sensitive to wealth shocks. Near retirement their welfare relies more heavily on accrued wealth and this explain the decreasing side of the hump. In our model, portfolio allocation depends always on marginal utility at retirement, which in turn depends on human wealth over the whole work-life, even when agents act as buffer-stock savers.

Thus a young worker chooses an highly aggressive financial portfolio and then, as she nears retirement, she has to diversify her wealth by putting eggs in the riskless pension scheme by opting in. Having this in mind, it is easy to understand the age effect on the opting out policy function: that is $\alpha^{*}_{t+1}(w_{t+1}; y_{t;} w^{p}_{t}; x_t; z_t)$ is slightly increasing in t up to age 30 and then it monotonically decreases.

Discretionary Wealth Effect To understand the effect of nonretirement wealth on the opting out policy is enough to notice that its rate of return is R, so that an higher value for w increases the absolute covariance between R and marginal utility during retirement. Therefore it crowds out the funded share of pension wealth. That is to say that $\alpha^{*}_{t+1}(w_{t+1}; y_{t;} w^{p}_{t}; x_t; z_t)$ depends negatively on w_{t+1}. This effect is slight when agents are buffer-stock savers, as shocks on discretionary wealth are absorbed by consumption, and it becomes stronger and stronger as the agent ages following the increasing substitutability between discretionary and retirement wealth.

Private and Public Retirement Wealth Effect Increasing x the amount of safe wealth is pushed upwards and therefore α is lowered; if we increase w^p, the amount of risky wealth becomes higher and this reduces α.

Therefore $\alpha^{*}_{t+1}(w_{t+1}; y_{t;} w^{p}_{t}; x_t; z_t)$ depends negatively on w^{p}_{t} and positively on x_t.

Permanent Labor Income Shock Effect The relationship between optimal share of contributions to put in the funded pillar and the permanent component of labor income has the same explanation given for the age effect. Since a higher value of z means a higher expected value for labor income and this isolate retirement consumption from risky investment, we have a positive link between z and α. There is also a second mechanism at work since a higher value for z reduces optimal discretionary wealth at retirement and this, in turn, reduces covariance between marginal utility at retirement and risky rate of return.

3.3 Results

Plotting the average benchmark simulation results for 10,000 agents against age, we see all the previously described forces at work on the path of opting out over the life-cycle.

We start our graphical summary from NDC (figure 1). The consumption profile has typical buffer-stock theory shape (Carroll (1997)). Young workers engage in some savings to isolate consumption from labor income shocks notwithstanding the sharp increase in earnings that they face: therefore consumption tracks wages. By accruing discretionary wealth and nearing their income peak, their consumption becomes a life-cycle one which is driven by the interaction of rate of return, psychological discount rate and uncertainty.

Consider now the graphic giving the title to this paper, where the optimal share of contributions put in the funded pillar is plotted against age. As we have already noticed in the previous subsection, age is a key variable in choosing retirement savings allocation and actually it has a dramatic effect. As expected the shift is from the private to the public pillar.

During first five years of work life the agent contributes almost every dollar to the private scheme as he can afford the attached risk because of the implicit insurance provided by future labor income. Then a transition towards the public pillar takes place, lasting some ten years, that leads to fifteen years of full opting-in. Notwithstanding the hump-shape in the age effect, the opting out strategy is monotone because of the simultaneous movements in w, w^p and x. This is a noteworthy difference from the discretionary wealth portfolio allocation theory that typically finds an hump-shape in the optimal risky share of wealth. Our finding is also different from those derived from models of money-loser opting out and safe return on savings market. Disney, Palacios and Whitehouse (1999) find that agents opt out up to five years before retirement, given a calibration which is similar to ours.

Considering a single agent the typical pattern even more dichotomous: the transition phase lasts at most some five years. That is to say that the optimal path is a sort of 0/1 strategy and the smoother pattern plotted in the graphic can be read as the percentage of agents switching completely towards the unfunded pillar. Actually, the typical choice given to workers in pension reform countries allowing lifetime switching (such as Great Britain, Colombia and Uruguay) is to allocate all contributions due in the year between the two pillars. Therefore introducing this further constraint is not likely to modify significantly our results.

In contrast to the opting out path, it is worth noting how smooth is the risky share pattern over the life-cycle. Actually, agent's choice is on retirement wealth allocation between two assets, but she can only act on the contribution flow. Since this has a mild effect on portfolio allocation, agent's choice on the flow are quite radical.

As a consequence of this opting-in pattern, private retirement wealth has a convex shape, where the increasing slope is initially due to contributions and then to the effect of compounded interest. When contributions to the public pillar start, this pension wealth (which is represented as the present value of accrued pension entitlements in our graphics) grows at a higher pace, that monotonically diminishes and converges to the funded wealth growth rate. On the edge of retirement, social security portfolio is 57 percent private. If we add discretionary wealth, the risky share in overall wealth raises to about 77 percent. Pension wealth provides 55 percent of last gross wage and 57 percent of consumption at age 60.

It is worth remarking that these results are optimal strategies for those who had the choice to opt out since the beginning of their work life. It is likely that a worker, who was constrained to contribute every dollar to the unfunded pillar before the choice has been given, would opt out of social security even near retirement.

In AW (figure 2), most of public pension entitlements are accrued late in work life due to the benefit mechanism which is not actuarially fair. Therefore constraint on x lowers the upper bound on α, especially early in work life: this causes the opting out strategy to be hump-shaped and the transition towards public pillar to last longer. Again, due to the different pension entitlement structure, on the edge of retirement private share in pension wealth is lower than the NDC benchmark. The overall risky share in wealth is still lower but discrepancy is reduced by higher discretionary wealth. Higher savings are due to both higher uncertainty on pension entitlements and lower substitution rate of pension wealth which is 53 percent of last gross wage and 55 percent of consumption at age 60.

FR (figure 3) seems to entail a much stronger propensity towards the private pillar but this is only a graphical illusion due to the fact that FR redistributes from high to low income people. Therefore higher proportion of opting out late in life is due to low income people who turn out to have a high public entitlements relative to their discretionary wealth. On the contrary high income people has low public pension wealth and opt fully in. Actually private share in pension wealth at the time of retirement is only slightly higher compared to NDC. Due to lower uncertainty on lifetime resources consumption is higher during mid work life and discretionary wealth lower.

