Pension funds performance evaluation: a utility based approach*

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Abstract

This paper uses a life cycle framework to derive a benchmark to evaluate pension funds. We present a model for optimal asset allocation, where the agents' labor income process is calibrated to capture a realistic pattern and the available financial assets include one riskless and two risky assets, with returns potentially correlated with labor income shocks. The optimal asset allocation is the benchmark for pension funds performance. Also, the welfare costs associated with the adoption of simple sub-optimal strategies ("age rule" and "$1/N$ rule") are computed, and a new welfare-based metric for pension fund evaluation are disussed.

Keywords: Pension funds, life-cycle portfolio choice, investor heterogeneity, performance evaluation

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1 Introduction

Methods for evaluating the performance of defined contribution (DC) pension funds are similar to those applied to mutual funds, and typically associate a higher return per unit of risk with better performance. These methods are adequate if a worker, or the pension fund acting on her behalf, has preferences defined exclusively over the mean and the variance of portfolio returns[1]. Ideally, though, a worker contributes to a pension fund in order to help stabilize consumption during retirement years, given that the yearly pension transfer granted by first-pillar schemes is lower than the last wage. Thus, the optimal asset allocation for a pension fund ought to take into account, along with the asset return distributions and the risk aversion parameter that enter a standard portfolio choice problem, both any pension transfer accruing after retirement as well as the worker's life expectancy. Since the pension transfer is usually a fraction of labor income earned during the last working year which in turn derives from the worker's professional history, the optimal asset allocation trades off the gains from investing in high risk premium assets with the need to hedge labor income shocks.

This paper proposes a method for evaluating the ability of delegated pension funds in performing such function, by tailoring asset allocation to the labor income shocks of its representative member, also taking in due count the characteristics of the first pillar. To this end, we discuss how the optimal asset allocation delivered by a life-cycle model - built on Campbell, Cocco, Gomes and Maenhout (2001), Cocco, Gomes and Maenhout (2005) and Gomes and Michaelides (2004, 2005) - can become a benchmark asset allocation for pension funds. And we put forward a welfare-based metric in order to evaluate the DC fund performance relative to this new benchmark. Thus, we depart from the performance evaluation literature that measures whether active portfolio strategies obtain higher return-to-risk relative to passive efficient benchmarks, assuming that investors implement the welfare-maximizing portfolio strategies supporting that benchmark. Here, we investigate whether pension funds implement or depart from an optimal passive portfolio strategy on behalf of its members.

Our approach to performance evaluation derives from the literature on strategic asset allocation, indicating that optimal portfolios for long-term investors may not be the same as for short-term investors - because of a different judgement of assets' riskiness which depends on the ratio of discounted expected future labor income (i.e. human wealth) to accumulated financial

[1] The investor may also have more elaborate preferences that, combined with investment opportunities, reduce to mean variance preferences.

wealth. This ratio changes over the investor's life in a way that simple assumptions on the stochastic process generating labor income are not capable to capture. Instead, a life-cycle model allows for a more realistic age profile of labor income, making human wealth increase relative to financial wealth in the early part of the working life to reach a peak, and then decline in the years towards retirement. Our life-cycle model features two risky and one riskless assets, which are parameterized by the first two moments of their return distribution, and correspond in our simulations to domestic stocks, bonds and bills. As in Bodie, Merton and Samuelson (1992) and Cocco, Gomes and Maenhout (2005), early in the worker's life the average asset allocation is tilted towards the high risk premium asset, because labor income provides an effective hedge against financial risks. On the contrary, in the two decades before retirement, it gradually shifts to less risky assets, because income profiles peak at around age 45.

We perform sensitivity analysis along several important dimensions. The first examines the reaction of optimal asset allocation to the labor income profile. For instance, a construction worker may face a higher variance of uninsurable labor income shocks than a teacher (Campbell, Cocco, Gomes and Menhout 2001); alternatively, the correlation between stock returns and labor income may be higher for a self-employed or a manager than for a public sector employee. If such differences had negligible effects on optimal asset allocation, the pension plan would offer the same option to all participants and save on management fees. Instead, in our simulations optimal portfolio shares are highly heterogeneous across coeval agents (despite their common life expectancy, retirement age and replacement ratios) due to such individual-specific labor income shocks. Dispersion decreases as workers approach retirement, the more so the higher is the labor income-stock return correlation: as this increases, the histories of labor incomes tend to converge over time and so do the optimal associated portfolio choices. These results suggest that the optimal allocation ought to be implemented through diversified investment options for most occupations and age brackets.

The pension transfer in our model is a fixed annuity (granted by an unmodelled first pillar or defined-benefit -DB- scheme),[2] proportional to labor income in the last working year. Replacement ratios vary widely across countries, as documented by OECD (2007), ranging from 34.4% in UK to 95.7% in Greece. Such differences also depend on the inflation coverage of pension annuities, which is often imperfect, implying a reduced average replacement ratio. By measuring the sensitivity of optimal portfolio composition with

[2]Koijen, Nijman and Werker (2006) argue that a fixed annuity is suboptimal relative to alternative annuity designs, despite its diffusion across pension systems.

respect to the replacement ratio, we understand whether optimal pension fund portfolio policies should vary across countries for given members' types. When the replacement ratio falls, simulations reveal that agents save more during their working life in anticipation of lower pension incomes, thus accumulating a higher level of financial wealth. This determines a lower optimal share of stocks at all ages and for all values of the labor income-stock return correlation, holding risk aversion fixed: with higher financial wealth, a given labor income becomes less apt to offset bad financial outcomes. In other words, our model indicates that asset allocation in low replacement ratio countries ought to be more conservative because workers' contributions to pension funds ought to be higher.

Computing the optimal life-cycle asset allocation allows us to use it as a performance evaluation benchmark, which explicitly accounts for the pension plan's role in smoothing participants' consumption risk. We then propose an indicator of pension funds' performance. This is the ratio of the worker's ex-ante maximum welfare under the optimal asset allocation to her welfare under the pension fund actual return distribution, which implies a different optimal consumption-savings path for the plan member. The higher the value of the ratio, the worse the pension fund performance. Importantly, worse performance may derive not only from a lower return per unit of financial risk earned by the pension fund manager - which is what previous methods look at - but also from a worse matching between the pension fund portfolio and its members' labor income and pension risks.

Since the seminal work of Brinson, Hood and Beebower (1986), it is customary to focus on the extra-return earned by portfolio managers, associating it to ability in selecting securities within each asset class, in timing the market and in choosing the asset allocation. Later studies of pension funds performance evaluation focus on market timing and security selection of delegated portfolio managers rather than on what we are interested in, namely the pension fund asset allocation. Borrowing from the mutual funds literature, they examine whether managers obtain higher performance with respect to passive investment strategies, after adjusting fund return for exposure to systematic risk. Many studies assess performance against a single factor benchmark, such as the S&P 500 (Ippolito and Turner 1987; Lakonishok, Shleifer and Vishny 1992) or multifactor benchmarks and style indices (Coggin, Fabozzi and Rahaman 1993; Busse, Goyal and Wahal 2008; Bauer and Frehen 2008).[3].

[3]Elton, Gruber and Blake (2006) examine the adequacy of the funds offered in 401(k) plans by testing whether they span the same return-risk possibilities that can be achieved out of other available funds.

Blake, Lehmann and Timmermann (1999) focus on longer term performance, but abstract from life-cycle considerations. They adopt the decomposition of Brinson et al. (1986) and identify the strategic asset allocation over different asset classes with the average allocation across pension funds. In turn, individual pension funds deviations from the average are considered as being driven by market timing and security selection bets. Interestingly, they find - like Brinson et al. - that the bulk of managed portfolio returns over the long run is attributable to asset allocation, because individual managers tend to deviate very little from the average. However, they do not evaluate the properties of such asset allocation against an alternative one, which is what we focus on.

To our knowledge, Antolin (2008) represents a unique attempt to evaluate the asset allocation chosen by pension plans. He constructs mean-variance efficient benchmarks with the available asset classes and compares their Sharpe ratio with that of pension funds. Our benchmark is not mean-variance efficient, as it trades off the benefits from maximizing return given risk with the benefits from hedging labor income shocks.

In all previous studies the metric for evaluating the performance of managed portfolios is return-based, including Jensen alpha, the Treynor and Mazuy alpha, the Battacharya and Pfleiderer measure and the Sharpe ratio. In our work, we do not aim at assessing the managerial skills in beating the market return, or the average pension fund return, or any other benchmark return. Rather, we are interested in the manager ability to help the pension member in hedging labor income - and the associated retirement income - risk. This is why we propose a welfare-based metric - like both Samwick and Skinner (2004) and Poterba, Rauh, Venti and Wise (2007). Their focus is on pension fund design rather than asset allocation, as they compare the life-time expected utility obtained by workers under an existing DB scheme against a newly introduced DC second pillar. Thus, while their benchmark is an existing DB plan, we adopt a life-cycle model to derive a benchmark strategic asset allocation against which to evaluate the performance of pension funds.

In principle return-based performance evaluation is appropriate also if the worker's preferences are defined over consumption and there are non-traded assets, as in our life-cycle model. An important requirement is that the benchmark portfolio must be the optimal portfolio for hedging fluctuations in the intertemporal marginal rates of substitution (MRS) of any marginal investor. If markets are complete, the MRS is captured by aggregate shocks and idiosyncratic risk does not affect risk prices or individual consumption. Even with incomplete consumption insurance and heterogeneous consumers equilibrium results may be similar to the complete-markets framework (Lucas

1991, and Telmer 1993). This is not always the case, however. Constantinides and Duffie (1996) argue that the MRS is also affected by the variance of the cross-sectional distribution of individual consumption growth, in the presence of incomplete consumption insurance and consumer heterogeneity. In a life-cycle setting with capital accumulation (Storesletten, Telmer and Yaron 2007) the distribution of employed and retired agents across the population affects the MRS. In practice, benchmarks in portfolio evaluation typically reflect the state of empirical asset pricing and constraints on available data (Lehmann and Timmermann, 2008), which makes pursuing alternative approaches worthwhile.

Unmodelled costs of tailoring portfolios to age, labor income risk and other worker-specific characteristics can be quite high for pension funds. This is why we assess the welfare costs of implementing two simpler strategies. The first is an "age rule", where the portfolio share allocated to risky assets decays deterministically with the worker's age. The second strategy is an equally-weighted portfolio of the three financial assets, which echoes the "$1/N$ rule" of DeMiguel, Garlappi and Uppal (2008) that outperforms several portfolio strategies in *ex post* portfolio experiments. The latter strategy performs consistently better than the "age rule", making it a better benchmark for evaluating the performance of pension funds when management costs are sufficiently high. Importantly, our numerical results suggest that this portfolio strategy is likely to be cost-efficient for both high-wealth and highly-risk-averse-average-wealth workers in medium-to-high replacement ratios countries. In these cases, the welfare costs of the suboptimal 1/3 rule are often lower than 50 basis points per annum in terms of welfare-equivalent consumption, which is likely to be lower than the management cost differential. Thus, 1/3 may well become the benchmark asset allocation for performance evaluation.

