Fixed versus Contingent Indexation: Welfare Implications *

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Abstract

Funded social security programs are particularly vulnerable to economic and financial market shocks. As a consequence of the recent crisis, many Dutch pension funds had to submit recovery plans that set out how the pension buffers will be restored over the next couple of years. Such plans will have to rely primarily on a mix of reduced benefit indexation and increased pension contributions. In view of these considerations, a discussion has emerged whether indexation should be differentiated across the various groups of participants in a pension fund. We investigate numerically this issue, developing an applied many-generation small open economy OLG model with heterogeneous agents. The pension system consists of a first pillar PAYG component and a funded second tier. In our stochastic simulations, we hit the economy with a variety of unexpected demographic, economic and financial shocks. We then compare welfare of different generations and (utilitarian) social welfare under different indexation schemes. The design of the indexation policy strongly affects welfare and the capability of the system to prevent underfunding. Overall, we find welfare improvement when indexation is linked to age or income. However, we observe large differences among the generations.

Keywords: indexation, funded Social Security, inter-generational welfare, pension buffer, stochastic simulations.
JEL codes: H55, I38, C61

1 Introduction

Funded social security programs are particularly vulnerable to economic and financial market shocks. The recent crisis in the financial markets has made this particularly clear for the Netherlands, where about half of the pension income is provided by pension funds. The combined effect of the fall in asset prices and the reduction in the long-run interest rate (used to discount future pension payments) has in a few months time reduced the funding ratios (the ratio of assets over

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liabilities) of most pension funds by at least a quarter to a third. As a result, a large proportion of the funds had to submit recovery plans that set out how over the next couple of years the pension buffers will be restored. Many pension funds will have to rely on a mix increasing pension contributions, reduced indexation and, in exceptional cases, a reduction in the pension rights that individuals have accumulated up to now.

This paper focuses on reductions in indexation as an instrument for the stabilisation of pension buffers. Indexation can take on two basic forms. One is the indexation to price inflation aimed at maintaining the purchasing power of the pensioneers. The other is the indexation to wage increases which is aimed at keeping the purchasing power of retired in line with that of workers. Reductions in the indexation of pension rights is uniform. Given that people accumulate nominal pension claims over their working life,[1] those that are retired or close to retirement and those who are in the highest income groups will be hurt most by a uniform reduction in indexation. Moreover, the retired and the older workers are left with little if no flexibility to make up for the lost indexation by working longer, while in addition a given loss of purchasing power has to be absorbed in a consumption reduction over a relatively short remaining lifetime. Hence, these groups are at particular risk under policies that resort to indexation as a way to keep pension buffers stable.

In view of these considerations, a discussion has emerged on whether the policy parameters should be differentiated across the various groups of participants in a pension fund. In fact, Hurst and Willen (2007) find that it is typically welfare improving to have pension contributions increase with the worker's age. However, to the best of our knowledge there are no results on how indexation should be ideally varied across the different participants in a pension fund.

We investigate numerically the welfare consequences of cohort-specific indexations. In particular, we compare three types of cohort-specific indexation: status-dependent indexation, in which the retired always receive full indexation to the general price level increase (but not more than that), while adjustments to indexation are proportional across all workers (who thus bear all the risk associated with the pension buffers); age-dependent indexation, in which the reduction in indexation in the case of underfunding is smaller for older than for younger cohorts; and income-dependent indexation, in which the reduction in indexation in the case of underfunding is smaller for low-income than for higher-income individuals.

We develop an applied many-generation small open economy OLG model with heterogeneous agents. The pension system consists of a first pillar PAYG component and a funded second tier. We calibrate the pension system in our model to the Dutch situation. However, for the remaining exogenous parameters we follow the standard literature and use the demographic, macroeconomic and financial market data over the past decades for the U.S.. In our stochastic simulations, we hit the economy with a variety of unexpected shocks. These may be broadly classified into three categories: demographic uncertainty (the size of newborn generations and survival probabilities that determine life expectancy), economic uncertainty (productivity growth and the inflation rate) and financial uncertainty (bond, equity and housing returns, and yield curve). We compare welfare of different generations and (utilitarian) social welfare under different indexation schemes.

We find that the design of the indexation policy strongly affects welfare and the capability of the system to prevent underfunding. We find some welfare gain when indexation is linked to skill or (especially) age. However, there are large differences among generations. Combining age- and

[1] In the Netherlands, pension rights are always expressed as the number of euros of pension one gets as of the retirement age. Each year of additional work adds an extra amount to the existing stock of pension rights, while usually this stock of rights is increased with the rate of price or wage inflation (indexation). However, indexation is not required by law and the board of the fund may index by less if this is deemed necessary to maintain a healthy pension buffer.

skill-dependent indexation policies the welfare gain is maximised.

The paper is organised as follows. Section 2 presents the theoretical framework. Section 3 describes the benchmark calibration, while Section 4 shows the main results from stochastic simulations based on the benchmark indexation policies, as well as alternative policies and a robustness check on the policy parameters. Section 5 concludes the main text. Finally, the Appendix provides further details on a number of aspects of the model and the simulations.

2 The model

There are a number of D cohorts alive in any given period t . Each cohort j $(= 1, ..., D)$ consists of $N_{j,t}$ individuals at time t, who are distributed in I equally-sized skill groups, $i = 1, ..., I$. A higher value of i denotes a higher skill level. The skill level of a person determines his income, given his age and the macroeconomic circumstances. Index $j = 1, ..., D$ indicates the age of the cohort, computed as the amount of time since entry into the labour force. Further, all individuals within a given group earn the same income. Finally, a period in our model corresponds to one year.

2.1 Cohorts and demography

We assume that each individual born in period $t - j + 1$ (that is, the person has age zero at the start of $t - j + 1$ and age one at the end of that period) has an exogenous marginal probability $\psi_{j,t-j+1} \in [0, 1]$ of reaching age j (at the end of period t) conditional on having reached age $j - 1$. For example, $\psi_{j,t-j+1} = 1$ means that an individual alive at age $j-1$ at the end of period $t-1$ will be alive with certainty at age j at the end of period t. Similarly, $\psi_{j,t-j+1} = 0$ implies that anyone alive at age $j - 1$ at the end of period $t - 1$ will surely die before the end of period t. Specifically, we assume that $\psi_{j,t-j+1} = 0$ for any $j \geq D + 1$. To be precise, we assume that individuals can die only at the start of a period, so that the survival of that moment implies that the person reaches the end of the period and receives an income and consumes during that period. We further assume that the cohort of newborn agents in period t is $1 + n_t$ times larger than the cohort of newborn agents in period $t - 1$:

$$N_{1,t} = (1 + n_t) N_{1,t-1}. \tag{1}$$

In general, we denote with $N_{j,t}$ the size of cohort j at time t. This size depends on the history of past survival probabilities. Indeed, for $j = 2, ..., D$:

$$N_{j,t} = N_{j-1,t-1}\psi_{j,t-j+1}.$$

2.2 Individuals

Individuals in the same cohort can only differ in terms of their income. Each individual in a given cohort belongs to some skill group i, with $i = 1, ..., I$. We assume that individuals remain in the same skill group over their entire life. Individuals work until the exogenous retirement age R and live for at most D years. During their working life $(j = 1, ..., R)$, they receive a labour income $y_{i,j,t}$ defined as follows:

$$y_{i,j,t} = e_i s_j z_t, \tag{2}$$

where e_i, $i = 1, ..., I$ is an efficiency index (linked to the skill level of class i), s_j, $j = 1, ..., R$, a seniority index (for given skill level income varies with age) and z_t is an exogenous income process:

$$z_t = (1 + g_t)\, z_{t-1}, \tag{3}$$

where g_t is the exogenous *nominal* growth rate of the process and $z_0 = 1$.

Average income across workers is defined as:

$$y_t = \frac{\sum_{j=1}^{R} \frac{N_{j,t}}{I} \sum_{i=1}^{I} y_{i,t,j}}{\sum_{j=1}^{R} N_{j,t}}. \tag{4}$$

If all workers have identical productivity (i.e. $e_1 = ... = e_I = s_1 = ... = s_I = 1$), then $y_t = z_t$. We make a distinction between y_t and z_t because the relative sizes of the cohorts may change over time, implying that the ratio y_t/z_t will fluctuate over time.

2.3 Social security and accidental bequests

Social security is based on a two-pillar system. The first pillar is a pay-as-you-go (PAYG) defined benefit (DB) program which pays a flat benefit to every retiree. It is organised by the government, which sets the contribution rate to ensure that the first pillar is balanced on a period-by-period basis. The second pillar is funded and may either be organised by the government or by the private sector. In reality, in the Netherlands some of the parameters of the second pillar are set by the government, while other parameters are set by the pension fund itself. Since we do not explicitly model the objectives of the different policymakers we do not need to make specific assumptions about who sets which parameters. Finally, the government redistributes the accidental bequests left by those who die.

2.3.1 The first pillar of the social security system

Each period, an individual of working age pays a mandatory contribution $p^F_{i,j,t}$ to the first pillar of the social security system. This contribution depends on the size of income $y_{i,j,t}$ relative to the thresholds $\delta^l y_t$ and $\delta^u y_t$:

$$p^F_{i,j,t} = \left\{ \begin{array}{ll} 0 & \text{if } y_{i,j,t} < \delta^l y_t \\ \theta^F_t \left(y_{i,j,t} - \delta^l y_t \right) & \text{if } y_{i,j,t} \in \left[\delta^l y_t, \delta^u y_t \right] \\ \theta^F_t \left(\delta^u y_t - \delta^l y_t \right) & \text{if } y_{i,j,t} > \delta^u y_t \end{array} \right\}, \quad j \leq R, \tag{5}$$

where δ^l, δ^u and θ^F_t are policy parameters. In period t the benefit received by an individual retiree is a fraction ρ^F of the average income in the economy:

$$b^F_t = \rho^F y_t. \tag{6}$$

Given the benefit formula in equation (6), each period the contribution rate θ^F_t adjusts such that aggregate contributions into the first pillar P^F_t equal aggregate first-pillar benefits B^F_t paid out to the retired:

$$P^F_t = B^F_t, \tag{7}$$

where

$$P_t^F = \sum_{j=1}^{R} \frac{N_{j,t}}{I} \sum_{i=1}^{I} p_{i,j,t}^F,$$

and

$$B_t^F = \sum_{j=R+1}^{D} \frac{N_{j,t}}{I} \sum_{i=1}^{I} b_t^F = b_t^F \sum_{j=R+1}^{D} N_{j,t}.$$

2.3.2 The second pillar of the social security system

The second pillar consists of a DB funded program. Each period, an individual of working age also pays a mandatory contribution $p_{i,j,t}^S$ to this second pillar if her income exceeds the franchise income level. Specifically, $p_{i,j,t}^S$ is a fraction of the labour income in excess of the franchise:

$$p_{i,j,t}^S = \theta_t^S \max\{0, y_{i,j,t} - \lambda y_t\}, \quad j \leq R, \tag{8}$$

where θ_t^S is a policy parameter.

A cohort entering retirement at age $R+1$ receives a benefit proportional to its average wage over the years worked. Period t benefits for an individual in skill group i of cohort j are given by:

$$b_{i,j,t}^S = M_{i,j,t}, \qquad j > R, \tag{9}$$

where the accumulated "stock of nominal rights" $M_{i,j,t}$ at the end of period t evolves as:

$$M_{i,j,t} = \left\{ \begin{array}{ll} (1-m_t) \left\{ \begin{array}{c} (1+\omega_{i,j,t}) M_{i,j-1,t-1} \\ +\mu \max[0, y_{i,j,t} - \lambda y_t] \end{array} \right\} & j \leq R \\ (1-m_t)(1+\omega_{i,j,t}) M_{i,j-1,t-1} & j > R+1 \end{array} \right\}, \tag{10}$$

where the coefficients μ and λ denote the annual accrual rate and franchise, respectively, as shares of average income and $\omega_{i,j,t}$ is the amount of indexation of nominal rights, which is allowed to be cohort- and skill-group specific. Indexation aims at following nominal wage growth. However, actual indexation may depend on the financial position of the pension fund, as explained in more detail below. Further, m_t captures a proportional reduction in nominal rights that may be applied when the funding ratio is so low that restoration using standard instruments is no longer possible (see below). We assume that $m_t > 0$ only when $\omega_{i,j,t} = 0$. Each individual enters the labour market with zero nominal claims. Hence, $M_{i,0,t-j} = 0$, where $M_{i,0,t-j}$ are nominal claims at the end of period $t-j$ or beginning of period $t-j+1$ when the generation enters the labour market at age 0.