Opting out is almost independent of underlying public pension scheme and therefore we will conduct sensitivity analyses only for NDC.

4 Sensitivity Analyses

4.1 Different Education Groups

Our baseline results are given for the High School Degree group. In figure 4 results for NDC opting out model are shown for College Degree and No High School Degree groups. Income processes are different in both deterministic and stochastic components. Specifically low education people has the flattest pattern, the highest variance in transitory shocks (.1056) and the lowest variance in the Ar(1) innovation (.0105). The opposite is true for high education people which has the steepest deterministic pattern, the lowest transitory variance (.0584) and the highest variance in Ar(1) innovation (.0169).

Nevertheless opting out behavior is always very similar. Low education people switch to the public pillar a couple of years before mid and high education groups due to their flatter income profile. For the same reason, they hold a flatter discretionary wealth profile. At the time of retirement risky share in pension and overall wealth is equal to the benchmark.

High education people has a riskier human wealth which pushes the private share in retirement wealth downwards relative to the benchmark. Therefore they have, on average, a lower substitution rate from mandatory contributions which is partially undone by higher saving propensity. As they exit the work force, their risky share in overall wealth is again equal to the baseline.

This result is interesting since it shows that even heavily different income stochastic processes do not bring about different opting out strategies. Probably, a major channel of differences in these optimal patterns would arise if we account for correlation between return and permanent income innovation, which is typically dependent on education.

4.2 Correlation between Labor Income and Stock Return

If we remove the simplifying assumption of stochastic independence between stock return and permanent labor income innovation -that is worth reminding it is not likely to be completely idiosyncratic- we move a step further towards reality. Actually, in Cocco, Gomes and Maenhout, estimates from PSID show a positive coefficient of correlation $\rho_{\xi R} = .3709$ for high school degree households, where $\log u_t = \xi_t + \psi_t$ -i.e. the sum of the aggregate and idiosyncratic components-.

In this calibration, we consider the extreme case where $\log u_t = \xi_t$ -all of the permanent income shock is aggregate- and its effect on optimal opting out strategies. Given this positive link between earnings and return on stocks, human capital becomes a worse substitute for riskless capital. If correlation was strong enough, age effect could even be negative, but in our calibration it is evident that this is not the case.

Since future labor income is still a better substitute for riskless savings, we still obtain the typical out/in pattern (figure 5), but reduced insurance properties of human wealth asks for longer contributions period in the public pension scheme than the benchmark -specifically twenty rather than fifteen years-. Moreover, weaker age effect entails a longer transition path, that actually lasts some fifteen years. Therefore we do not observe a phase of full opting out even at the very beginning of work life; nevertheless the partial contributions to the private pillar, during the transition path, accounts for a high 34 percent of mandatory wealth at the time of retirement, due to the effect of compounded interest. Reduced opting out, in turn, lowers replacement provided by pension wealth that is some 0.43 if compared to last labor income and to 0.45 if compared to consumption at 60. Discretionary wealth is slightly higher all over the work life because of both reduced replacement ratio and lower insurance from savings. Therefore consumption profile is lower than the benchmark until age 55.

As this calibration shows, the demand for a riskless asset heavily depends on correlation between physical and human capital by introducing a negative hedging demand for risky investment. Moreover this correlation is typically dependent on the sector of employment and therefore it is a major source of heterogeneity in optimal pension arrangements.

4.3 Higher Psychological Discounting

If we increase psychological discount rate to $(0.9)^{-1}$ per year (figure 6), this more impatient agent chooses a less steep consumption profile over the life-cycle, but in the very first years when it follows closely increasing net earnings. In turn, this dynamic entails much lower discretionary wealth at any given age: in the first half of working life, the individual creates a buffer-stock equal to approximately one year labor income to protect herself from shocks and, as she nears retirement, she does not need much additional savings because of lower optimal consumption in old age.

Actually, as noted in the previous section, lower discretionary saving crowds in risky investment in retirement wealth that earns higher return on average. Therefore, while the opting-in pattern is still there, the phase of contributions to the private pillar lasts longer, and the number of years of contributions to the public pillar is shortened: more impatient agents are those with more aggressive portfolio because of low wealth they voluntarily keep. Thus, at the end of working life, replacement ratio is approximately 60 per cent of last wage, the risky share in mandatory wealth is higher than in the benchmark case, while the overall share is almost equal.

4.4 Higher Risk Aversion

Results in figure 7 are shown for a more prudent agent whose constant degree of risk aversion is set to 6. From this parametrization, we would expect a tilt in portfolio composition towards riskless asset. Since human wealth still substitutes for risk-free wealth, the qualitative pattern is expected to be unchanged.

This prediction is correct and the effect of 50 percent increase in risk aversion is extremely strong. Even the youngest do not fully contribute to the private pillar and at age 35 they are already completely within the public scheme. The risky share in pension wealth drops from 98 percent at 25 to some 27 percent on the edge of retirement; overall risky share moves down to 71 percent.

Because of longer contribution to the riskless pillar, the replacement ratio is 40 percent of last gross wage, compared to the 55 percent benchmark. This is the reason for higher discretionary wealth at the time of retirement -specifically thirty percent higher and equal to ten years income-, while higher precautionary motives shift wealth upwards early in work life. The higher voluntarily wealth held at any given age is in turn a second reason for shorter opting-out phase since it increases covariance between marginal utility of consumption during retirement and rate of return on risky asset. As a counterpart of wealth profile, consumption is lower than in the benchmark for great part of work life.

4.5 Mixed Portfolio

We have already observed how disturbing is to assume stock as the only financial asset. To address this undesirable feature, but avoiding the higher complexity arising from a second portfolio choice, we calibrated this simulation reducing both mean and variance of funded wealth yield; specifically we set $R \sim N(1.03; 0.1)$. This is the simplest way to represent a portfolio made up of both stocks and public debt. Obviously, this assumes implicitly that discretionary and private pension wealth are invested in the same way and portfolio allocation is kept constant[7].

In figure 8, the first effect of these different parameters that we can observe is that as agents become life-cyclers they engage in a flatter and lower consumption strategy due to lower return on savings market. As a counterpart, discretionary wealth is lower.

Although opting out strategy is very close to the benchmark, lower return makes the pension wealth substitution rate drop to 40 percent of last gross income and to 48 percent of consumption the year before retirement.