The rest of the paper is organized as follows. Section 2 presents a simple operative life-cycle model, and Section 3 shows how it can be calibrated to deliver quantitative predictions on optimal portfolio allocation. The welfare metric for pension funds' performance evaluation is discussed in Section 4. A final section summarizes the main conclusions.

2 The life-cycle model

We model an investor that maximizes the expected discounted utility of consumption over her entire life. Though the maximum length of the life span is T periods, its effective lenght is governed by age-dependent life expectancy. At each date t, the survival probability of being alive at date $t+1$ is p_t, the conditional survival probability at t. The investor starts working at age t_0 and retires with certainty at age $t_0 + K$. Investor's i preferences at date t are described by a time-separable power utility function:

$$\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0}\left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=0}^{j-1} p_{t_0+k}\right) \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma}\right]$$

where C_{it} is the level of consumption at time t, $\beta < 1$ is an utility discount factor, and γ is the constant relative risk aversion parameter.[4] We rule out utility derived from leaving a bequest, introduced by Cocco, Gomes and Maenhout (2005). Moreover, we do not model labor supply decisions, whereby ignoring the insurance property of flexible work effort (allowing investors to compensate for bad financial returns with higher labor income), as in Gomes, Kotlikoff and Viceira (2008).

2.1 Labor and retirement income

Available resources to finance consumption over the life cycle derive from accumulated financial wealth and from the stream of labor income. At each date t during the working life, the exogenous labor income Y_{it} is assumed to be governed by a deterministic age-dependent growth process $f(t, \mathbf{Z}_{it})$, and is hitted by both a permanent u_{it} and a transitory n_{it} shock, the latter being uncorrelated across investors. Formally, the logarithm of Y_{it} is represented by

$$\log Y_{it} = f(t, \mathbf{Z}_{it}) + u_{it} + n_{it} \qquad t_0 \leq t \leq t_0 + K \tag{1}$$

More specifically, $f(t, \mathbf{Z}_{it})$ denotes the deterministic trend component of permanent income, which depends on age t and on a vector of individual characteristics $\mathbf{Z}_{it}$, such as gender, marital status, houseohld composition and education. Uncertainty of labor income is captured by the two stochastic processes, u_{it} and n_{it}, driving the permanent and the transitory component respectively.[5] Consistently with the available empirical evidence, the perma-

[4] Assuming power utility with relative risk aversion coefficient γ constrains the intertemporal elasticity of substitution to be equal to $1/\gamma$. Moreover, γ also governs the degree of relative "prudence" of the consumer RP, related to the curvature of her marginal utility and measured by

$$RP = -\frac{CU'''(C)}{U''(C)} = 1 + \gamma$$

Relative prudence is a key determinant of the consumer's optimal reaction to changes in the degree of income uncertainty.

[5] The permanent-transitory model is the simplest model of the earnings structure. We could consider more realistic processes, such as the one with autocorrelated transitory disturbances which proves to be a good characterization of the earnings process in the US.

nent disturbance is assumed to follow a random walk process:

$$u_{it} = u_{it-1} + \varepsilon_{it} \tag{2}$$

where ε_{it} is distributed as $N(0, \sigma_\varepsilon^2)$ and is uncorrelated with the idiosyncratic temporary shock n_{it}, distributed as $N(0, \sigma_n^2)$. Finally, the permanent disturbance ε_{it} is made up of an aggregate component, common to all investors, $\xi_t \sim N(0, \sigma_\xi^2)$, and an idiosyncratic component $\omega_{it} \sim N(0, \sigma_\omega^2)$ uncorrelated across investors:

$$\varepsilon_{it} = \xi_t + \omega_{it} \tag{3}$$

As specified below, we allow for correlation between the aggregate permanent shock to labor income ξ_t and innovations to the risky asset returns.

During retirement, income is certain and equal to a fixed proportion λ of the permanent component of the last working year income:

$$\log Y_{it} = \log \lambda + f\left(t_{0+K}, \mathbf{Z}_{it_{0+K}}\right) + u_{it_{0+K}} \qquad t_0 + K < t \leq T \tag{4}$$

where the level of the replacement ratio λ is meant to capture at least some of the features of Social Security systems. Other, less restrictive, modelling strategies are possible. For example, Campbell, Cocco, Gomes and Maenhout (2001) model a system of mandatory saving for retirement as a given fraction of the (stochastic) labor income that the investor must save for retirement and invest in the riskless asset, with no possibility of consuming it or borrowing against it. At retirement, the value of the wealth so accumulated is transformed into a riskless annuity until death.

2.2 Investment opportunities

We allow savings to be invested in a short-term riskless asset, yielding each period a constant gross real return R^f, and in two risky assets, called *stocks* and *bonds*. The risky assets yield stochastic gross real returns R_t^s and R_t^b respectively. We maintain that the investment opportunities in the risky assets do not vary over time and model excess returns of stocks and bonds over the riskless asset as

$$R_t^s - R^f = \mu^s + \epsilon_t^s \tag{5}$$
$$R_t^b - R^f = \mu^b + \epsilon_t^b \tag{6}$$

where μ^s and μ^b are the expected stock and bond premia, and ϵ_t^s and ϵ_t^b are normally distributed innovations, with mean zero and variances σ_s^2 and σ_b^2 respectively. We allow for the two disturbances to be correlated, with correlation ρ_{sb}. Moreover, we let the innovation on the stock return be correlated with the aggregate permanent disturbance to the labor income, and denote this correlation by ρ_{sY}. We do not allow for excess return predictability and other forms of changing investment opportunities over time, as in Michaelides (2002) and Koijen, Nijman and Werker (2008). While both papers document market timing effects on asset allocations when parameters of the return distributions are known with certainty, there is still considerable debate as to the ex-post value of market timing (De Miguel, Garlappi and Uppal 2008) and return predictability in general (Goyal and Welch 2008; Fugazza, Guidolin and Nicodano 2008) when such parameters are estimated by an asset manager.

At the beginning of each period, financial resources available for consumption and saving are given by the sum of accumulated financial wealth W_{it} plus current labor income Y_{it}, that we call *cash on hand* $X_{it} = W_{it} + Y_{it}$. Given the chosen level of current consumption, C_{it}, next period cash on hand is given by:

$$X_{it+1} = (X_{it} - C_{it})R_{it}^{P} + Y_{it+1} \tag{7}$$

where R_{it}^{P} is the portfolio return

$$R_{it}^{P} = \alpha_{it}^{s} R_{t}^{s} + \alpha_{it}^{b} R_{t}^{b} + \left(1 - \alpha_{it}^{s} - \alpha_{it}^{b}\right) R^{f} \tag{8}$$

with α_{it}^{s}, α_{it}^{b} and $\left(1 - \alpha_{it}^{s} - \alpha_{it}^{b}\right)$ denoting the shares of the investor's portfolio invested in stocks, bonds and in the riskless asset respectively. We do not allow for short sales and assume that the investor is liquidity constrained, so that the nominal amount invested in each of then three financial assets are $F_{it} \geq 0$, $S_{it} \geq 0$ and $B_{it} \geq 0$ respectively for the riskless asset, stocks and bonds, and the portfolio shares are non negative in each period.

The focus of this paper is on optimal asset allocation and savings until retirement, which however also depend on investment opportunities during the retirement period. The simulations presented below concern the case when the pension fund continues to optimally invest the retiree's savings into the same three assets. However, the results concerning asset allocation appear to be qualitatively similar in (unreported) simulations based on the assumption that retirees invest in the riskless asset only.

2.3 Solving the life cycle problem

In this standard intertemporal optimization framework, the investor maximizes the expected discounted utility over life time, by choosing the consumption and portfolio rules given uncertain labor income and asset returns.

Formally, the optimization problem is written as:

$$\max_{\{C_{it}\}_{t_0}^{T-1},\left\{\alpha_{it}^s,\alpha_{it}^b\right\}_{t_0}^{T-1}} \left(\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0} \left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=0}^{j-1} p_{t_0+k} \right) \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} \right] \right) \tag{9}$$

$$s.t. \quad X_{it+1} = (X_{it} - C_{it}) \left(\alpha_{it}^s R_t^s + \alpha_{it}^b R_t^b + \left(1 - \alpha_{it}^s - \alpha_{it}^b\right) R^f \right) + Y_{it+1}$$

with the labor income and retirement processes specified above and short sales and borrowing constraints.

Given its intertemporal nature, the problem can be restated in a recursive form, rewriting the value of the optimization problem at the beginning ot period t as a function of the maximized current utility and of the value of the problem at $t+1$ (Bellman equation):

$$V_{it}\left(X_{it,}u_{it}\right) = \max_{\{C_{it}\}_{t_0}^{T-1},\left\{\alpha_{it}^s,\alpha_{it}^b\right\}_{t_0}^{T-1}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta p_t E_t \left[V_{it+1}\left(X_{it_{+1},}u_{it+1} \right) \right] \right) \tag{10}$$

At each time t the value fucntion V_{it} describes the maximized value of the problem as a function of the two state variables: the level of cash on hand at the beginning of time t (X_{it}), and the level of the stochastic permanent component of income at beginning of $t.(u_{it}$).

In order to reduce the dimensionality of the original problem, obtaining a problem with one state variable, we exploit the homogeneity of degree $(1-\gamma)$ of the utility function, and normalize the entire problem by the permanent component of income u_{it}. Thus, we can rewrite (10) as

$$V_{it}\left(X_{it}\right) = \max_{\{C_{it}\}_{t_0}^{T-1},\left\{\alpha_{it}^s,\alpha_{it}^b\right\}_{t_0}^{T-1}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta p_t E_t \left[V_{it+1}\left(X_{it+1}\right) \right] \right) \tag{11}$$

This problem has no closed form solution, hence the optimal values for consumption and portfolio allocation at each point in time have to be derived numerically. To this aim, we apply a backward induction procedure and obtain optimal consumption and portfolio rules in terms of the state variable starting form the last (possible) period of life T.