For a given accrual rate μ and franchise λ, each period the government chooses the contribution rate θ_t^S and the indexation parameters $\{\iota_t, \kappa_t\}$ in the benefit formula in equations (9)-(10). The choice of these policy parameters will depend on the level of the nominal funding ratio F_t, which is the ratio between the pension fund's assets, A_t, and its liabilities, L_t:

$$F_t = \frac{A_t}{L_t} \tag{11}$$

At the end of period t the pension fund's assets are the sum of the second-pillar contributions from workers in period t *minus* the second-pillar benefits paid to the retirees in period t *plus* the pension fund's assets at the end of period $t-1$ grossed up by their return in the financial markets:

$$A_t = \left(\sum_{j=1}^{R} \frac{N_{j,t}}{I} \sum_{i=1}^{I} p_{i,j,t}^S - \sum_{j=R+1}^{D} \frac{N_{j,t}}{I} \sum_{i=1}^{I} b_{i,j,t}^S \right) + (1+r_t^g) A_{t-1}, \tag{12}$$

where

$$1+r_t^g=\left(1-z^e-z^h\right)\left(1+r_t^{lb}\right)+z^e\left(1+r_t^e\right)+z^h\left(1+r_t^h\right) \tag{13}$$

is the gross nominal rate of return on the pension fund's asset portfolio with an exogenous and constant share z^e invested in equities, an exogenous and constant share z^h invested in the housing market and the remainder in long-term bonds. In view of their obligations, pension funds have a preference for investing long-term debt. Here, we assume that the entire fixed-income part of the pension fund's portfolio consists of 10-year zero coupon bonds. Further, the net returns on the long-term bonds (r_t^{lb}), equity (r_t^e) and housing (r_t^h) are exogenous.

Our assumption that the pension fund always holds 10-year bonds, implies that at the end of each year bonds of 9-year maturity are sold to purchase new 10-year bonds. In more detail, the fund's annual portfolio rebalancing operation works as follows. In year $t-1$, say, the pension fund buys 10-year zero-coupon bonds for an amount of B_{t-1}. Denoting the return on 10-year bonds by $r_{10,t-1}^b$, the value at maturity of the bonds is

$$P_{t+9}=B_{t-1}\left(1+r_{10,t-1}^b\right)^{10}. \tag{14}$$

Hence, the present value B_{t-1} of the bond holdings in year $t-1$ is:

$$B_{t-1}=\frac{P_{t+9}}{\left(1+r_{10,t-1}^b\right)^{10}}.$$

In year t, only 9 years of maturity are left, and the bond return is $r_{9,t}^b$. The present value B_t is then

$$B_t=\frac{P_{t+9}}{\left(1+r_{9,t}^b\right)^{9}}$$

Combining with (14) we obtain the following expression:

$$B_t=B_{t-1}\frac{\left(1+r_{10,t-1}^b\right)^{10}}{\left(1+r_{9,t}^b\right)^{9}}=B_{t-1}r_t^{lb}.$$

The fund's liabilities are the sum of the present values of current and future rights already accumulated by the cohorts currently alive:

$$L_t=\sum_{j=1}^{D}\frac{N_{j,t}}{I}\sum_{i=1}^{I}L_{i,j,t}. \tag{15}$$

The expected present value at time t of current and future benefits of a cohort j in skill group i is

$$L_{i,j,t}=\begin{array}{ll} E_t\left[\sum\limits_{l=R+1-j}^{D-j}\frac{1}{\psi_{j,t-j+1}}\left(\prod\limits_{k=0}^{l}\psi_{j+k,t-j+1}\right)\frac{1}{\left(1+r_{l,t}^s\right)^l}M_{i,j,t}\right] & j\le R \\ E_t\left[\sum\limits_{l=0}^{D-j}\frac{1}{\psi_{j,t-j+1}}\left(\prod\limits_{k=0}^{l}\psi_{j+k,t-j+1}\right)\frac{1}{\left(1+r_{l,t}^s\right)^l}M_{i,j,t}\right] & j>R \end{array}. \tag{16}$$

where future benefits are discounted using the swap curve $\left\{r_{k,t}^s\right\}_{k=1}^D$. Note that $\psi_{j,t-j+1}$ cancels out in the above equation. When $j\le R$, furthermore, we discount all future benefits to the current year t, but of course they will only have to be paid out once individuals have retired.

2.3.3 Accidental bequests

Accidental bequests do not have any significant bearing on our results. Their only role is to ensure that resources do not "disappear" because people die. The financial assets (wealth) left by those who die are all collected by the government. The aggregate of these accidental bequests in the economy amounts to:

$$H_t = \sum_{j=2}^{D} \left(1 - \psi_{j,t-j+1}\right) \frac{N_{j-1,t-1}}{I} \sum_{i=1}^{I} a_{i,j,t} = \sum_{j=2}^{D} \frac{\left(N_{j-1,t-1} - N_{j,t}\right)}{I} \sum_{i=1}^{I} a_{i,j,t},$$

where $a_{i,j,t}$ are the assets accumulated by each individual in cohort j in skill class i at the end of period $t-1$ and which become available for collection by the government at the start of period t.

The government redistributes H_t equally over all individuals alive at time t, resulting in an individual transfer

$$h_t = \frac{H_t}{\sum_{j=1}^{D} N_{j,t}}. \tag{17}$$

2.4 The individual decision problem

In a given period t an individual of skill group i in cohort j chooses a sequence of nominal consumption levels for the rest of her life. Savings are then invested in a portfolio of bond, equity and housing assets. Hence, the individual solves:

$$V_{i,j,t} = \max_{\{c_{i,j+l,t+l}\}_{l=0}^{D-j}} E_t \left[\sum_{l=0}^{D-j} \frac{\beta^l}{\psi_{j,t-j+1}} \left(\prod_{k=0}^{l} \psi_{j+k,t-j+1} \right) u \left(\frac{c_{i,j+l,t+l}}{\prod_{k=0}^{l} \left(1 + \pi_{t+k}\right)} \right) \right],$$

where $u(.)$ is the period utility function, which we assume to be of the conventional CRRA format with coefficient of relative risk aversion $\gamma > 0$,

$$u(x) = \frac{x^{1-\gamma} - 1}{1 - \gamma}$$

subject to equations (1)-(17), and the intertemporal budget constraint

$$a_{i,j+l+1,t+l+1} = \left\{ \begin{array}{ll} \left(1 + r_{t+l+1}\right)\left(a_{i,j+l,t+l} - c_{i,j+l,t+l}\right) & \\ +y_{i,t+l+1,t+l+1} - p^F_{i,t+l+1,t+l+1} - p^S_{i,t+l+1,t+l+1} + h_{t+l+1} & \text{if } j+l < R \\ \left(1 + r_{t+l+1}\right)\left(a_{i,j+l,t+l} - c_{i,j+l,t+l}\right) & \\ +b^F_{t+l+1} + b^S_{i,t+l+1,t+l+1} + h_{t+l+1} & \text{if } j+l \geq R \end{array} \right\},$$

where $a_{i,j+l,t+l}$ are the assets (wealth plus income) in year $t+l$ of an individual in skill group i of cohort $j+l$ and

$$1 + r_{t+l+1} = \left(1 - x^e_{j+l} - x^h_{j+l}\right)\left(1 + r^{sb}_{t+l+1}\right) + x^e_{j+l}\left(1 + r^e_{t+l+1}\right) + x^h_{j+l}\left(1 + r^h_{t+l+1}\right),$$