5 Conclusions

Although opting out is a politically appealing strategy to downsize public pension schemes, it has still missed a microeconomic analysis dealing with investment risk attached to private pillar wealth.

This paper models a break-even opting out system in that an opted out dollar is run into a debt earning the rate of return on public debt which we assume to be riskless. At the same time, this dollar is put into an individual account which earns a risky and higher rate of return. We have built a partial equilibrium life-cycle model to address the optimal pattern of opting out of social security when workers are allowed to switch contributions all over their work life.

Our main finding is that workers opt out only when young and reduce monotonically their risk exposure by opting in. This age pattern is not completely consistent with most recent portfolio allocation theory where the youngest are not those investing most aggressively because of high consumption sensitivity to wealth shocks. The difference is due to the wealth considered: here we have examined retirement wealth -which cannot be used to smooth consumption during work life-, portfolio allocation literature dealt with discretionary wealth.

Given our benchmark parameter set, some ten years are devoted to private contribution and the following twenty five to the public scheme. At the time of exiting work life, private share in retirement wealth is approximately 0.5.

This quantitative finding is very different from that obtained assuming a safe rate of return on private pillar investment and a money-loser opting out mechanism. there agents stay out of the public pillar for great part of their life and switch in only a few years before retirement in order to maximize available resources to finance old age consumption at expenses of social security budget. In this literature, opting out responds mainly to the benefit accrual mechanism considered, here, on the contrary, results are almost independent of the public pension scheme.

Since our quantitative findings rely on a parameter set, we have engaged in some sensitivity analyses both on the behavioral and on the economic side. While the qualitative result is maintained, what is responsive to calibration is the optimal length of years within and outside the public pillar.

Here two findings are particularly interesting. First an agent whose degree of relative risk aversion is 6 -rather a typical assumption in portfolio allocation literature- spends almost her whole life in the public pillar. Secondly, as we augment the basic model to account for correlation between permanent income shocks and private wealth return, there is a significant tilt towards public pillar. Since this correlation is highly dependent on employment sector, it is apparent that heterogeneity affects heavily optimal opting out.

The main weakness in this paper is the partial equilibrium approach: introducing opting out raises capital accumulation and requires the Government to increase public debt at least during the transition to the new steady state. These phenomena are likely to modify risk and return of financial assets and labor income.

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Notes

[1]Miles (1999) shows that life-span risk impacts heavily on the optimal size of flat rate public pension scheme considering different hypoteses on annuity market imperfections. Since we want to find a lower bound for opting in, we decided not to include this feature, which increases propensity to the public pillar.

[2]Our results will not change significantly by assuming that GDP growth rate is higher than the rate of return on public debt.

[3]In a non-life-cycle framework a notable exception is Viceira (1997).

[4]Anyway, this assumption is not novel: see Miles (1999) who set a single asset on discretionary wealth with a rate of return between 2 and 6 percent and a standard deviation ranging from 0.1 to 0.175. Dutta, Kapur and Orzag (1999) use -as a first round proxy- stocks as the unique asset for funded and discretionary savings.

[5]For high covariance between R and u, labor income becomes a closer substitute for risky asset, thus entailing a reversed hump-shape age effect in stocks demand. Empirically this seems not to be the case: among the others Bertaut and Haliassos (1997).

[6]The claim that risky stocks is good for the young and not for the elderly was rejected by Samuelson. But his finding relied on the assumption of no labor income. Anyway this seminal result has proved extremely useful in rejecting the vulgar insight that holding stocks for long time would reduce risk, which was a wrong case for the Law of Large Numbers-.

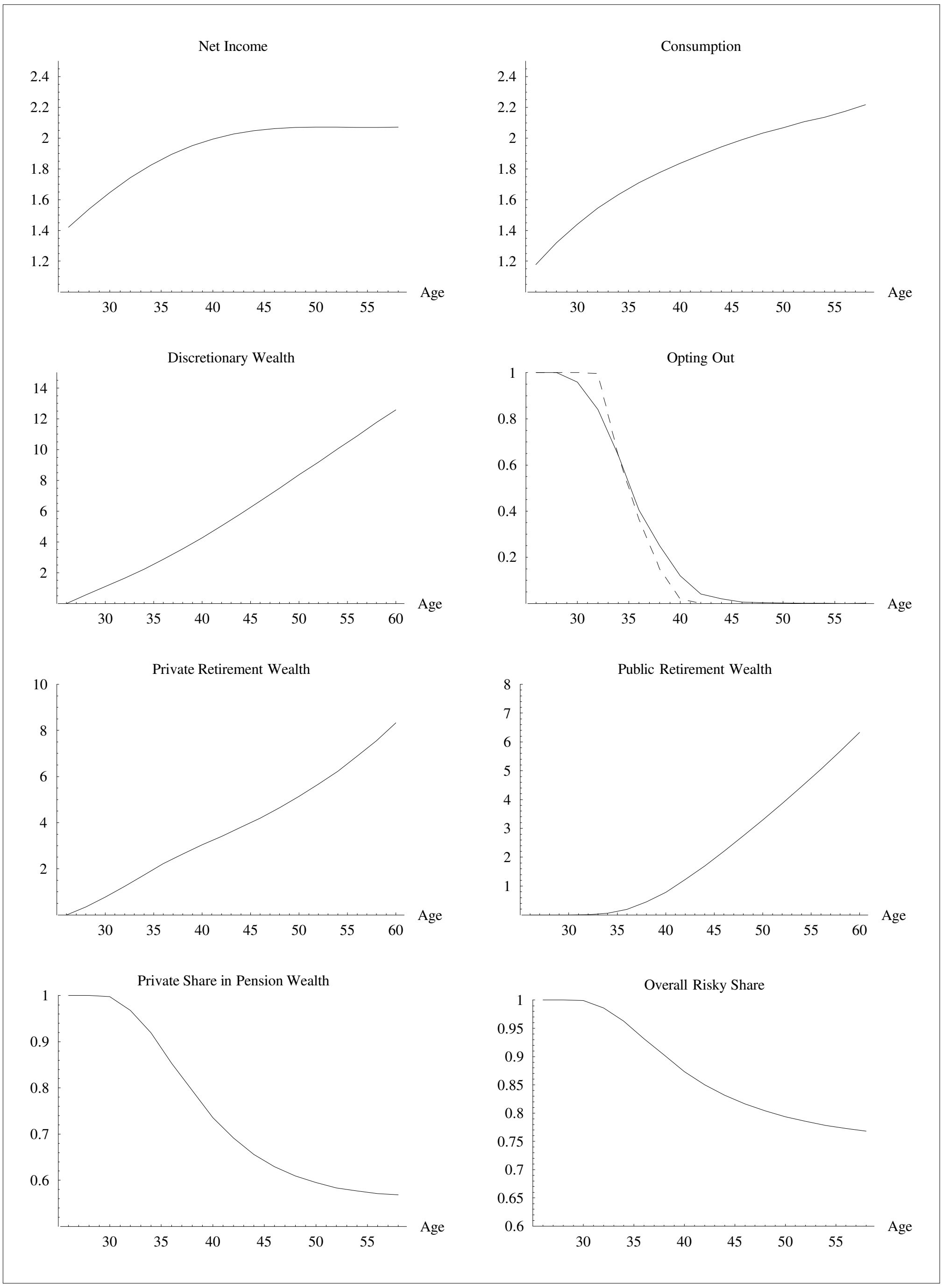
[7]Making portfolio choice on funded wealth endogenous is likely to provide a higher mean-variance combination early in life and a lower one in the second half of working phase. Moreover this assumption is strong, since public pillar investment is crowds out from retirement portfolio public debt.