In particular, the solution for period T is trivial, considering that, as we do not allow for positive bequest, it is optimal to consume all the available resources (i.e., $C_{iT} = X_{iT}$) implying that

$$V_{iT}\left(X_T\right) = \frac{X_{iT}^{1-\gamma}}{1-\gamma} \tag{12}$$

The value function at T coincides with the direct utility function over the cash on hand available at the beginning of the period. Then, going backwards, for every period, $t = T-1, T-2, ..., t_0$, and for each possible value of the state variable (the initial level of cash on hand at t) the optimal rules for consumption and the assets' portfolio shares are obtained from the Bellman equation (10) using the grid search method.[6] From the Bellman equation, for each level of the state variable X_{it}, the value function at the beginning of time t, $V_{it}(X_{it})$, is obtained by picking the level of consumption and of portfolio shares that maximizes the sum of the utility from current consumption $U(C_{it})$ plus the discounted expected value from continuation, $\beta p_t E_t V_{it+1}(X_{it+1})$. The latter value is computed using $V_{it+1}(X_{it+1})$ obtained from the previous iteration. In particular, given $V_{it+1}(X_{it+1})$, the expectation term is evaluated in two steps. We use numerical integration perfomed by means of the standard Gaussian Hermite quadrature method to approximate the distribution of shocks to labor income and asset returns. Then, cubic spline interpolation is employed to evaluate the value function at points that do not lie on the state space grid.

3 Simulation results

The numerical solution method briefly outlined above yields, for each set of parameters chosen, the optimal policy functions for the level of consumption and the shares of the financial portfolio invested in the riskless asset, stocks and bonds as functions of the level of cash on hand. Using those optimal rules, it is then possible to simulate the life-cycle consumption and asset allocation choices of a large number of agents. In this section, we describe results obtained from this procedure, focussing first on a benchmark case and then presenting extensions along various dimensions.

3.1 Calibration

Parameter calibration concerns the investor's preferences, the features of the labor income process during working life and retirement, and the moments of the risky asset returns. To obtain results for a benchmark case, we chose plausible sets of parameters referred to the US and based on Cocco, Gomes and Maenhout (2005) and Gomes and Michaelides (2004, 2005).

[6] According to this method, the problem is solved over a grid of values covering the space of the state variables and the controls, to ensure that the solution found is a global optimum.

The investor begins her working life at the age of 20 and works for (a maximum of) 45 periods (K) before retiring at the age of 65. After retirement, she can live for a maximum of 35 periods until the age of 100. In each period, we take the conditional probability of being alive in the next period p_t from the life expectancy tables of the US *National Center for Health Statistics*. As regards to preferences, we set the utility discount factor $\beta = 0.96$, and the coefficient of relative risk aversion $\gamma = 5$ (capturing an intermediate degree of risk aversion).

The labor income process is calibrated using the estimated parameters for US households with high-school education (but not a college degree) in Cocco, Gomes and Maenhout (2005). The age-dependent trend is captured by a third-order polynomial in age, delivering the typical hump-shaped profile until retirement depicted as the dash-dotted line in Figure 1. After retirement income is a constant proportion λ of the final (permanent) labor income, with $\lambda = 0.68$. The continuous line in the figure protrays the whole deterministic trend $f(t, \mathbf{Z}_{it})$, used in the simulations below, that allows also for other personal characteristics. In the benchmark case, the variances of the permanent and transitory shocks (ε_{it} and n_{it} respectively) are $\sigma_\varepsilon^2 = 0.0106$ and $\sigma_n^2 = 0.0738$; in some of the extensions below we let those parameters vary (to explore the effects of increasing labor income uncertainty) but keep the permanent-transitory ratio roughly constant at the 0.14 level. The riskless (constant) interest rate is set at 0.02, with expected stock and bond premia μ^s and μ^b fixed at 0.04 and 0.02 respectively. The standard deviations of the returns innovations are set at $\sigma_s = 0.157$ and $\sigma_b = 0.08$; in the benchmark case, we fix their correlation at a positive but relatively small value: $\rho_{sb} = 0.2$, a value calibrated on the historical annual correlation in the US and close to the choice of Gomes and Michaelides (2004). Finally, we set $\rho_{sY} = 0$ in the benchmark case, imposing a zero correlation between stock return innovations and labor income disturbances.

3.2 Benchmark results

In all simulations we took cross-sectional averages of 10,000 agents over their life cycle. Figure 2 displays the simulation results for the pattern of consumption, labor income and accumulated financial wealth for the working life and the retirement period in the benchmark case. The typical life-cycle profile for consumption is generated. Binding liquidity constraints make consumption closely track labor income until the 35-40 age range, when the consumption path becomes less steep and financial wealth is accumulated at a faster rate. After retirement at 65, wealth is gradually decumulated and consumption decreases to converge to retirement income in the last possible period of life.

Before presenting the age profile of optimal portfolio shares, Figures 3 and 4 display the optimal policy rules for the risky asset shares α_{it}^{s} and α_{it}^{b} as functions of the level of (normalized) cash on hand (the problem's state variable); in each figure the optimal fraction of the portfolio invested in stocks and bonds is plotted against cash on hand for investors of four different ages (20, 30, 55 and 75). The basic intuition that should guide the interpretation of those optimal policies, on which the following simulation results are based, is that labor income is viewed by the investor as an implicit holding of an asset. Although in our setting labor income is uncertain (its process being hit by both permanent and transitory shocks), as long as the correlation of asset returns' innovations and labor income disturbances is not too large, labor income is more similar to the risk-free than to the risky assets;[7] therefore, when the present discounted value of the expected future labor income stream (i.e. human wealth) constitutes a sizeable portion of overall wealth, the investor is induced to tilt her portfolio towards the risky assets. The proportion of human out of total wealth is widely different across investors of different age and is one of the main determinants of their chosen portfolio composition.

Looking at Figures 3 and 4, in the case of an investor of age 75, the certain retirement income acts as a holding of the riskless asset and the relatively poor investors (with a small amount of accumulated wealth and current income) will hold a financial portfolio entirely invested in stocks.[8] Wealthier investors will hold a lower portfolio share in stocks (and increase their holdings of bonds), since for them the proportion of the overall wealth implicitly invested in the riskless asset (i.e. human wealth) is lower. At age 55, the investor still has a decade of relatively high expected labor income before retirement, and she will tend to balance this implicit holding of a low-risk asset with a financial portfolio more heavily invested in risky stocks than older investors: her optimal policies in Figures 3 and 4 are shifted outwards with respect to the 75-year-old investor for all levels of cash on hand.[9] The same intuition applies to earlier ages, for which the optimal stock and bond policies shift gradually outwards as younger investors are considered. The only exception to this pattern occurs for the very young investors (approximately in the 20-25 age range), for whom the labor income

[7] We recall here that in our benchmark case, there is a zero correlation between stock return and labor income innovations: $\rho_{sY} = 0$.

[8] The portfolio shares of the risky assets are not defined for extremely low values of cash on hand since the investor (of any age) has no savings in this case.

[9] The step-wise appearence of the policy rules is due to the choice of the grid in the numerical solution procedure. The use of a finer grid would deliver smoother policies, at the cost of additional computing time.

profile is increasing very steeply, making it optimal to hold portfolios more invested in stocks (in the figures, the policy functions shift outward in the 20-25 age range).

On the basis of such optimal investment policies, the mean portfolio shares of stocks and bonds across 10,000 agents have been obtained by simulation and plotted in Figure 5(a) against age. The age profiles for stock and bonds are mainly determined by the fact that over the life cycle the proportion of overall wealth implicitly invested in the riskless asset through expected labor incomes varies, being large for young investors and declining as retirement approaches. In fact, younger agents invest their entire portfolio in stocks until approximately the age of 40. Middle-age investors (between 40 and the retirement age of 65) gradually shift the composition of their portfolio away from stocks and into bonds, to reach shares of 60% and 40% respectively at the retirement date. After retirement, income becomes certain and the proportion of implicit holdings of the safe asset increases again; moreover, previously accumulated financial wealth is run down quickly to support a relatively stable consumption level (see Figure 2). Consequently, the share of stocks starts increasing, at the expense of bonds, to compensate for it. Throughout, the holdings of the riskless asset are kept at a minimum (very often zero); only very young investors keep a small fraction of their portfolio in the riskless asset.

Overall, the popular financial advice of holding a portfolio share of risky stocks equal to 100 minus the investor's age (so that $\alpha^s_{age} = (100-\text{ age})/100$), implying a gradual shift toward bonds over life, is not completely at variance with optimally designed investment policies. However, in the benchmark case above the decumulation of stocks is not linear (as suggested by the simple age-dependent rule, according to which the stock share should be run down from 80% at the age of 20 to reach 35% at retirement). A more rigorous comparison of the optimal investment policy with the simple "age rule" will be provided below.

3.3 The sensitivity of optimal asset allocation to labor income risk

To evaluate the robustness of the above results, and to explore the sensitivity of optimal asset allocation to changes in the main parameters of the model, the benchmark case can be modified along a number of dimensions, including varying degrees of risk aversion, different shapes of the labor income process, and different assumptions on the moments of the asset returns' distributions. In this subsection, we focus on two important dimensions (and their interactions), concerning the correlation between stock return innovations and the shock to labor income (ρ_{sY}), set to zero in the benchmark case, and the variances of the permanent and transitory disturbances driving (the stochastic part of) labor income ($\sigma_{\varepsilon}^2 = 0.0106$ and $\sigma_n^2 = 0.0738$ in the benchmark case), to capture changes in labor income uncertainty. Figure 5 displays the mean share age profiles for stocks and bonds over the accumulation period, ranging from the beginning of the working life at the age of 20 to the age of 99.

First, we let the stock return innovations be positively correlated with the innovations in labor income. Empirical estimates of this correlation for the US include values not significantly different from zero as in Cocco, Gomes and Michaelides (2005) for households with any level of educational attainment, and the relatively high values reported by Campbell, Cocco, Gomes and Maenhout (2001) and Campbell and Viceira (2002), ranging from 0.33 for households with no high-school education to 0.52 for college graduates. Since our calibration of the labor income process reflects the features of households with high-school education, we choose an intermediate value of $\rho_{sY} = 0.4$, close to the value of 0.37 used by Campbell and Viceira (2002). Figure 5(b) displays the optimal portfolio shares of stocks and bonds when $\rho_{sY} = 0.4$. The general pattern of asset allocation obtained in the benchmark case (Figure 5(a)) is confirmed for middle-aged workers, whereas for younger workers (in the 20-40 age range) optimal portfolio shares differ sharply. In fact, the positive correlation between labor income shocks and stock returns makes labor income closer to an implicit holding of stocks rather than of a riskless asset. Younger investors, for whom human capital is a substantial fraction of overall wealth, are therefore heavily exposed to stock market risk and will find it optimal to offset such risk by holding a relatively lower fraction of their financial portfolio in stocks. This effect decreases as workers get older, determining a gradual increase in the portfolio share of stocks until around the age of 40. Finally, as the retirement age approaches, the size of human capital decreases and the investor shifts her portfolio composition again towards safer bonds; this yields a hump-shaped profile for the optimal share of stocks during working life.