is the overall return on her asset portfolio in period $t+l+1$, the composition of which is age-specific and characterised by the exogenous weights $\left\{x^e_{j+l}, x^h_{j+l}\right\}$ at the end of period $t+l$. The individual earns returns on the investments in short-maturity bonds r^{sb}_{t+l+1}, equities r^e_{t+l+1}, and housing market r^h_{t+l+1}. Note that in contrast to the pension fund's portfolio, the individual's portfolio does not include holdings in long-maturity bonds. The exclusion of long-term bonds from the individual's portfolio has no consequences for the results. Also note that the individual's portfolio varies with age, but for given age is assumed to be fixed across skill groups. The end of next period's assets equal the gross return on this period's assets minus consumption, plus "net income". For the workers, net income is labour income minus social security contributions plus the accidental bequest, while for the retired net income equals the sum of the social security benefits plus the accidental bequest. Note that only the second-pillar benefit is cohort- and skill-specific.

The Euler equation for this problem is

$$u'\left(c_{i,j+l,t+l}\right) = \beta\psi_{j+l+1,t-j+1}E_{t+l}\left[\frac{1+r_{t+l+1}}{1+\pi_{t+l+1}}u'\left(\frac{c_{i,j+l+1,t+l+1}}{1+\pi_{t+l+1}}\right)\right]. \tag{18}$$

2.5 Shocks

We assume that there are no individual-specific shocks. In our model, eight types of exogenous macro-economic shocks hit the economy. Specifically, we consider demographic shocks (to the growth rate of the cohort of the newborn cohort and to the survival probabilities), productivity shocks (to the income growth rate), inflation rate shocks and financial market shocks (to equity returns, housing returns, the bond yield curve and the swap curve). All these shocks are collected in the vector $\omega_t = \left[\epsilon^n_t, \epsilon^\psi_t, \epsilon^g_t, \epsilon^\pi_t, \epsilon^e_t, \epsilon^h_t, \epsilon^b_{1,t}, ..., \epsilon^b_{D,t}, \epsilon^s_{1,t}, ..., \epsilon^s_{D,t}\right]$ with elements

- ϵ^n_t: shock to the newborn cohort growth rate, n_t
- ϵ^ψ_t: shock to the set of survival probabilities, $\left\{\psi_{j,t-j+1}\right\}^D_{j=1}$
- ϵ^g_t: shock to the nominal income growth rate, g_t
- ϵ^π_t: shock to the inflation rate, π_t
- ϵ^e_t: shock to the nominal equity return, r^e_t
- ϵ^h_t: shock to the housing return, r^h_t
- $\epsilon^b_{k,t}, k = 1, 2, ..., D$: shock to the nominal bond return at maturity k, $r_{k,t}$
- $\epsilon^s_{k,t}, k = 1, 2, ..., D$: shock to the swap return at maturity k, $r_{k,t}$

All these shocks affect the size of the funding ratio (equation (11)), whereas only demographic shocks affect the first-pillar pension system (equation (7)). As a consequence, the key parameters of the pension system have to be adjusted to restore the balance in the first pillar and to maintain sustainability of the second pillar.

Each demographic shock is distributed independently of all other shocks. The growth rate n_t of the newborn cohort depends on deterministic and random components:

$$n_t = n + \epsilon^n_t,$$

with n the mean and ϵ^n_t the innovation at time t, which follows an AR(1) process with parameter φ:

$$\epsilon^n_t = \varphi\epsilon^n_{t-1} + \eta^n_t, \quad \eta^n_t \sim N\left(0, \sigma^2_n\right).$$

The survival probabilities evolve according to a Lee-Carter model (see Appendix Section 6.2.2 for details):

$$\ln\left(1-\psi_{j,t-j+1}\right)=\ln\left(1-\psi_{j,t-j}\right)+\tau_j\left(\chi+\epsilon^{\psi}_{t-j+1}\right),\quad \epsilon^{\psi}_{t-j+1}\,\tilde{}\,N\left(0,\sigma^2_{\psi}\right),\quad j=1,...,D.$$

with τ_j an age-dependent coefficient, χ a constant growth factor (to describe the historical trend increase in survival probabilities) and ϵ^{ψ}_{t-j+1} an innovation at time $t-j+1$ that follows an i.i.d. process with variance σ^2_{ψ}.

We allow the shocks to the inflation rate, the nominal income growth, the one-year bond return, the equity return, and the housing return to be correlated with each other and over time. These variables feature the following multivariate process:

$$\begin{pmatrix}\pi_t\\ g_t\\ r^b_{1,t}\\ r^e_t\\ r^h_t\end{pmatrix}=\begin{pmatrix}\pi\\ g\\ r^b_1\\ r^e\\ r^h\end{pmatrix}+\begin{pmatrix}\epsilon^{\pi}_t\\ \epsilon^{g}_t\\ \epsilon^{b}_{1,t}\\ \epsilon^{e}_t\\ \epsilon^{h}_t\end{pmatrix},$$

with means $\left(\pi,g,r^b_1,r^e,r^h\right)'$ and innovations $\left(\epsilon^{\pi}_t,\epsilon^{g}_t,\epsilon^{b}_{1,t},\epsilon^{e}_t,\epsilon^{h}_t\right)'$ following a VAR(1) process,

$$\begin{pmatrix}\epsilon^{\pi}_t\\ \epsilon^{g}_t\\ \epsilon^{b}_{1,t}\\ \epsilon^{e}_t\\ \epsilon^{h}_t\end{pmatrix}=\mathbf{B}\begin{pmatrix}\epsilon^{\pi}_{t-1}\\ \epsilon^{g}_{t-1}\\ \epsilon^{b}_{1,t-1}\\ \epsilon^{e}_{t-1}\\ \epsilon^{h}_{t-1}\end{pmatrix}+\begin{pmatrix}\eta^{\pi}_t\\ \eta^{g}_t\\ \eta^{b}_{1,t}\\ \eta^{e}_t\\ \eta^{h}_t\end{pmatrix}, \tag{19}$$

with

$$\begin{pmatrix}\eta^{\pi}_t\\ \eta^{g}_t\\ \eta^{b}_{1,t}\\ \eta^{e}_t\\ \eta^{h}_t\end{pmatrix}\tilde{}\,N\left(\mathbf{0},\Sigma_f\right).$$