Table 1: Baseline Parameters	
Description	Parameter Value
Entrance age in workforce	26
Retirement age (K)	60
Death age (T)	74
Discount factor (¯)	0.95
Risk aversion (±)	4
Riskless rate (G)	1.015
Mean risky rate (R)	1.05
Std risky asset return (¾$_R$)	0.157
Social Security tax rate (¿)	0.10
AW Substitution rate (Ã)	.3453
FR Pension (þ)	.735

Table 2: Log Labor Income Process

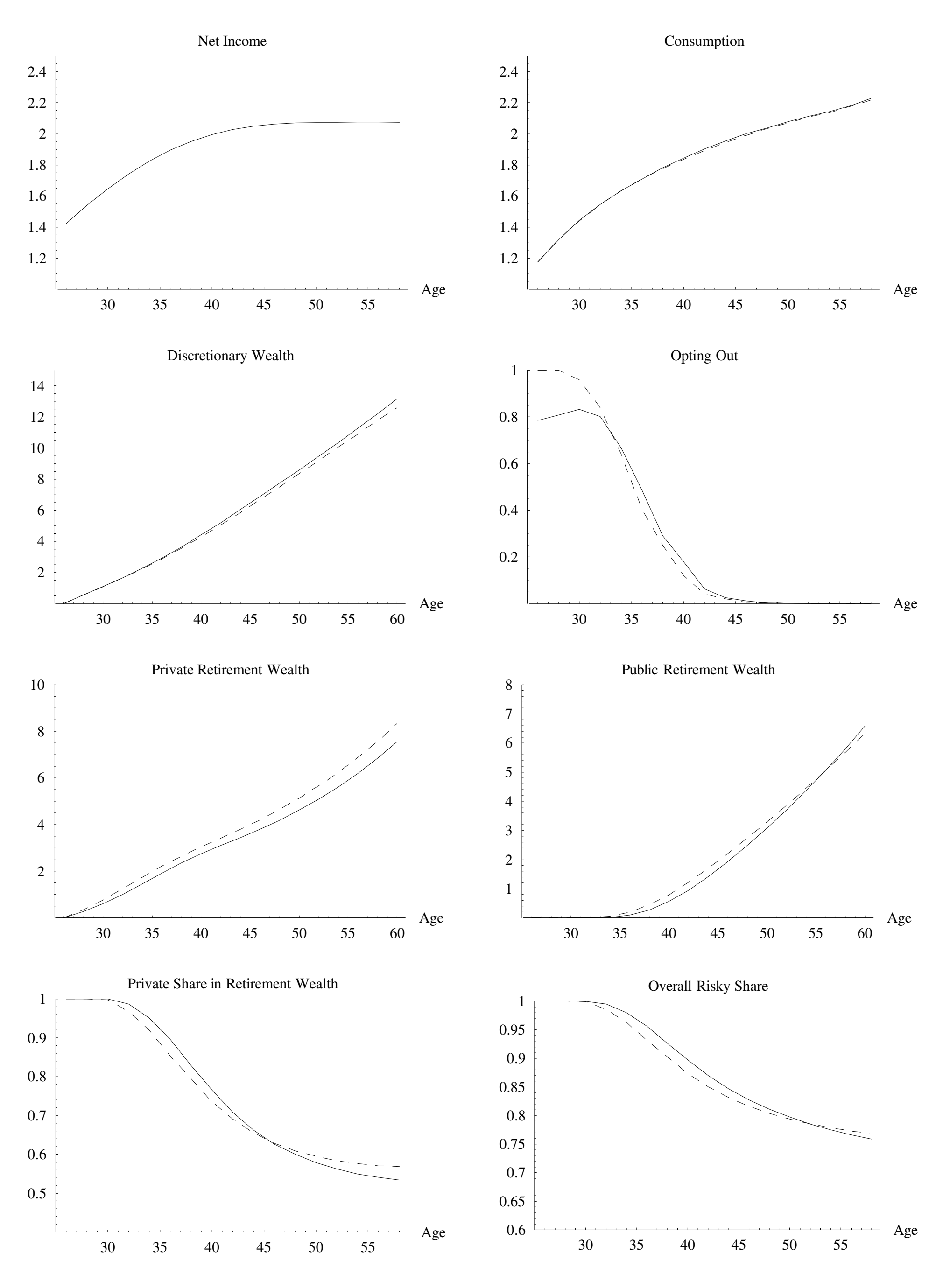
Coefficients in the age polynomial	High School Degree	College Degree	No High School Degree
Constant	-2.1700	-4.3148	-2.1361
Age	0.1682	0.3194	0.1684
Age2/10	-0.0323	-0.0577	-0.0353
Age3/100	0.0020	0.0033	0.0023
Variance			
Variance of transitory shocks (σ^2_ε)	0.0738	0.0584	0.1056
Variance of permanent shocks (σ^2_u)	0.0106	0.0169	0.0105