The effects of increasing labor income risk on optimal asset allocation over the working life are portrayed in Figures 5(c) and 5(d). In both sets of simulations we increase the variance of both the permanent and the transitory stochastic components of the labor income process, setting now $\sigma_{\varepsilon}^2 = 0.0408$ and $\sigma_n^2 = 0.269$, keeping their ratio approximately equal to that used in the benchmark case. Panel (c) plots the results for $\rho_{sY} = 0$ as in the benchmark case, whereas panel (d) shows optimal portfolio shares when $\rho_{sY} = 0.4$. When there is no correlation between labor income and stock returns the effect of increasing labor income risk is more evident: higher labor income risk reduces the optimal share of stocks in the portfolio at any age. As panel (c) shows, the (average) investor holds a diversified portfolio of risky assets even at a very young age, and starts decumulating stocks and increasing the bond share from the age of around 40. At retirement, the share of stocks is much lower than in the benchmark case, reaching around 0.4, with a correspondingly higher fraction invested in bonds.

A similar effect is detected also in the case of positive correlation between stock returns and labor income shocks ($\rho_{sY} = 0.4$). Comparing the portfolio shares in panel (d) (with high labor income risk) with those in panel (b) (with low income risk), the investor chooses a lower portfolio share of stocks at any age, and at retirement the share of stocks is significantly lower than in the case of reduced labor income risk.

3.4 The heterogeneity of optimal portfolio shares

So far, we presented simulation results in terms of the average optimal portfolio shares across the investors' population. However, in our framework the presence of idiosyncratic labor income shocks may generate substantial heterogeneity in the pattern of financial wealth accumulation over time, and consequently a potentially wide dispersion of the optimal portfolio shares across individuals of the same age but with different levels of accumulated wealth. The degree of heterogeneity in the optimal asset allocation may be an important element in evaluating the performance of pension funds managing individual accounts, whereby each member's asset allocation is adjusted over time on the basis of age and of the history of individual labor income. For this reason, in exploring the sensitivity of the benchmark results to variations in risk aversion (γ), the replacement ratio (λ),[10] and the correlation between permanent labor income shocks and stock return innovations (ρ_{sY}), we focus on the main features of the distribution of optimal portfolio shares across the investors' population: for each age, Figures 6-10 display the median and the 5th and 95th percentiles of the distribution of optimal stock and bond portfolio shares.

In Figure 6, panels (a) and (b) present the distribution of portfolio shares for the benchmark values of risk aversion ($\gamma = 5$), the replacement ratio ($\lambda = 0.68$), and the two values of the labor income-stock return correlation ($\rho_{sY} = 0$ and 0.4) already used in Figure 5(a)-(b). Panel (c) highlights the role of the correlation between labor income shocks and stock returns by assuming $\rho_{sY} = 1$. Note that even this extreme value for ρ_{sY} does not imply

[10] We do not analyse changes in retirement age, referring the reader to Bodie, Detemple, Otruba and Walter (2004) who investigate this in a general life-cycle setting with stochastic wage, labour supply flexibility, and habit formation.

a (counterfactually) high correlation between the stock return innovation and the growth rate of individual labor income, since the latter includes a sizeable idiosyncratic component which is uncorrelated with stock returns.[11]

The results confirm that as ρ_{sY} increases young workers invest less in stocks, gradually raising the share of the riskier asset until the age of 40, to start decumulating towards retirement (Benzoni, Collin-Dufresne and Goldstein 2007; Benzoni 2008); in the case of $\rho_{sY} = 1$ the highest stock share in the financial portfolio never exceeds 80%. In all panels the distribution of portfolio shares is highly heterogeneous due to the presence of idiosyncratic labor income shocks (with the exception of young workers in the case of $\rho_{sY} = 0$, who invest the entire portfolio in stocks to compensate for the relatively riskless nature of their human capital). However, some interesting patterns can be detected. The dispersion among workers decreases as they approach retirement, the more so the higher is the labor income-stock return correlation: as ρ_{sY} increases, the histories of labor incomes and the optimal associated portfolio choices tend to converge over time. We also observe a high dispersion of portfolio shares after retirement; in fact, at the retirement age the dispersion of accumulated financial wealth is maximal and those agents with highest wealth at 65 are likely to earn a higher pension income as well, which boosts heterogeneity among retirees.

The effects of a high risk aversion ($\gamma = 15$) are explored in Figure 7. As expected, the share of stocks is significantly reduced at all ages and for all values of the labor income-stock return correlation. The hump-shaped pattern of the optimal stock share during working life now appears also in the case of $\rho_{sY} = 0$. In order to assess the effects on optimal asset allocation of the generosity of the first-pillar pension system (whose features are summarized by the level of the replacement ratio λ, set at 0.68 in the benchmark case), Figures 8 and 9 display portfolio shares for two different values of the replacement ratio, 0.40 and 0.80 respectively (and for the benchmark risk

[11] In fact, using (1), (2) and (3) we can express the correlation between the growth rate of individual labor income ($\Delta \log Y_{it}$) and the stock return innovation (ε_t^s) in terms of ρ_{sY} and the variances of the aggregate and idiosyncratic labor income shocks as:

$$corr(\Delta \log Y_{it}, \varepsilon_t^s) = \frac{1}{\sqrt{\sigma_n^2 + \frac{\sigma_\varepsilon^2}{\sigma_n^2}}} \cdot \rho_{sY} < \rho_{sY}$$

Using our benchmark value for $\sigma_n^2 = 0.0738$, we derive an upper bound for $corr(\Delta \log Y_{it}, \varepsilon_t^s)$:

$$corr(\Delta \log Y_{it}, \varepsilon_t^s) \leq 0.28 \cdot \rho_{sY}$$

Therefore, the values for ρ_{sY} used in our simulations (0.4 and 1) correspond to (relatively low) values for $corr(\Delta \log Y_{it}, \varepsilon_t^s)$ of (at most) 0.11 and 0.28, respectively.

aversion $\gamma = 5$). When the replacement ratio is 0.40, anticipating relatively low pension incomes, agents choose to save more during their working life, accumulating a higher level of financial wealth. This determines a lower optimal share of stocks at all ages and for all values of the labor income-stock return correlation.

In our framework, all income flows are expressed in real terms: this amounts to an implicit assumption of full indexation of pension income. Often the pension annuity is instead only partially indexed to inflation. A simple way to accomodate this case is to allow for a fixed percent decrease in the replacement ratio λ[12]. When we set such percent equal to 2, the benchmark replacement rate of 0.68 at age 65 reduces to reach 0.34 at age 100. The 95% percentile of the distribution of portfolio shares at retirement drops from 0.8 in Figure 6 to 0.6 in Figure 10. The similarity with the case of a reduction in the replacement rate portrayed in Figure 8 is not accidental: a 2% yearly loss in purchasing power of retirement income amounts to having an average anticipated replacement rate of about 45% throughout retirement years. Such simple modelling does not account for the effects of inflation uncertainty on optimal precautionary savings, which will tend to increase the wealth to labour income ratio further reducing the optimal portfolio share allocate to stocks.

The asset allocation simulations presented here overlook health shocks during retirement years, which may abruptly reduce disposable income net of health-care expenses in countries without complete public coverage. Yogo (2007) analyzes consumption and portfolio decisions during retirement, modelling health as a durable good and health expenses as investment in health. He finds a sizeable reduction in the optimal share invested in risky assets with respect to models that ignore health expenses. One simple way to address this problem is to reduce the yearly pension annuity by the cost of a complementary health-insurance policy, which is likely to be rising in age along with morbility probability.

From Figures 6-7 and 8-9 a general pattern emerges as to the dispersion in the portfolio shares, which decreases as the retirement age approaches, and the more so the higher is the risk aversion parameter and the lower is the replacement ratio. Indeed, the higher the risk aversion and the lower the replacement ratio, the higher is saving and the the larger is the accumulation of financial wealth over the working life; this, according to the policy functions shown in Figure 3, implies a reduced sensitivity of portfolio composition

[12] Such simple modelling overlooks the effects of inflation uncertainty on optimal precautionary savings, which will tend to increase the optimal welath to labour income ratio further reducing the optimal portfolio share allocate to stocks.

to the level of human capital. This insensitivity is stronger the closer is the worker to the retirement age, when financial wealth reaches the maximum level.

A general pattern emerges as to the dispersion in the portfolio shares of the retirees too. From Figures 6-7 and 8-9 we note that an increasing risk aversion and a falling replacement ratio determine a reduction in the retirees' optimal portfolio shares dispersion. Indeed, the higher the risk aversion and the lower the replacement ratio, the higher is saving and the the larger is the accumulation of financial wealth; this, according to the policy functions shown in Figure 3, implies a reduced sensitivity of portfolio composition to the level of human capital. This insensitivity is stronger the closer is the worker to the retirement age, when financial wealth reaches the maximum level. Another general feature of the portfolio shares displayed in Figures 6 to 10 is that, for retired individuals, their distribution is very similar across the three panels, characterized by increasing values of the labor income-stock return correlation, since during retirement pension income is not risky. Since in many instances portfolio allocations are quite different over the last part of the working life, a sharp portfolio reallocation may occur at age 65. This is partcularly evident in the case of high risk aversion (Figure 7), and in the case of $\lambda = 0.40$ (Figure 8), for non-zero values of ρ_{sY}. Indeed, at age 65 uncertainty over pension income in the next 35 years is resolved. The lower is λ, the lower is the ratio of (now riskless) human wealth to (risky) financial wealth during retirement, and the lower is the desired share of stocks in the pension years, which is attained through a portfolio reallocation at the retirement age.