We finally turn to the swap curve $\left\{r^s_{k,t}\right\}^D_{k=1}$ and the bond yield curve $\left\{r^b_{k,t}\right\}^D_{k=1}$. Consistent with the prevailing literature (see, e.g., Evans and Marshall, 1998; Dai and Singleton, 2000) we assume that the swap curve follows the process:

$$\begin{pmatrix}r^s_{1,t'}\\ r^s_{2,t'}\\ \vdots\\ r^s_{D,t'}\end{pmatrix}=\begin{pmatrix}r^s_1\\ r^s_2\\ \vdots\\ r^s_D\end{pmatrix}+\begin{pmatrix}\epsilon^s_{1,t'}\\ \epsilon^s_{2,t'}\\ \vdots\\ \epsilon^s_{D,t'}\end{pmatrix}, \tag{20}$$

where t' indicates the month[2] and the vector of innovations follows a vector autoregressive distributed lag (VADL) process of order 1,

[2] Notice that here one period is a month rather than one year. We need a higher frequency to obtain a large enough data set.

$$\begin{pmatrix} \epsilon^s_{1,t'} \\ \epsilon^s_{2,t'} \\ \vdots \\ \epsilon^s_{D,t'} \end{pmatrix} = \mathbf{\Gamma}_0 + \mathbf{\Gamma}_1 \begin{pmatrix} \epsilon^s_{1,t'-1} \\ \epsilon^s_{2,t'-1} \\ \vdots \\ \epsilon^s_{D,t'-1} \end{pmatrix} + \mathbf{\Gamma}_2 \begin{pmatrix} \epsilon^\pi_{t'-1} \\ \epsilon^g_{t'-1} \\ \epsilon^b_{1,t'-1} \\ \epsilon^e_{t'-1} \\ \epsilon^h_{t'-1} \end{pmatrix} + \begin{pmatrix} \eta^s_{1,t'} \\ \eta^s_{2,t'} \\ \vdots \\ \eta^s_{D,t'} \end{pmatrix}, \tag{21}$$

and

$$\begin{pmatrix} \eta^s_{1,t'} \\ \eta^s_{2,t'} \\ \vdots \\ \eta^s_{D,t'} \end{pmatrix} \tilde{} N\left(\mathbf{0}, \Sigma^s\right).$$

At every time t', the swap return at maturity k, $r^s_{k,t'}$, is given by the sum of the average value r^s_k and the innovation $\epsilon^s_{k,t'}$. The innovation is a linear combination of deterministic and random components. The deterministic part is a function of several variables at time $t'-1$: the innovations at all maturities k, and the innovations in (19) converted to monthly frequency (see Appendix Section 6.2.4). The random part is given by the shock $\eta^s_{k,t'}$ (correlated across maturities).

Since $E\left[r^s_{k,t'}\right] = E\left[r^s_{k,t'-1}\right]$, because of stationarity, the average swap curve is given by the expression

$$\begin{pmatrix} \overline{r}^s_1 \\ \overline{r}^s_2 \\ \vdots \\ \overline{r}^s_D \end{pmatrix} = \begin{pmatrix} r^s_1 \\ r^s_2 \\ \vdots \\ r^s_D \end{pmatrix} + \left(I - \mathbf{\Gamma}_1\right)^{-1} \mathbf{\Gamma}_0. \tag{22}$$

The bond yield curve $\left\{r^b_{k,t}\right\}_{k=1}^D$ is constructed analogously. The one-year bond interest rate $r^b_{1,t}$ is already determined via the VAR process (19). The remaining parts of the curve are modelled analogously to the swap curve:

$$\begin{pmatrix} r^b_{2,t'} \\ r^b_{3,t'} \\ \vdots \\ r^b_{D,t'} \end{pmatrix} = \begin{pmatrix} r^b_2 \\ r^b_3 \\ \vdots \\ r^b_D \end{pmatrix} + \begin{pmatrix} \epsilon^b_{2,t'} \\ \epsilon^b_{3,t'} \\ \vdots \\ \epsilon^b_{D,t'} \end{pmatrix}$$

with $\begin{pmatrix} \epsilon^b_{2,t'} & \epsilon^b_{3,t'} & \ldots & \epsilon^b_{D,t'} \end{pmatrix}'$ following a VADL(1) process similar to (21). Appendix 6.2.5 provides details on the computation of the parameters of the bond yield curve. Realisations of the 9- and 10-year bond returns determine the return on long-term bonds of the fund's portfolio, $r^{lb}_t = \frac{\left(1+r^b_{10,t-1}\right)^{10}}{\left(1+r^b_{9,t}\right)^9}$; realisations of the 1-year bond returns determine the return on short-term bonds of an individual's portfolio, $1 + r^{sb}_t = 1 + r^b_{1,t}$.

The simulations conducted below take place at the annual frequency. Therefore, in those simulations we apply (21) (and the corresponding model for the bond yields) twelve times for a given year t, and use the last realisations as our annual swap curve $\left\{r^s_{k,t}\right\}_{k=1}^D$ and annual bond yield curve $\left\{r^b_{k,t}\right\}_{k=1}^D$. For more details. see the Appendices 6.2.4 and 6.2.5.

2.6 Policy interventions

We assume that the government automatically adjusts the contribution rate $\theta_t^F \in [0,1]$ to maintain (7) and thus a balanced first pillar of the social security system. On average, this contribution rate increases over the years along with population ageing.

As far as the second pillar is concerned, policy works as follows (the rule is formally described in Appendix 6.1). We define a state variable $\tau \in \{0,1\}$ to indicate whether policy intervention takes place or not. When the funding ratio (11) lies between $1+\xi^m$ and $1+\xi^u$, $\tau = 0$ and no intervention is undertaken. When the ratio falls below $1+\xi^m$ or goes above $1+\xi^u$, $\tau = 1$ and intervention takes place. In particular, when the ratio falls below $1+\xi^m$ a long-term restoration plan is started, while when it falls below $1+\xi^l$ ($\xi^l < \xi^m$), a situation referred to as "underfunding", a short-term restoration plan is started. When the ratio exceeds $1+\xi^u$ ($\xi^u > \xi^m$), measures are taken to reduce the funding ratio. Hence, policy is aimed at keeping the funding ratio within the interval $[1+\xi^m, 1+\xi^u]$. This is achieved by following an "indexation policy", of which the primary instruments are changes in the parameters $\iota_t \in [0,1]$ and $\kappa_t \in [0,1]$ that capture the degree of indexation to real income growth, $\frac{1+g_t}{1+\pi_t} - 1$, and inflation, π_t, respectively. In the case of a short-term or a long-term restoration plan, first productivity indexation ι_t is reduced up to a minimum of zero. Then, if necessary, price indexation κ_t is reduced up to a minimum of zero, followed by an increase in the contribution rate θ_t^S up to a maximum $\theta^{S,\max}$. If this is still not enough in the case of underfunding, then nominal claims are scaled back by whatever amount is necessary to eliminate the underfunding within the allowed restoration period. In the case of a long-term restoration plan, no further action is undertaken and so nominal pension claims are unaltered. When the fund is in a situation of "overfunding", i.e. the funding ratio is above $1+\xi^u$, first any reductions in nominal pension rights are given back. Then, if the funding ratio allows this, any missed price indexation is restored, followed by the restoration of any missed productivity indexation. Then, if the funding ratio is still not expected to return to below $1+\xi^u$ within the allowed time span, the contribution rate is reduced up to a minimum of zero.