Figure 1: Notional Defined Contribution



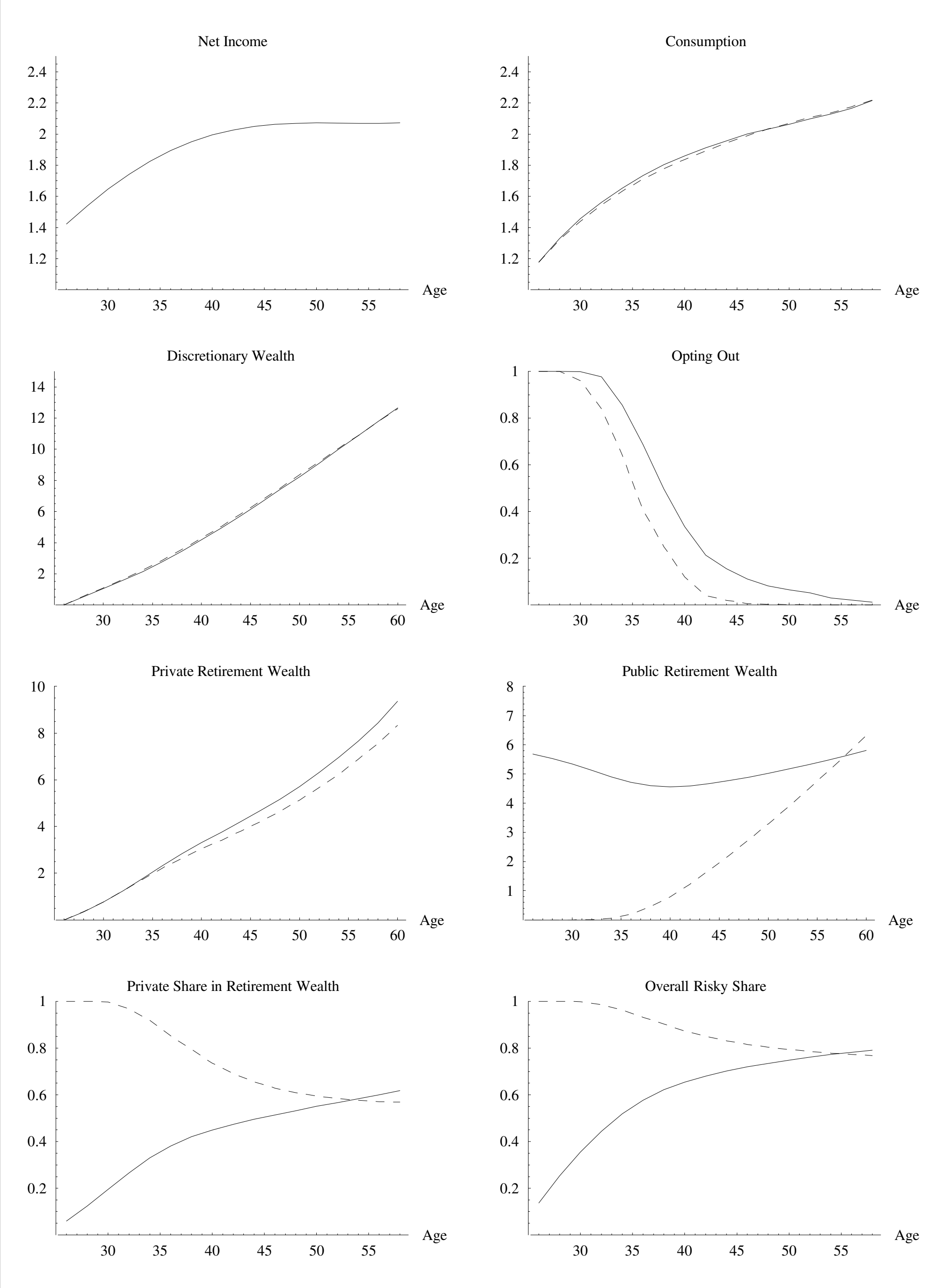
Solid Line: Mean values for 10,000 agents plotted against age. Dashed Line: values for a single agent receiving average draws

Figure 2: Average Wage



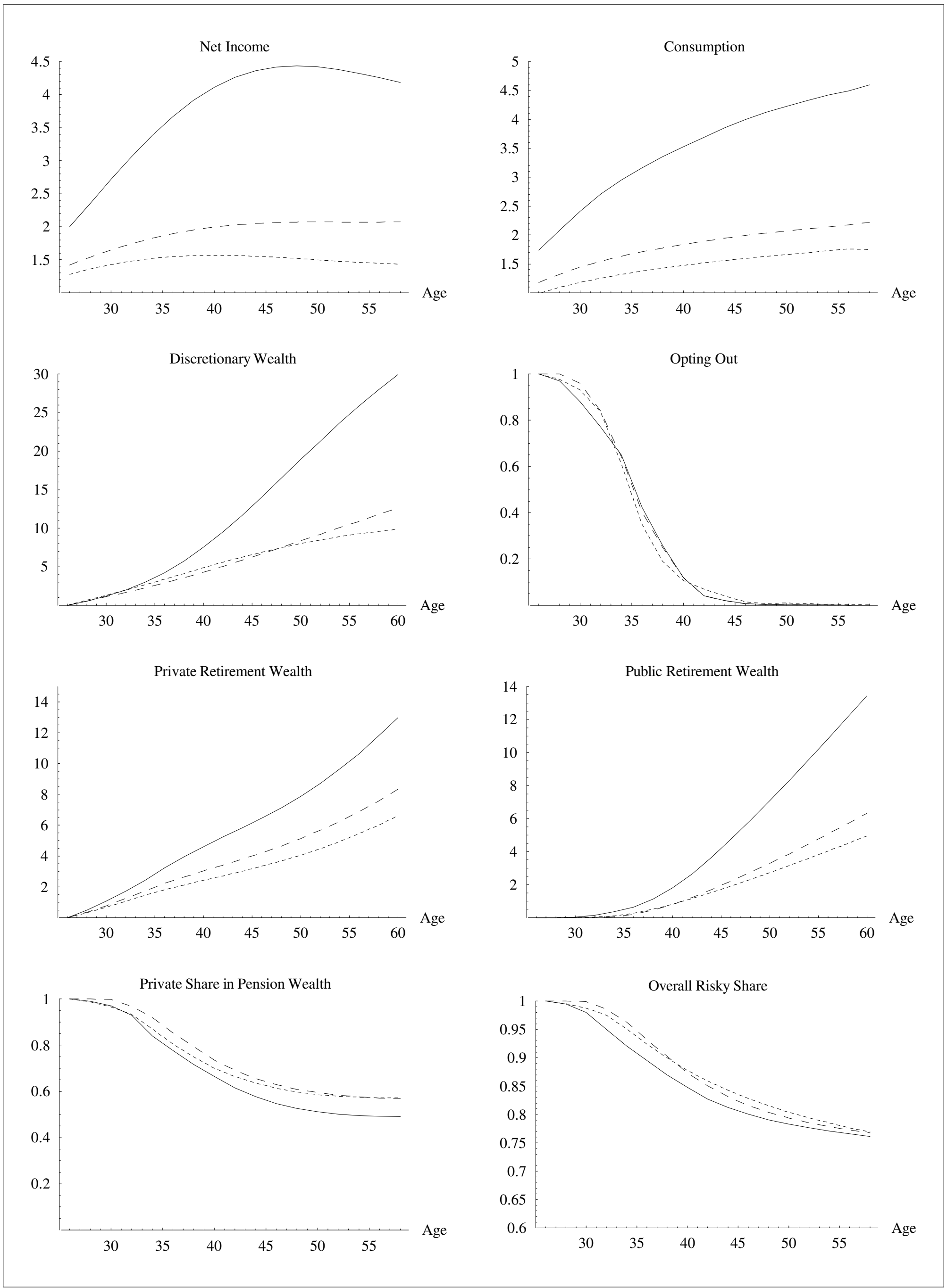
Solid Line : Mean values for 10,000 agents plotted against age . Dashed Line : benchmark calibration .