3.5 Welfare costs of suboptimal asset allocations

Tailoring asset allocations to the specificities of workers' income stories may involve considerable management fees that are not included in our model. To practically assess the welfare gains from optimal asset allocation relative to simpler alternative investment strategies, we present in Table 1 the welfare gains of the optimal strategy computed as the yearly percentage increase in consumption granted by the optimal asset allocation. The first alternative strategy is an "age rule", whereby the risky portfolio share is set at (100-age)% and equally allocated between stocks and bonds. This mirrors the empirical relationship between the average proportion invested in stocks and the fund's horizon for Target Date Funds, which is approximately linear with a slope of -1. Bodie and Treussard (2008) adopt another variant of this formula: starting the process of saving for retirement 40 years before the target retirement date, they set the initial proportion invested in equity to

80% letting it fall to 40% at the target date. Thus the formula for the equity percentage T years from the target date is $40 + T$. The second alternative strategy fixes portfolio shares at 1/3 for each financial asset in our model: this mirrors the $1/N$ rule of DeMiguel, Garlappi and Uppal (2008), that systematically outperforms several optimal asset allocation strategies in *ex post* portfolio experiments.

The table shows the welfare cost of each sub-optimal strategy for the two values of risk aversion ($\gamma = 5$ and 15) and the three values of ρ_{sY} considered in the simulations above. For each parameter combination, the table reports the mean welfare cost for the overall population and the welfare costs corresponding to the 5th, 50th and 95th percentiles of the distribution of accumulated financial wealth at age 65.

Several results stand out. First, the magnitude of the mean welfare costs is broadly in the range of 1-3%, consistently with Cocco, Gomes and Maenhout (2005). Second, welfare costs fall as risk aversion increases, because high risk aversion implies reduced optimal exposure to the stock market, and risky asset in general. Looking at the cost distribution conditional on wealth, welfare costs increase as financial wealth falls, because a high human to financial wealth ratio implies a relatively high optimal exposure to the stock market. Third, higher welfare costs are associated to lower values of the labor income-stock return correlation, due to the more important role of the stock market in hedging background risk.

Last but not least, the $1/N$ strategy performs consistently better than the "age rule", showing lower mean welfare costs for all parameter combinations. Tabulated results suggest that an unconditional $1/N$ asset allocation is likely to be cost efficient for high wealth worker in medium to high replacement ratio countries. Note, indeed, that the $1/N$ rule implies a reduction of 49 basis points per annum in terms of equivalent consumption for a highly risk averse worker with median wealth and intermediate labor income correlation with stock returns. According to Blake (2008), the annual fee for active portfolio management charged by pension funds ranges between 20 to 75 basis point per year depending on assets under management. Thus, the reduction in welfare (as measured by equivalent consumption flow) due to a sub-optimal asset allocation is lower than the maximum management fee. Moreover, such reduction refers to the benchmark of an optimal asset allocation chosen by an investor who knows precisely the distribution of both labor income and asset returns. On the contrary, asset managers typically make mistakes when estimating the parameters of such distributions, a fact that explains why an equally wighted, $1/N$ allocation usually outperforms optimal strategies in ex post experiments. Individual accounts are instead likely to be cost efficient for low-wealth workers, especially in low replacement ratio countries.

4 Welfare Ratios for Performance Evaluation

Standard methods for evaluating defined contribution pension funds are similar to those used for measuring mutual funds performance. Performance evaluation is based either on the return of the managed portfolio relative to that of an appropriate benchmark or directly on portfolio holdings (see Ferson and Khang, 2002). The investor horizon is usually assumed to be short, and when it is relatively long, as in Blake, Lehmann and Timmermann (1999) the question being asked concerns whether performance is due to strategic asset allocation, as opposed to short-term market timing and security selection. Rarely do studies assess performance at the pension plan level, paying attention instead to delegated fund managers.[13]

Computing the optimal life-cycle asset allocation allows to evaluate pension funds' performance with reference to a benchmark that explicitly accounts for the pension plan's role in smooting consumption risk. For instance, we can take the ratio of the worker's *ex-ante* maximum welfare under optimal asset allocation, $V_0\left(R_{it}^{P*}\right)$, by her welfare level under the actual pension fund asset allocation, $V_0\left(R_{it}^{PF}\right)$:

$$WR \equiv \frac{V_0\left(R_{it}^{P*}\right)}{V_0\left(R_{it}^{PF}\right)} \tag{13}$$

where R_{it}^{P*} and R_{it}^{PF} are the optimal and actual portfolio return - net of management costs - for member i at time t. More precisely, R_{it}^{PF} are simulated returns which are extracted from the estimated empirical distribution of pension fund returns. Similarly, $V_0\left(R_{it}^{PF}\right)$ results by simulation of optimal consumption and savings decisions for pension members, without optimizing for the asset allocation. The higher the value of the welfare ratio WR, the worse the pension fund performance. Importantly, a lower ratio may be due not only to a higher return per unit of financial risk earned by the pension fund, but also to a better matching between the pension fund portfolio and its members' labor income and pension risks.

Note that we need not know the pension fund actual investment policy, as we let the worker optimize savings over the life cycle conditional on pension funds returns. Note also that this evaluation process assumes that the optimal savings coincides with the actual savings by plan members, which need note be the case. By relaxing this assumptions we could also evaluate the adequacy of members' contributions to pension funds.

[13] Recently, Bauer and Frehen (2008) evaluate US pension funds plans against their internal benchmark portfolios.

Table 2 displays the welfare ratios WR computed for various combinations of risk aversion, the replacement ratio, and the correlation between shocks to labor income and stock returns. In the table, it is assumed that the fund follows a suboptimal strategy (the age rule) that is insensitive to members' incomes and replacement ratios, yielding a Sharpe ratio equal to 0.34. The average Sharpe ratio of the optimal rule is consistently lower, from a minimum of 0.24 for $\lambda = 0.8$ and $\rho_{sY} = 0$ to a maximum of 0.31 for $\lambda = 0.4$ and $\rho_{sY} = 1$. Thus, performance evaluated according to a standard return-to-risk metric is worse for the optimal than for the age rule. The picture changes when we look at the proposed welfare metric, that always exceeds 1 -indicating a higher welfare associated with the optimal asset allocation.

We can also note how performance evaluation is affected by both institutional design and investor heterogeneity. In fact the higher values for the welfare ratio (1.1) obtain for the fifth income percentile and $\lambda = 0.40$ or 0.68. Such figures are associated to $\rho_{sY} = 0$, i.e. a case where the absence of correlation between income and stock returns allows for a better hedging of labour income shocks. Thus, the value of pension funds in smoothing consumption risk tends to be higher the lower are both the member's income and the country's replacement ratio.

This metric allows for cross-country performance comparisons, along the lines of Antolin (2008), even if countries differ in labor income profiles, replacement ratios, inflation protection for pension annuities and life expectancy. These parameters enter both the numerator and the denominator of WR; thus the cross-country distribution of this ratio is only affected by how well pension funds perform their consumption smoothing role. It is well known that the investible asset menu in certain countries is restricted by regulation (see Antolin, 2008). If this is the case, the numerator of the welfare ratio ought to be computed conditional on the country investable asset menu so as to evaluate the pension fund manager's ability, and the metric can be used to assess the costs from restricting the asset menu for retirees.

Finally, the previous section argues that the $1/N$ strategy dominates the optimal asset allocation when the costs of tailoring the asset allocation to workers' profiles exceed their benefits, i.e. the differential in management fees is sufficiently high. In such a case, it could be appropriate to substitute the numerator with the *ex ante* welfare achieved when portfolio returns are associated with the $1/N$ strategy, obtaining the welfare ratio computed against this latter.

5 Conclusions

Modern finance theory suggests that the features of the labor income stream over the investor's life cycle are a crucial determinant of her optimal investment policy. Models that, incorporating those characteristics, generate realistic patterns of life-cycle consumption and wealth accumulation can advance the design of appropriate investment policies of defined-contribution pension funds. In this paper, we take this line of reasoning one step further and propose to base the assessment of DC funds by benchmarking their asset allocation to the one implied by a life-cycle model. Thus, we abandon standard performance evaluation practice relying on the idea that a higher return-to-risk differential maps into better performance, overlooking the pension fund ability in hedging labor income risk and pension risk of plan participants.

To illustrate our idea, we compare our benchmark asset allocation to the one of an imaginary pension fund which consistently outperforms the benchmark in terms of Sharpe ratio. Thus, evaluation according to a standard return-to-risk metric ranks the pension fund performance better than the benchmark. The welfare based metric on the contrary ranks the benchmark higher, because it optimally smoothes consumption risk. This simulation also exemplifies how performance evaluation is affected by both institutional design and investor heterogeneity. For instance, the role of pension funds in optimally smoothing consumption risk tends to be higher for lower income members and replacement ratios. One limit of our welfare ratio is that it is conditional on a specific utility function. Further work might assess the sensitivity of our method to alternative characterizations of preferences.

The results on asset allocation sensitivity to changes in labor income profiles suggest that pension plans ought to offer different investment options for workers depending on their age and past income. It is however possible to evaluate the performance and the associated participants' welfare costs of simple rules - implementable by pension funds at lower costs - that partially account for the heterogeneity of optimal portfolio shares, e.g. by grouping members into age classes and applying the optimal "median" share to all members in a specified class. In this respect, further research may be carried out to scrutinize our conclusion that the $1/N$ portfolio strategy is likely to be cost efficient for both high wealth and highly-risk-averse-average-wealth workers in medium-to-high replacement ratios countries.

Our simulations are calibrated on US data. Clearly, the model can be used for assessing pension fund performance in other countries as well, conditionally on availability of labor income profiles. See Borella (2004) who estimate various earning structure models in addition to the permanent-transitory model.

In our simulation exercise, the available financial assets include a riskless short-term asset, a high risk premium asset and a low risk premium asset, with potentially correlated returns. The calibrated version of the model uses US stock index and bond index returns. However, any pair of assets (or baskets of assets, such as the Fama-French portfolios) can be accomodated, to the extent that their mean returns, their variances and covariances can be estimated precisely. Returns on foreign assets ought to be expressed in foreign currency, with currency risk fully hedged, since there is no explicit dynamics of the exchange rate in this simple version of the model. Furthermore, the model can be used in its current version in economies where inflation is not highly volatile, as the model assumes constant inflation.

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Table 1 Welfare Gains

Mean welfare gains for the overall population are reported.

Moreover, we report welfare gains corresponding to percentiles of financial wealth accumulated at age 65

	Risk aversion 5		Risk aversion 15	
	ρ_{sy}=0			
	WelfareCosts		*WelfareCosts*	
	(100-age)/2	*1/3*	*(100-age)/2*	*1/3*
Mean	0.021	0.018	0.015	0.013
5th percentile	0.032	0.027	0.043	0.037
50thpercentile	0.024	0.021	0.012	0.011
95thpercentile	0.004	0.003	0.004	0.004
	ρ_{sy}=0.4			
	WelfareCosts		*WelfareCosts*	
	(100-age)/2	*1/3*	*(100-age)/2*	*1/3*
Mean	0.012	0.012	0.012	0.011
5th percentile	0.027	0.022	0.036	0.034
50thpercentile	0.013	0.011	0.006	0.005
95thpercentile	0.001	0.001	0.002	0.001
	ρ_{sy}=1			
	WelfareCosts		*WelfareCosts*	
	(100-age)/2	*1/3*	*(100-age)/2*	*1/3*
Mean	0.012	0.011	0.011	0.010
5th percentile	0.025	0.023	0.032	0.031
50thpercentile	0.012	0.011	0.005	0.005
95thpercentile	0.002	0.001	0.002	0.002

Table 2 Welfare Ratios
Mean welfare ratio for the overall population are reported.
Moreover, we report welfare ratios corresponding to percentiles of financial wealth accumulated at age 65.