Policymakers start period t with a combination of parameters $\left\{\theta_t^S, \kappa_t, \iota_t\right\}$ determined in the preceding period and a funding ratio $F_t = A_t/L_t$. When $t = 1$, $\left\{\theta_t^S, \kappa_t, \iota_t\right\} = \left\{\theta^S, \kappa, \iota\right\}$ and the funding ratio F_t is assumed to be between the boundaries $1+\xi^m$ and $1+\xi^u$. The exact policy parameter combination $\left\{\theta_{t+1}^S, \kappa_{t+1}, \iota_{t+1}\right\}$ for year $t+1$ is determined from a projection $\widetilde{F}_{t+1}$ of the funding ratio at time $t+1$, computed from the fund's levels of assets A_t and liabilities $L_{j,t} = \frac{1}{I}\sum_{i=1}^{I} L_{i,j,t}$ to the various cohorts in year t (averaged over the skill groups), and under the assumption of no further shocks $\left(i.e., \omega_{t+1} = \underset{(2D+6)\times 1}{\mathbf{0}}\right)$.

Indexation We consider different indexation schedules for the nominal second pillar pension rights. Under "uniform" indexation, indexation rates are identical to everybody. Under "status-dependent" indexation, retirees always receive full income indexation. Finally, under "contingent" indexation, the actual indexation rates vary across cohorts and/or skill groups.

Baseline: uniform indexation

In a given year indexation rates are identical across all individuals. That is,

$$1+\omega_{i,j,t} = \left(1+\iota_t\left(\frac{1+g_t}{1+\pi_t} - 1\right)\right)(1+\kappa_t\pi_t). \tag{23}$$

Status-dependent indexation

Now the amount of indexation depends on the "status" of the individual, worker or retiree. Retirees always receive full (price plus productivity) indexation, whatever is the size of the funding ratio. Hence, the purchasing power of their pension grows at the real-income growth rate. By contrast, indexation for workers is subject to uncertainty and can vary between zero (no indexation) and full indexation for both price and productivity increases. Hence,

$$1+\omega_{i,j,t}=\left\{\begin{array}{ll}\left(1+\iota_t\left(\frac{1+g_t}{1+\pi_t}-1\right)\right)(1+\kappa_t\pi_t) & \textbf{if } j\leq R\\ 1+g_t & \textbf{if } j>R\end{array}\right\}. \tag{24}$$

Contingent indexation

In this case, different groups in the population receive a different indexation, according to the formula

$$1+\omega_{i,j,t}=(1+g_t)+\left(1+(\iota_t-1)\left(\frac{1+g_t}{1+\pi_t}-1\right)\right)(1+(\kappa_t-1)\,\pi_t)\,f\,(i,j,\tau) \tag{25}$$

When real productivity growth and inflation are both positive and indexation is not full ($\iota_t<1$ or $\kappa_t<1$) then the term in square brackets on the right-hand side is negative, implying that $\omega_{i,j,t}<g_t$. The short-fall of actual indexation from nominal income growth may be differentiated across skill-groups and cohorts through the term $f\,(i,j,\tau)$.

According to equation (25), indexation depends on two components. A target level given by nominal income growth, $1+g_t$, and a correction that is necessary to keep the funding ratio in the interval $[1+\xi^m,1+\xi^u]$. The correction depends on the policy parameters ι_t and κ_t that are equal for all individuals and on a rescaling function $f\,(i,j,\tau)$ specific to each cohort and skill group, defined as follows:

$$f\,(i,j,\tau)=\left\{\begin{array}{cc}1 & \tau=0\\ g\,(i,j) & \tau=1\end{array}\right\} \tag{26}$$

The idea behind equation (26) is to make the correction larger for some population groups. Notice that there is no correction when $\iota_t=\kappa_t=1$, and that indexation is identical for all the cohorts when $\tau=0$, i.e., when the funding ratio is between $1+\xi^m$ and $1+\xi^u$ and there is no policy intervention. The indexation formula (25) is aimed at avoiding systematic redistribution of benefits across groups, but allows for more vigorous responses across groups of the degree of indexation to the pension funding ratio.

We concentrate on two specifications for the function $g\,(i,j)$: (1) an *age-dependent* function, where the correction is differentiated across cohorts, while all the skill groups within a skill group receive the same amount of indexation and (2) an *income-dependent* function, where the correction is differentiated across skill groups, while all the cohorts of working age within the same skill group receive the same amount of indexation.

With the age-dependent function, each cohort j receives a different amount of indexation depending on the average stock of nominal rights $\overline{M}_j$ it has accumulated so far,

$$g\,(i,j)=g^a\,(i,j)=\left\{\begin{array}{cc}\alpha_1-\alpha_2\frac{\overline{M}_j}{\overline{M}} & j\leq R\\ 1 & j>R\end{array}\right\}, \tag{27}$$

where $\alpha_1>1$ and $\alpha_2>0$ are two exogenous parameters, $\overline{M}_j$ are the average amount of nominal rights accumulated by all the skill groups belonging to cohort j,

$$\overline{M}_j = \frac{1}{I}\sum_{i=1}^{I}\overline{M}_{i,j}$$

Here, $\overline{M}$ is the average across the $\overline{M}_j$ of the working population,

$$\overline{M} = \frac{1}{\sum_{j=1}^{R} N_{j,1}}\sum_{j=1}^{R} N_{j,1}\overline{M}_j,$$

and the skill- and cohort-specific average stock of nominal rights, $\overline{M}_{i,j,1}$, is computed from equation (10) under the assumption that at $t = 1$, we have $\iota_t = \kappa_t = 1, m_t = 0$ and there are no shocks in the economy – that is, all the variables denoted by an ϵ symbol in Section 2.6 are equal to 0. Hence, the formulation with (27) ensures that for cohorts with a larger stock of nominal rights (the older workers) indexation reacts less vigorously to movements of the funding ratio outside the range $[1 + \xi^m, 1 + \xi^u]$. In particular, if the funding ratio falls below $1 + \xi^m$ and therefore (in the next period) $\iota_t < 1$ or $\kappa_t < 1$, then the shortfall of $\omega_{i,j,t}$ from g_t will be smaller for those cohorts. However, when the funding ratio rises above $1 + \xi^u$ and (to restore lost indexation in the past) the fund sets $\iota_t > 1$ or $\kappa_t > 1$, then the increase of $\omega_{i,j,t}$ beyond g_t will also be smaller for these groups.