Figure 3: Flat Rate



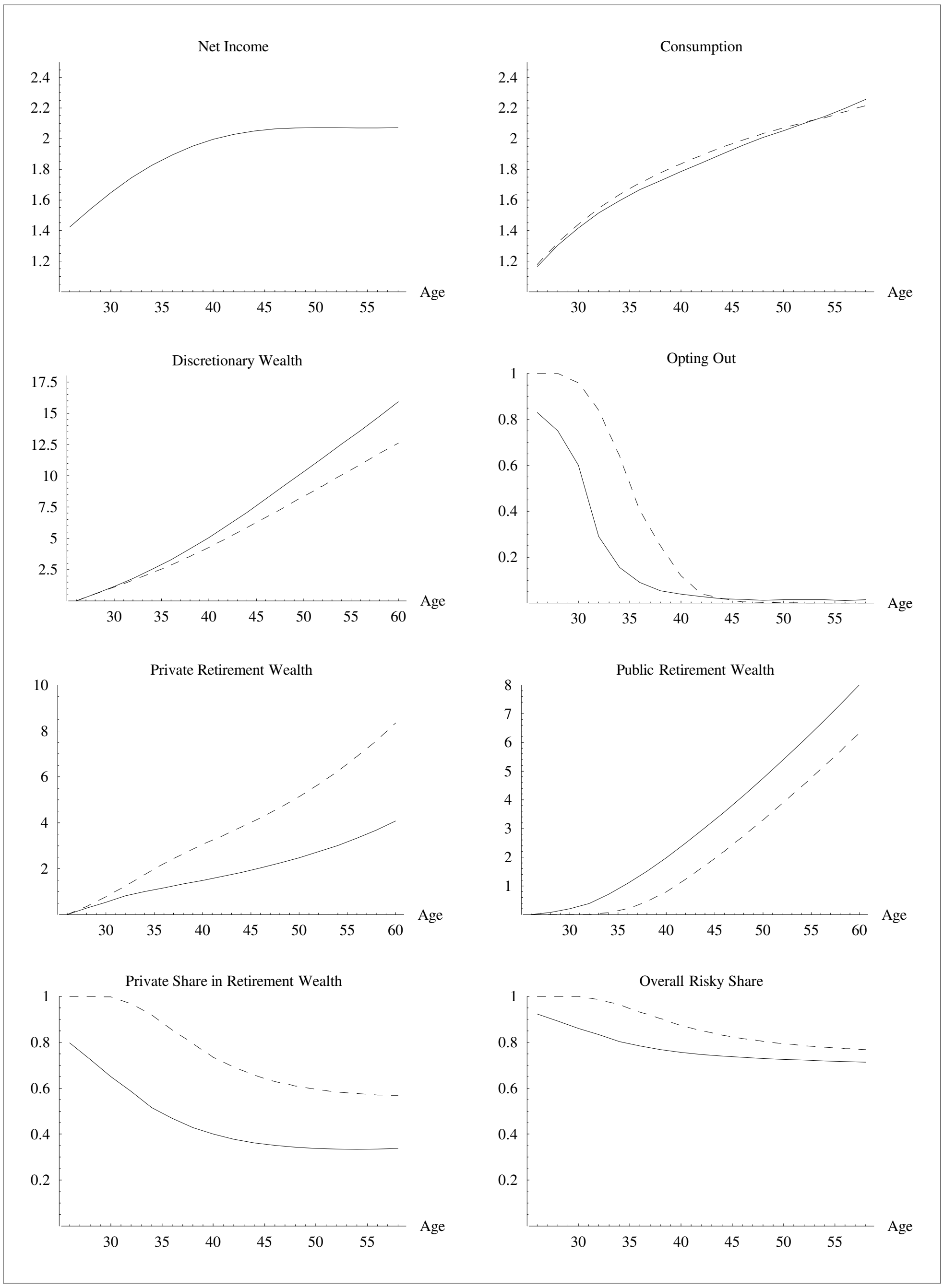
Solid Line: Mean values for 10,000 agents plotted against age. Dashed Line: benchmark calibration.