	Risk aversion 5		
Replacement ratio	**0.68**	**0.4**	**0.8**
		ρ_{sy}=0	
Sharpe ratio			
Optimal	0.260	0.286	0.244
Age rule	0.337	0.337	0.337
Welfare Ratio			
Mean	1.051	1.096	1.044
5th percentile	1.101	1.096	1.048
50thpercentile	1.056	1.074	1.057
95thpercentile	1.014	1.011	1.007
		ρ_{sy}=0.4	
Sharpe ratio			
Optimal	0.273	0.310	0.257
Age rule	0.337	0.337	0.337
Welfare Ratio			
Mean	1.033	1.049	1.028
5th percentile	1.049	1.076	1.028
50thpercentile	1.040	1.054	1.032
95thpercentile	1.009	1.012	1.007
		ρ_{sy}=1	
Sharpe ratio			
Optimal	0.296	0.314	0.264
Age rule	0.337	0.337	0.337
Welfare Ratio			
Mean	1.025	1.037	1.020
5th percentile	1.042	1.029	1.032
50thpercentile	1.031	1.043	1.021
95thpercentile	1.002	1.009	1.002

Figure 1 Labor income process
The figure reports the fitted polynomial in age and personal characteristics derived according Cocco et al. (2005) calibrations for households with high school education.

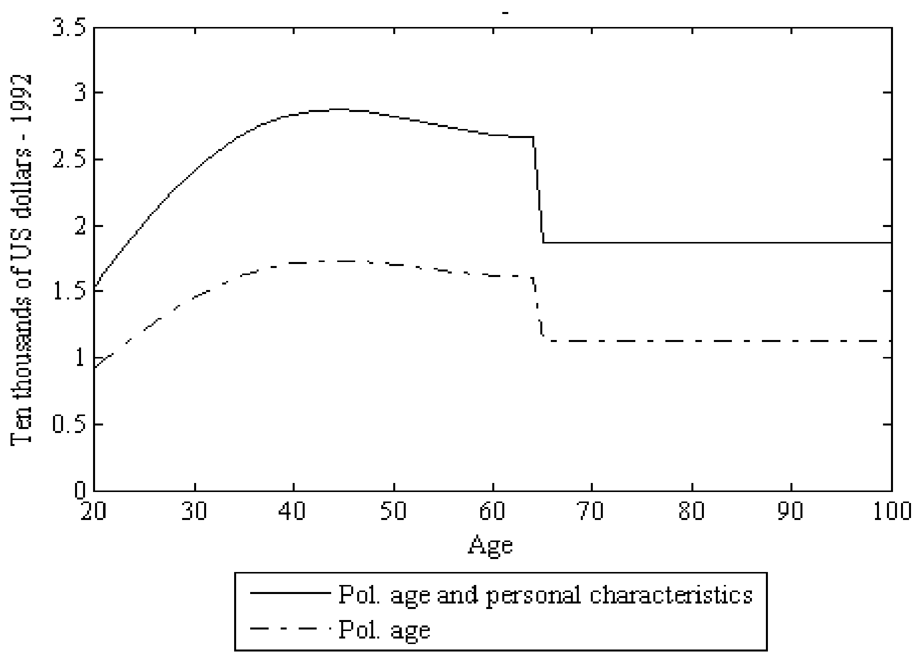

Figure 2 Life cycle profiles of consumption, income and wealth
The figure reports simulated consumption, income and wealth profile for the benchmark case.

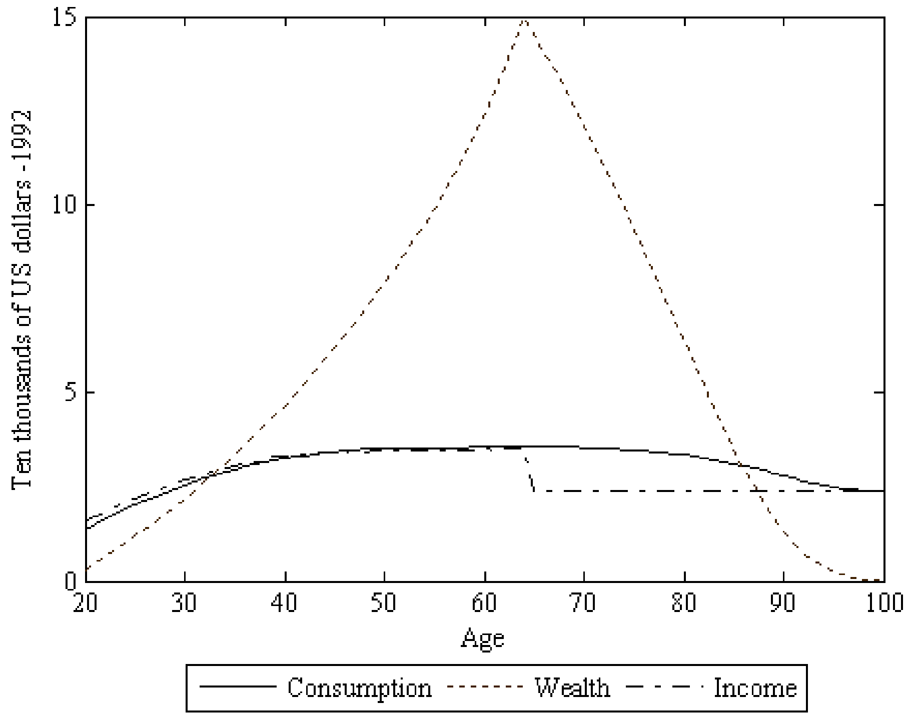

Figure 3 Policy functions
The figure reports the policy functions for the portfolio shares invested in stocks at different ages.

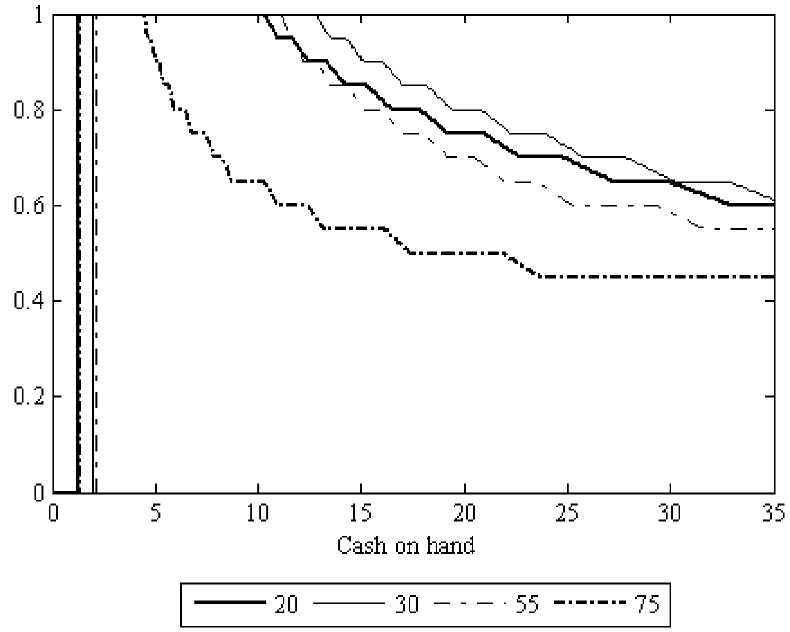

Figure 4 Policy functions
The figure reports the policy functions for the portfolio shares invested in bonds at different ages.

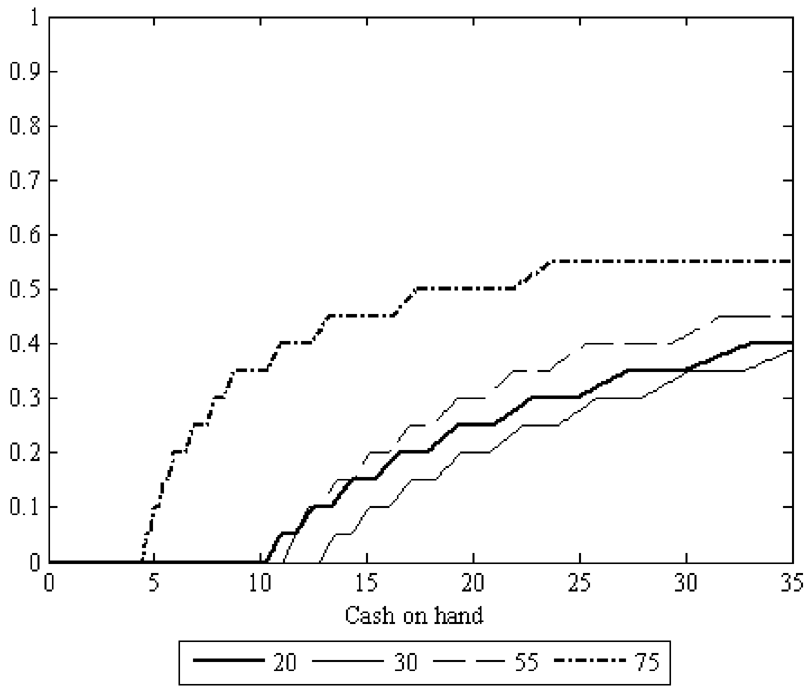

Figure 5
The figure reports mean share profiles, as a function of age, for stocks and bonds. The replacement ratio is equal to 0.68, the correlation between stock and bond returns is set to 0.2 while the one between stocks and labour income is 0 and 0.4. The variance of permanent and transitory shocks in the benchmark case is 0.0106 and 0.0738 in panels (a) and (b), while it is 0.0408 and 0.269 in panels (c) and (d).