With the income-dependent function, each skill class i receives a different amount of indexation:

$$g(i,j) = g^s(i,j) = \left\{ \begin{array}{cc} \nu_1 - \nu_2(10 - i) & j \leq R \\ 1 & j > R \end{array} \right\} \tag{28}$$

where $\nu_1 > 1$ and $\nu_2 > 0$ are two exogenous parameters. Hence, the higher is the skill level of the individual (i is higher), the more vigorously the indexation of his pension rights reacts to movements of of the funding ratio outside the range $[1 + \xi^m, 1 + \xi^u]$.

In both equations (27) and (28) $g(i,j) = 1$ for $j > R$. Contingent indexation is abolished in favour of uniform indexation when the funding ratio is restored to a level within the region $[1 + \xi^m, 1 + \xi^u]$. We want the funding ratio to be unaffected by the specific indexation formula in this situation. Indexation enters the funding ratio equation through total benefits and the present value of liabilities. The requirement that $g(i,j) = 1$ for $j > R$ implies that total benefits coincide under uniform and contingent indexation.

We also choose the parameters $\{\alpha_1, \alpha_2\}$ and $\{\nu_1, \nu_2\}$ in such a way to generate a present value of liabilities identical to the one under uniform indexation (see the discussion of the calibration in Section 3).

2.7 Welfare measures

We consider three measures of welfare, one is cohort- and skill-specific and the other two are economy-wide. The first is the intertemporal utility function $V_{i,j,t}$ for skill class $i \in \{1, ..I\}$, cohort $j \in \{1, ..D\}$ in year t. The second measure, S_t^A, is defined as the unweighted average of the intertemporal utilities of all individuals alive in period t:

$$S_t^A = \sum_{j=1}^{D}\frac{N_{j,t}}{\sum_{j=1}^{D} N_{j,t}}\frac{1}{I}\sum_{i=1}^{I} V_{i,j,t}.$$

The third measure, S_t^T, is defined as the unweighted average of the intertemporal utilities of all alive and unborn individuals:

$$S_t^T = \sum_{j=1}^{D} \frac{N_{j,t}}{\sum_{j=1}^{D} N_{j,t}} \frac{1}{I} \sum_{i=1}^{I} V_{i,j,t} + \sum_{s=1}^{\infty} \frac{N_{1,t+s}}{\sum_{j=1}^{D} N_{j,t}} \frac{\frac{1}{I}\sum_{i=1}^{I} V_{i,1,t+s}}{(1+q)^s}. \tag{29}$$

where q is the rate at which future generations' intertemporal utilities are discounted. In the simulations, we truncate the computation of welfare to 250 unborn generations, as the discounted welfare of subsequent generations is negligible in equation (29). Note that in equation (29) the size of any unborn generation is normalised to the size of the population alive in year t.

To ease the interpretation of the three measures $V_{i,j,t}$, S_t^A and S_t^T, we report them in terms of constant consumption flows. As regards the cohort-specific measure $V_{i,j,t}$, we define "certainty equivalent consumption" $CEC_{i,j,t}$ for skill class $i \in \{1,..I\}$, cohort $j \in \{1,..D\}$ in year t, as the certain, constant consumption level over the remainder of the individual's lifetime that yields a utility level equal to the utility level under the relevant scenario. Hence,

$$CEC_{i,j,t} = u^{-1}\left(\frac{V_{i,j,t}}{E_t\left[\sum_{l=j}^{D} \frac{\beta^{l-j}}{\psi_{j,t-j+1}} \left(\prod_{k=0}^{l-j} \psi_{j+k,t-j+1}\right)\right]} \right) \tag{30}$$

Analogously, for the economy-wide measures we define the constant consumption flows

$$C_t^{\Upsilon} = u^{-1}\left(\frac{S_t^{\Upsilon}}{\sum_{l=\overline{J}+1}^{D} \frac{\beta^{l-(\overline{J}+1)}}{\psi_{\overline{J}+1,t-\overline{J}}} \left(\prod_{k=0}^{l-(\overline{J}+1)} \psi_{(\overline{J}+1)+k,t-\overline{J}}\right)} \right), \quad \Upsilon = A, T, \tag{31}$$

of an agent with the average age $\overline{J}$ in the economy in year t,

$$\overline{J} = \text{integer}\left[\frac{\sum_{j=1}^{D} j N_{j,t}}{\sum_{j=1}^{D} N_{j,t}} \right],$$

where integer [.] is the function that generates the largest integer smaller than or equal to the number inside the square brackets. Note that this is the constant consumption stream of a person of age $\overline{J}$ that gives her utility equal to social welfare S_t. It is *not* the constant consumption stream that gives a person of age $\overline{J}$ the utility level that he has under the relevant policy. There is a one-to-one relationship between the levels of C_t and S_t as calculated in (31).

We also define the skill-group i specific counterparts $S_{i,t}^A$ and $S_{i,t}^T$ to S_t^A and S_t^T:

$$S_{i,t}^A = \sum_{j=1}^{D} \frac{N_{j,t}}{\sum_{j=1}^{D} N_{j,t}} V_{i,j,t} \text{ and } S_{i,t}^T = \sum_{j=1}^{D} \frac{N_{j,t}}{\sum_{j=1}^{D} N_{j,t}} V_{i,j,t} + \sum_{s=1}^{\infty} \frac{N_{1,t+s}}{\sum_{j=1}^{D} N_{j,t}} \frac{V_{i,1,t+s}}{(1+q)^s}.$$

Using $S_{i,t}^A$ and $S_{i,t}^T$, we calculate (analogous to (31)) skill-group specific constant consumption flows $C_{i,t}^A$ and $C_{i,t}^T$.

2.7.1 Comparison of policy scenarios

We evaluate welfare under some scenario A relative to the welfare under some scenario B. The scenarios differ in the way they distribute indexation among the individuals. In both scenarios the parameters are initially identical and equal to those in the benchmark calibration. They are kept unchanged in the ensuing years as long as the funding ratio remains between $1+\xi^m$ and $1+\xi^u$. Once the funding ratio falls below $1+\xi^m$ or goes above $1+\xi^u$, the policy parameters change according to the relevant indexation policy.