Figure 4: Different Education Groups



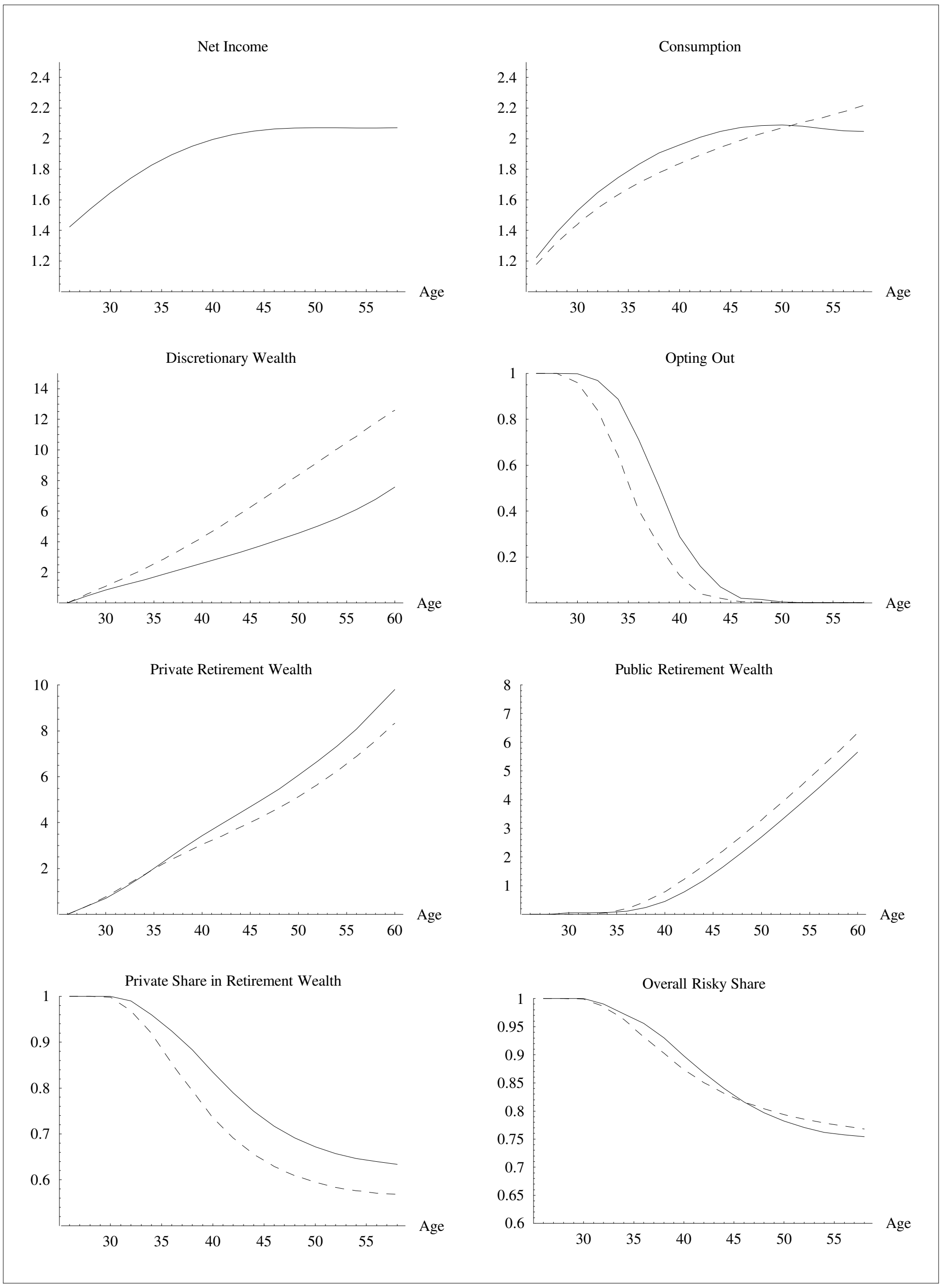
Solid Line: Mean values for 10,000 College Degree agents plotted against age. Dashed Line: benchmark calibration (High School Degree). Short Dashed Line: Mean values for 10,000 No High School Degree agents plotted against age

Figure 5: Income/Stock return correlation



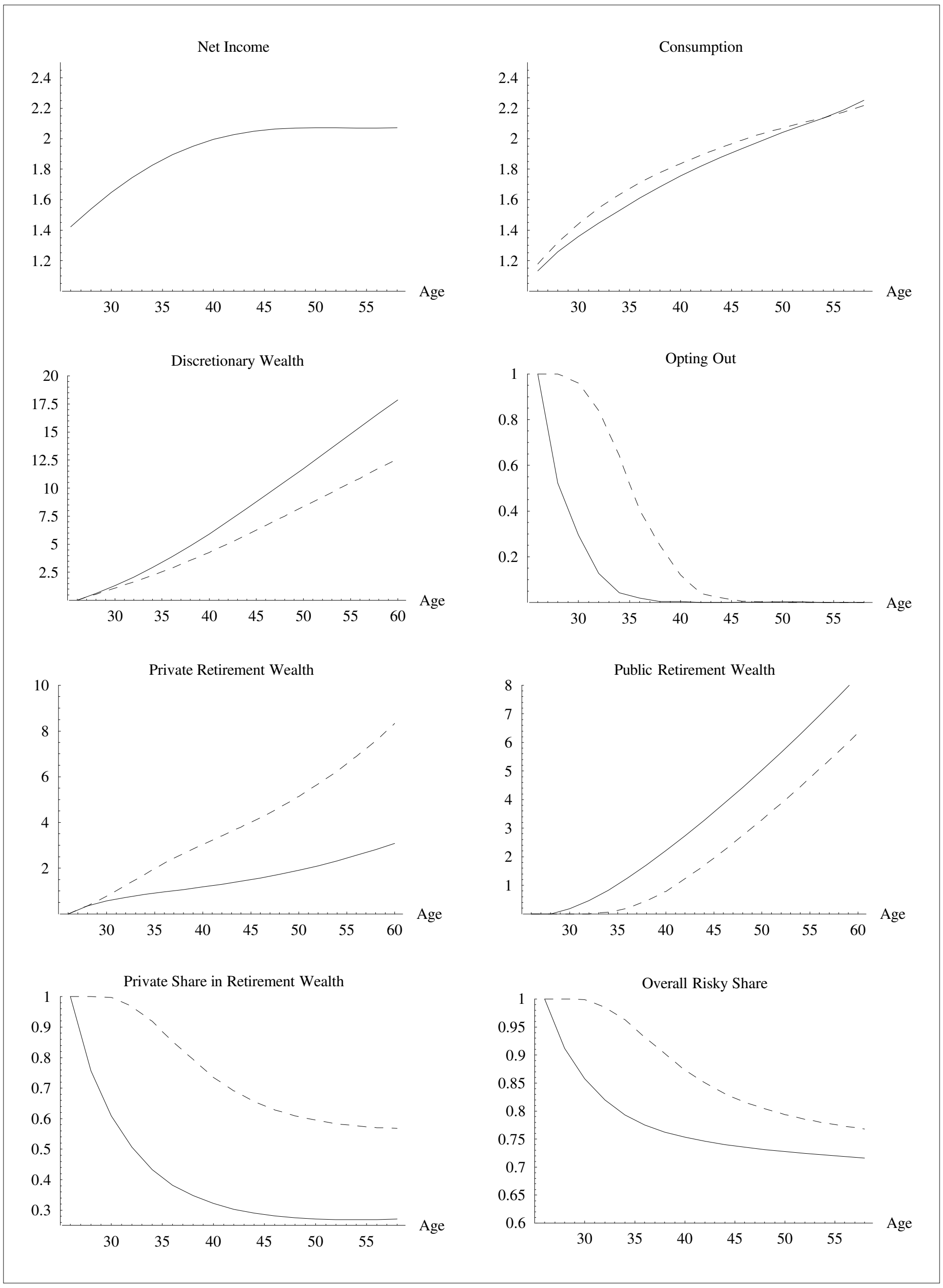
Solid Line : Mean values for 10,000 agents plotted against age. All parameters as in the benchmark , but the correlation coefficient between innovation in Ar(1) component of labor income and return on risky asset which is set to .3707 . Dashed Line : benchmark calibration .