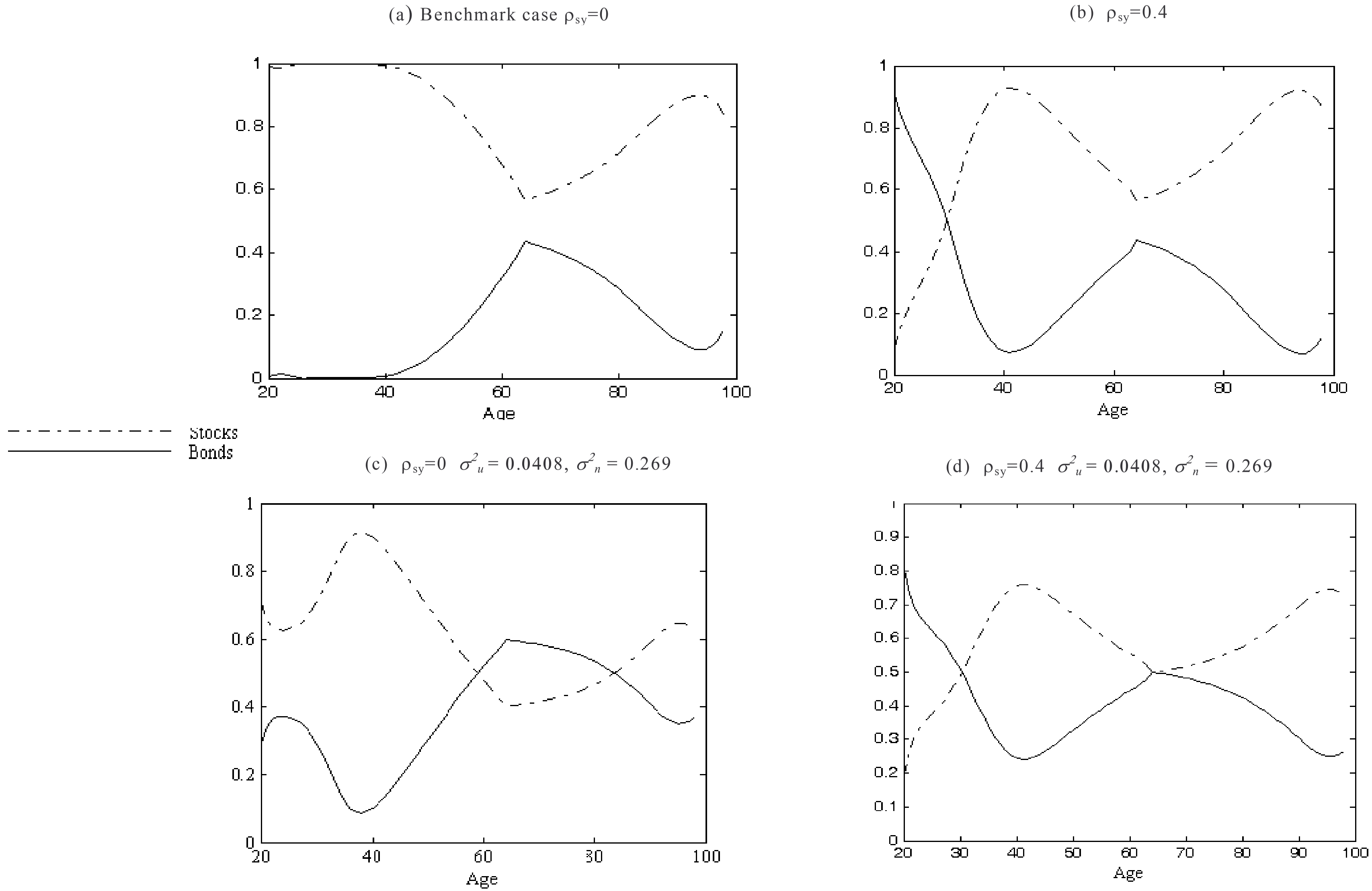

Figure 6
This figure reports share profiles, as a function of age, for stocks and bonds. The solid line represents the shape of the median portfolio share, while the (dotted) dashed refer to the (5^{th}) 95^{th} percentiles. The replacement ratio is equal to 0.68, the correlation between stock and bond returns is set to 0.2 while the one between stocks and labour income varies between 0 and 1.

Risk aversion 5

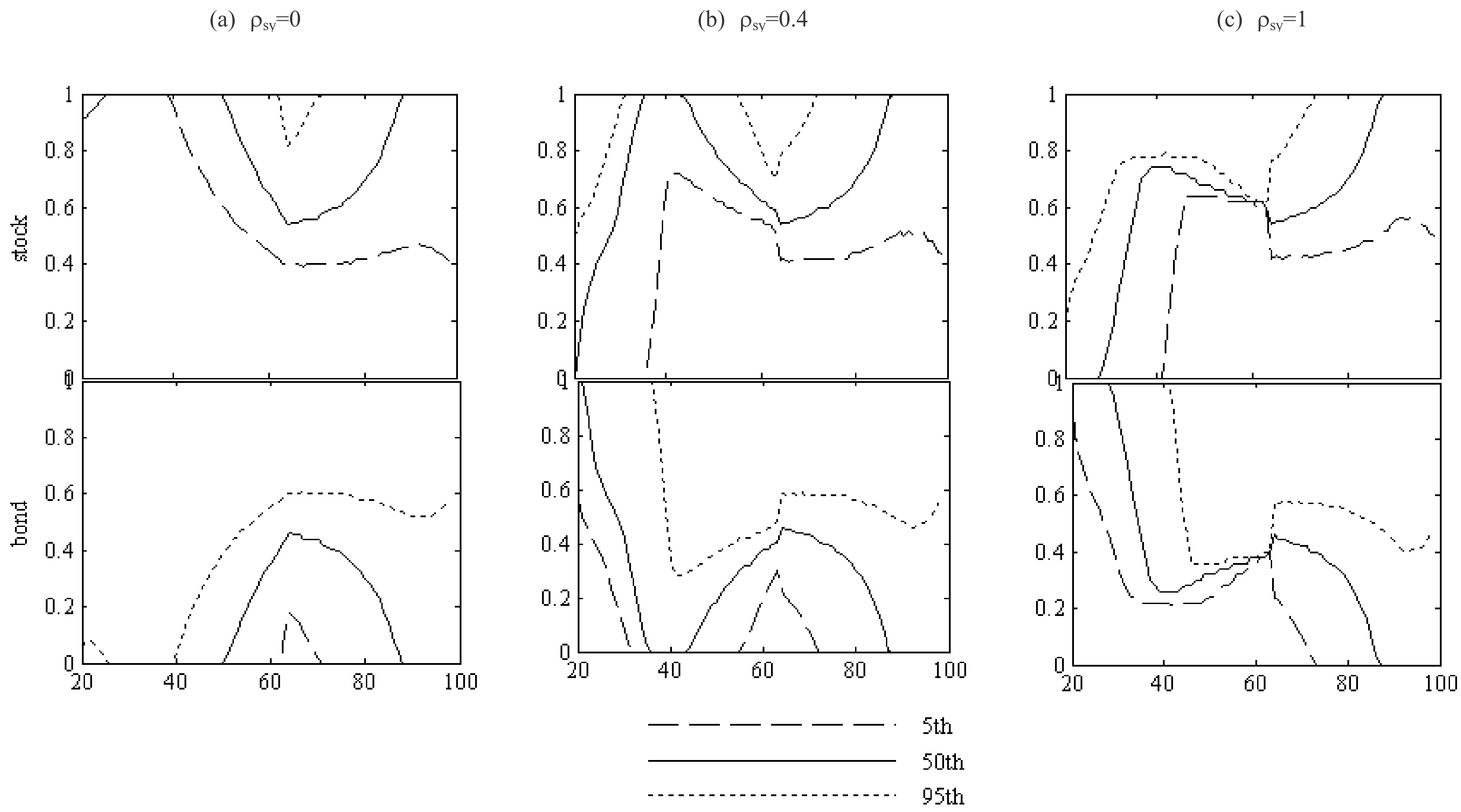

Figure 7
This figure reports share profiles, as a function of age, for stocks and bonds. The solid line represents the shape of the median portfolio share, while the (dotted) dashed refer to the (5^{th}) 95^{th} percentiles. The replacement ratio is equal to 0.68, the correlation between stock and bond returns is set to 0.2 while the one between stocks and labour income varies between 0 and 1.

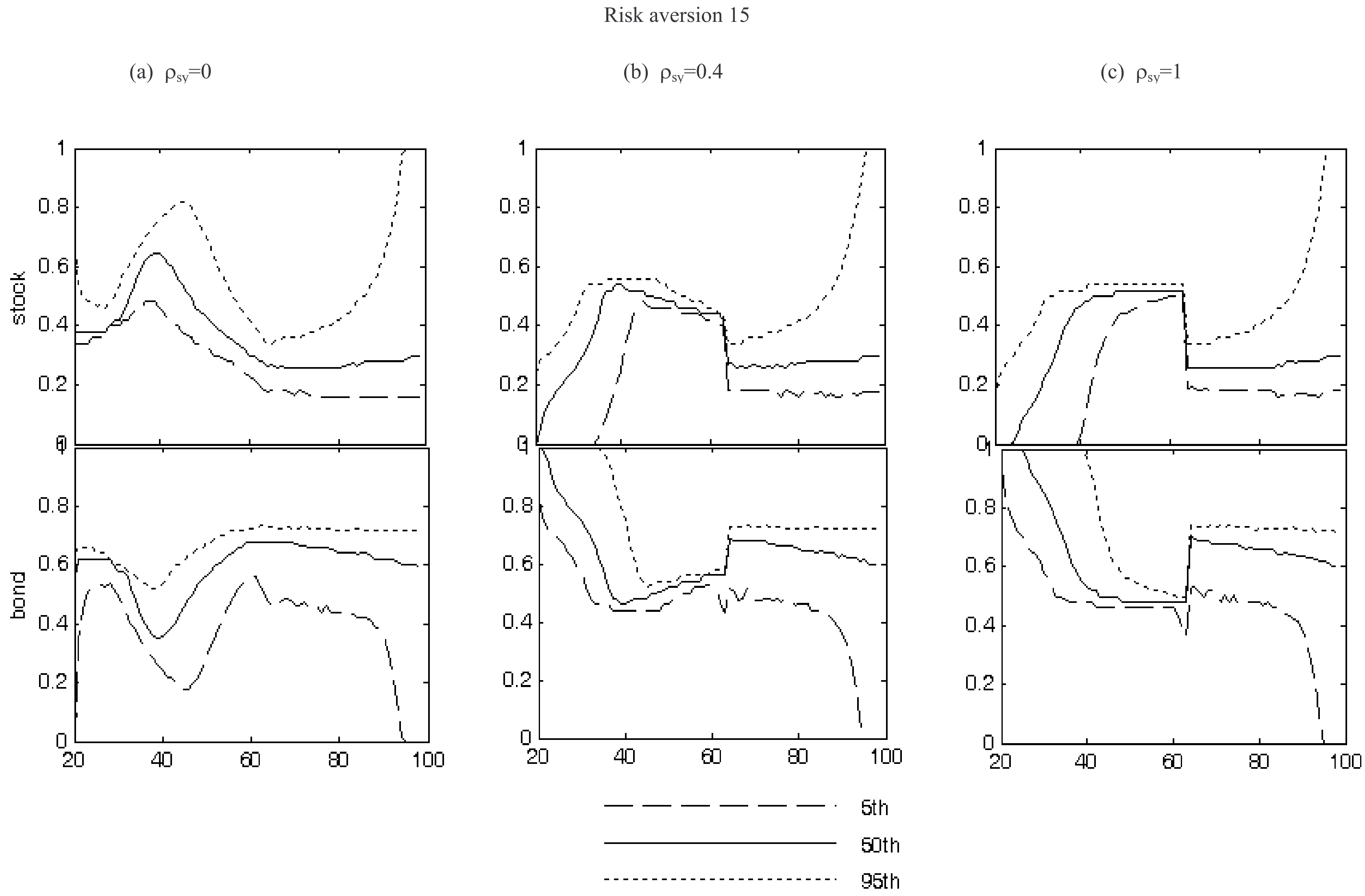

Figure 8
This figure reports share profiles, as a function of age, for stocks and bonds. The solid line represents the shape of the median portfolio share, while the (dotted) dashed refer to the (5th) 95th percentiles. The replacement ratio is equal to 0.40, the correlation between stock and bond returns is set to 0.2 while the one between stocks and labour income varies between 0 and 1.

Risk aversion 5 Replacement Ratio 0.40

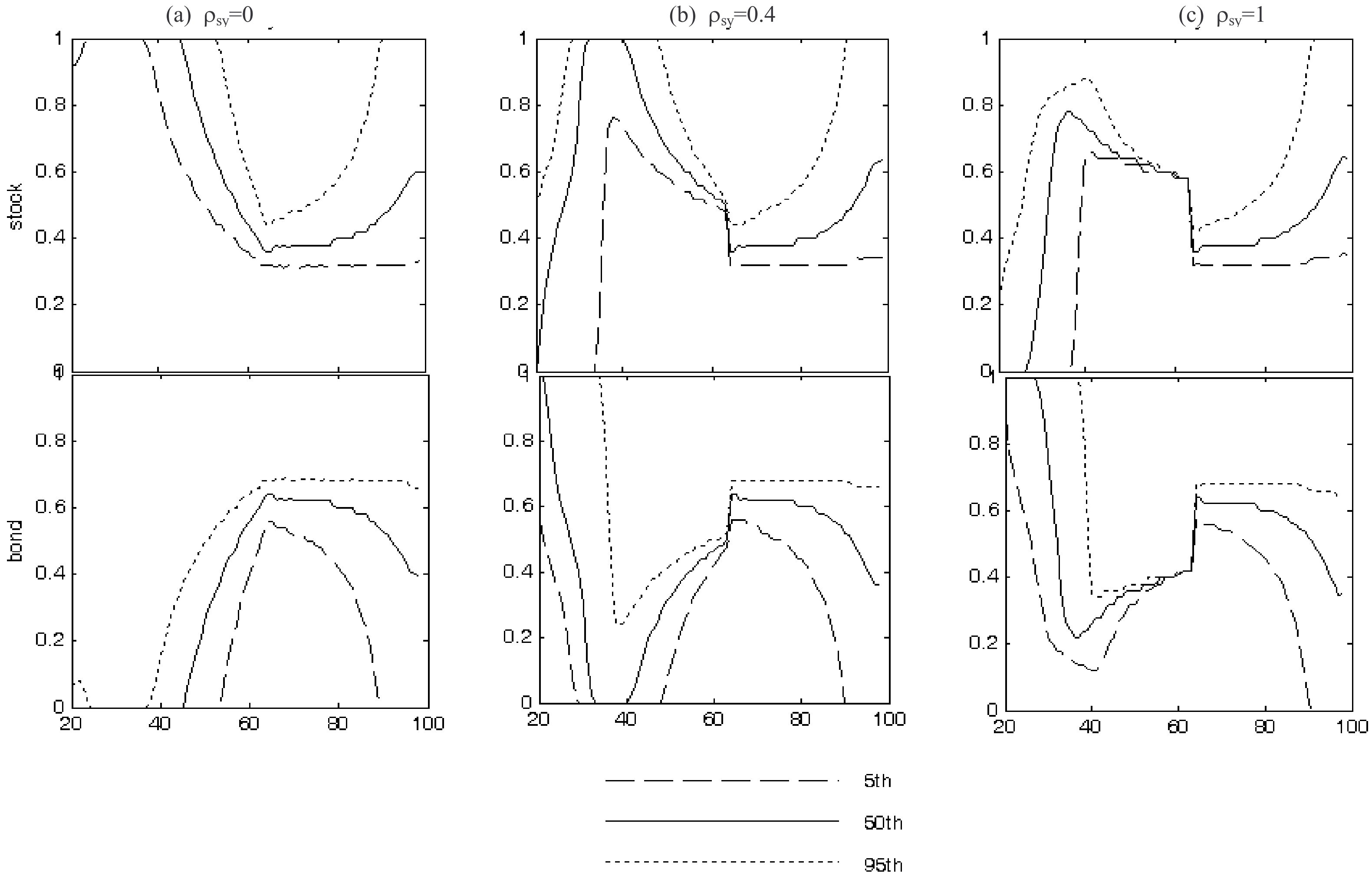

Figure 9
This figure reports share profiles, as a function of age, for stocks and bonds. The solid line represents the shape of the median portfolio share, while the (dotted) dashed refer to the (5[th]) 95[th] percentiles. The replacement ratio is equal to 0.80, the correlation between stock and bond returns is set to 0.2 while the one between stocks and labour income varies between 0 and 1.

Risk aversion 5 Replacement Ratio 0.80

(a) $\rho_{sy}=0$ (b) $\rho_{sy}=0.4$ (c) $\rho_{sy}=1$

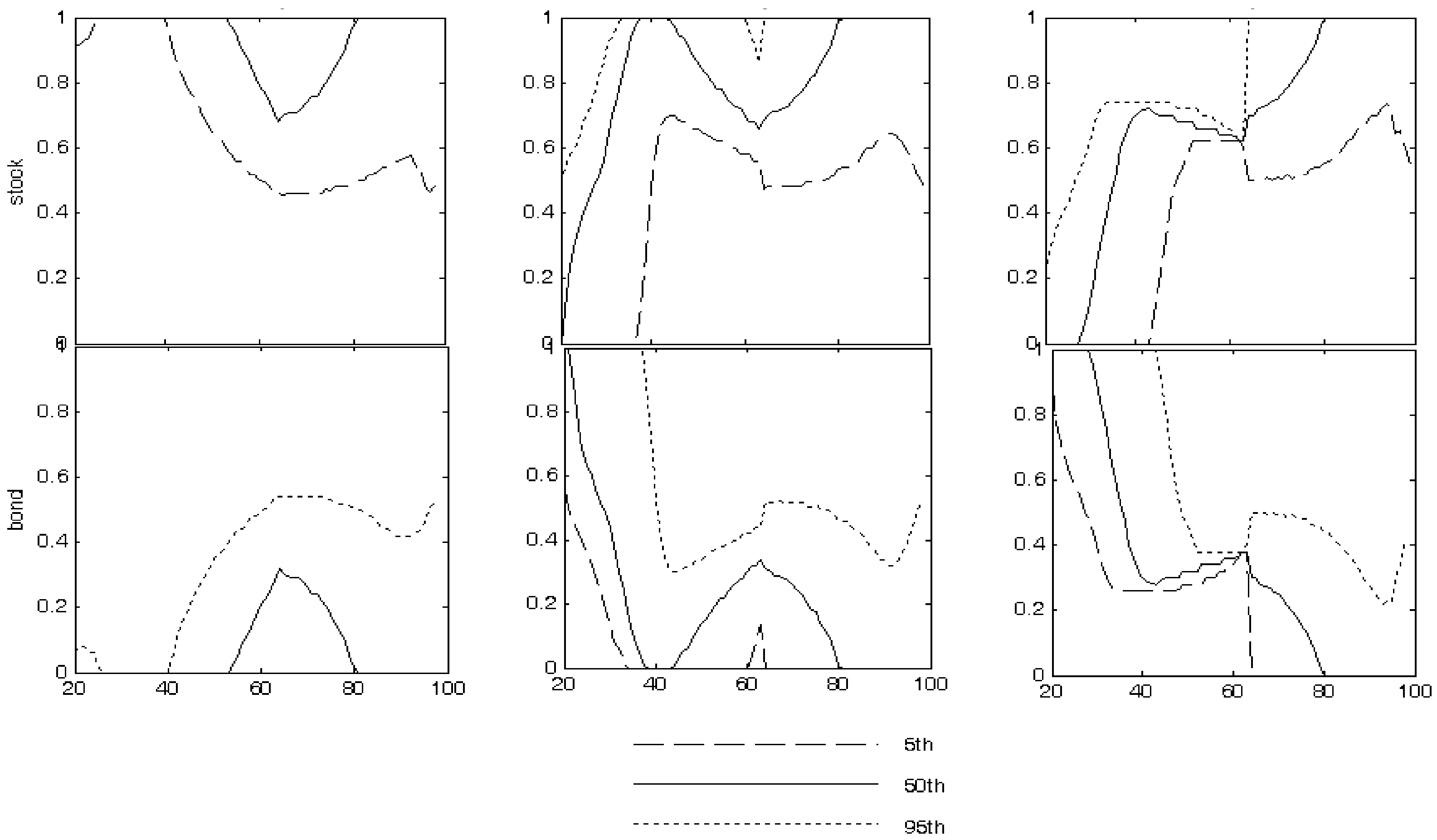

Figure 10
This figure reports share profiles, as a function of age, for stocks and bonds. The solid line represents the shape of the median portfolio share, while the (dotted) dashed refer to the (5^{th}) 95^{th} percentiles. pension treatments at the beginning of retirement correspond to a replacement ratio of 0.68 and then decrease by 0.02 per year due to inflation (which implies an average replacement ratio of about 0.45 over all retirement ages). The correlation between stock and bond returns is set to 0.2 while the one between stocks and labour income varies between 0 and 1.

Risk aversion 5

(a) $\rho_{sy}=0$ (b) $\rho_{sy}=0.4$ (c) $\rho_{sy}=1$