Figure 6: Higher Psychological Discount Rate



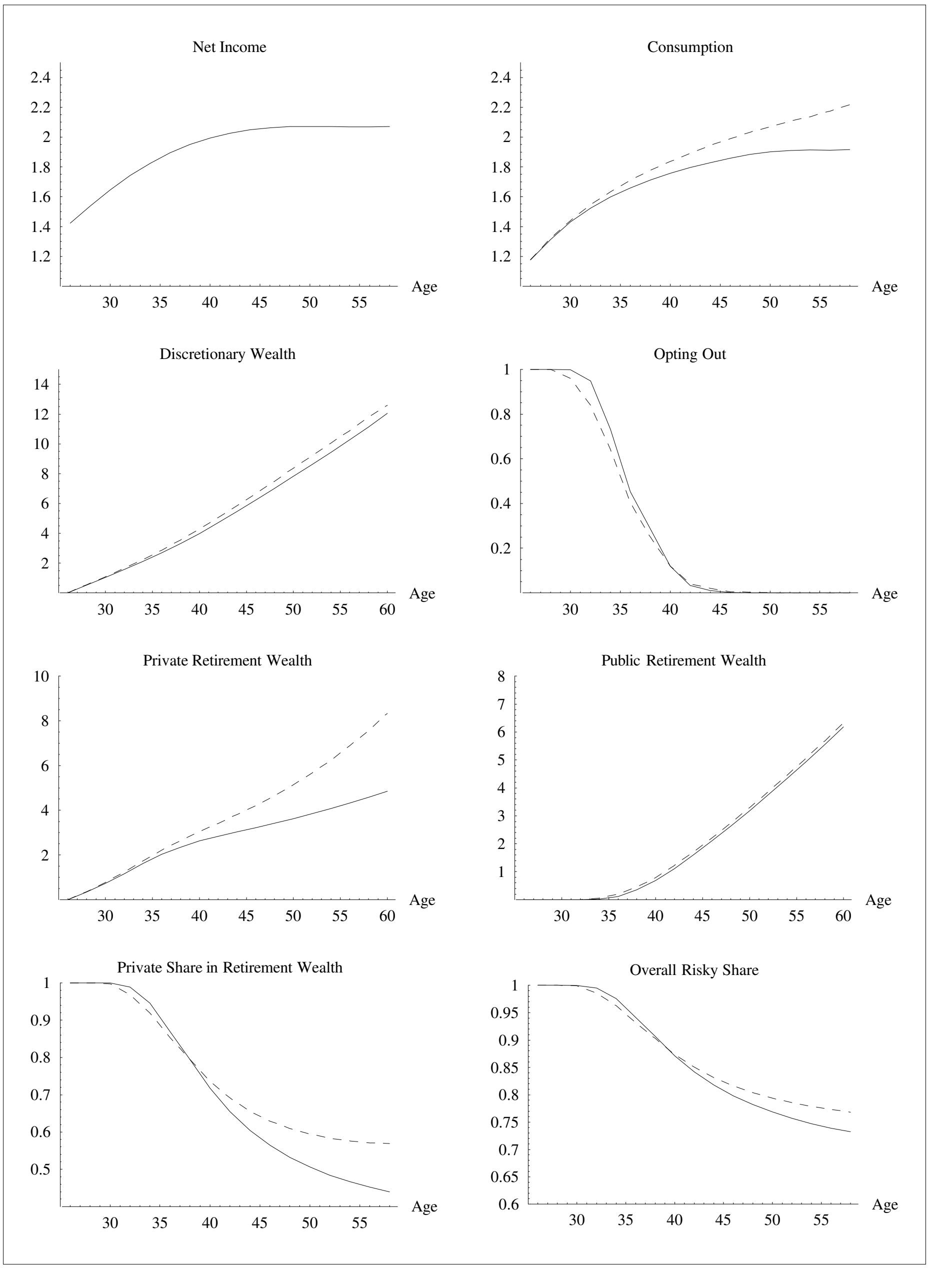
Solid Line: Mean values for 10,000 agents plotted against age. All parameters as in the Notional Defined Contribution Baseline Calibration, except for psychological discount rate set to 0.9^ 1. Dashed Line: benchmark calibration.

Figure 7: Higher Risk Aversion



Solid Line : Mean values for 10,000 agents plotted against age. All parameters as in the Notional Defined Contribution Baseline Calibration , except for degree of relative risk aversion set to 6. Dashed Line : benchmark calibration .

Figure 8: Mixed Portfolio



Solid Line : Mean values for 10,000 agents plotted against age. All parameters as in the Notional Defined Contribution Baseline Calibration , except for rate of return on funded wealth which is set to 1.03 with .1 standard deviation . Dashed Line : benchmark calibration .