We consider four measures of welfare comparison between the two policies. A first natural measure is the constant percentage difference in certainty equivalent consumption between the two scenarios. For each skill class and cohort in a given period this measure is computed as:

$$\Delta CEC_{i,j,t}(A,B) \equiv \frac{CEC_{i,j,t}(B) - CEC_{i,j,t}(A)}{CEC_{i,j,t}(A)}, \tag{32}$$

where $CEC_{i,j,t}(s)$ denotes the value of $CEC_{i,j,t}$ under scenario $s \in \{A,B\}$. We use three further measures to compare welfare between the two policies in a single number. One is the "majority support" for policy B, that is the share of people that are better off under B rather than under A:

$$D_t(A,B) \equiv \frac{1}{\sum_{j=1}^{D} N_{j,t}} \sum_{j=1}^{D} \frac{N_{j,t}}{I} \sum_{i=1}^{I} \mathbf{1}\left\{CEC_{i,j,t}(B) > CEC_{i,j,t}(A)\right\}, \tag{33}$$

where $\mathbf{1}\{.\}$ is an indicator function that assigns a value of one (zero) if the condition inside the curly brackets holds (does not hold). The final two measures of welfare comparison are the "social welfare gain" from using policy B rather than policy A, excluding, respectively including, the welfare of the unborn generations:

$$\Delta C_t^{\Upsilon}(A,B) \equiv \frac{C_t^{\Upsilon}(B) - C_t^{\Upsilon}(A)}{C_t^{\Upsilon}(A)}, \quad \Upsilon = A, T. \tag{34}$$

3 Calibration and details on the simulation

We follow the standard literature and calibrate the exogenous parameters of the model to reproduce the main features of the US economy. However, the pension arrangement is calibrated to the Dutch situation. Tables 1 and 2 summarise our benchmark calibration.

We assume that the economically active life of an agent starts at age 25. Individuals work for $R = 40$ years until they reach age 65. They live for at most $D = 75$ years, until age 100. Their coefficient of relative risk aversion γ is set to 2, in line with a large part of the macro-economic literature. The discount factor β is set to 0.98, slightly above the usual choice of 0.96 because individuals also take into account their survival probabilities. To compute the welfare measure (29) we try several discount rates q for the utility of unborn generations. However, qualitatively the results are insensitive to the specific value of q and we simply set $q = 4\%$. The age-dependent portfolio composition $\left\{x_j^e, x_j^h\right\}_{j=1}^{D}$ is taken from the mean values of the 2007 wave of the Survey of Consumer Finances (SCF, 2009).[3] Portfolio composition is reported by age groups and we interpolate the data using the spline method. We keep the portfolio weights constant as of age 90. The efficiency index $\{e_i\}_{i=1}^{I}$ is given by the income deciles in the U.S. for the year 2000 taken by the World Income Inequality Database (WIID, version 2.0c, May 2008). We normalise the index to

[3] We aggregate assets into three categories: bonds (transaction accounts, certificates of deposit, savings bonds, and bonds), equities (stocks, investment funds, cash value of life insurance, other assets) and housing (residental property).

have an average of 1. The seniority index $\{s_j\}_{j=1}^{I}$ uses the average of Hansen's (1993) estimation of median wage rates by age group. We take the average between males and females and interpolate the data using the spline method.

The exogenous social security parameters are specifically calibrated to the Dutch situation. For the first social security pillar we set the benefit scale factor $\rho^F = 0.17$. The Dutch Tax Office ("Belastingdienst") reports for 2008 a maximum income assessable for first-pillar contributions of EUR $3,850.40$ per month. We therefore set our upper income threshold for contributions $\delta^u = 1.10$, roughly equal to $3,850.40 * 12/42,403$, where EUR $42,403$ is our imputation of the economy's average income as of 2008.[4] The lower income threshold is set to $\delta^l = 0.33$, in such a way as to generate a starting contribution rate near 16% (it is $\theta_1^F = 16.42\%$), consistent with the reality. For the second social security pillar, historically the accrual rate has been between 1.5 and 2% and most frequently at 1.75%. We therefore consider $\mu = 0.0175$ and set the franchise to $\lambda = 0.33$ in order to generate a realistic average replacement rate of 37.60%. In the simulations we consider a short-term restoration period of length $K^s = 5$ when the pension buffer falls below $1 + \xi^l$ and a long-term restoration period of length $K^l = 15$ when the pension buffer falls below $1 + \xi^m$, but remains at or above $1 + \xi^l$. Further, we set the boundaries $\left\{\xi^l, \xi^m, \xi^u\right\}$ for the buffer at $\left\{\xi^l, \xi^m, \xi^u\right\} = \{5\%, 25\%, 60\%\}$.

In the benchmark simulations we set $\left\{z^e, z^h\right\} = \{45\%, 5\%\}$ for any level of F_{t-1}. Our choice corresponds to the balance sheet average for Dutch pension funds over the period 1996 - 2005 (source: DNB, 2009). Because the various assets in the pension fund's portfolio generally have different realised returns, at the end of each period t its portfolio is reshuffled such that the system enters the next period $t + 1$ again with the original portfolio weights $\left\{z_{t+1}^e, z_{t+1}^h\right\} = \{45\%, 5\%\}$.

We set the starting levels of the indexation parameters at $\kappa_1 = \iota_1 = 100\%$. Hence, the pension fund starts by providing full indexation to nominal wages. To measure the funding ratio consistently across the policies, the parameters $\{\alpha_1, \alpha_2\}$ and $\{\nu_1, \nu_2\}$ in equations (27) and (28) must be chosen so that the contingent indexation policy generates the same level of total liabilities as the policy with uniform indexation. Among the infinite combinations of parameters with this property, in the benchmark case we choose parameter combinations such that the maximum correction is always three times larger than minimum indexation. This yields $\{\alpha_1, \alpha_2\} = \{2.1183, 0.4209\}$ and $\{\nu_1, \nu_2\} = \{1.1524, 0.0854\}$.

Figure 1 shows the functions $g^a(i, j)$ and $g^s(i, j)$ obtained with these parameters by age and for some skill classes. The figure informs that under age-dependent indexation cohorts aged 25 (in any skill group i) rescale the indexation correction by $g^a(i, 1) = 209.28\%$, whereas cohorts aged 64 (the last working year in the model) rescale the correction by $g^a(i, 1) = 69.76\%$, so one-third of that of the cohorts aged 25. Under income-contingent indexation, cohorts of any age $j < R$ belonging to the third skill group rescale the correction by 55.49%, whereas those belonging to the tenth skill groups rescale the correction by 115.24%, three times the rescaling factor adopted for the first skill group, 38.41%. However, we do not report in the figure the rescaling function of classes 1 and 2 as their income is on average below the franchise λ, which means that these classes contribute to the second pillar only when exogenous shocks bring their income above the franchise. Hence, their benefits are negligible (on average null) in the computation of the fund's liabilities.

[4]In Eurostat the most recent number on average income in the Netherlands refers to year 2005. The same source also provides the minimum income until year 2008. Exploiting the correlation between average and minimum income, we run an OLS regression of average income over year and minimum income. As a result, we predict the average income of year 2008 to be EUR $42,403$.

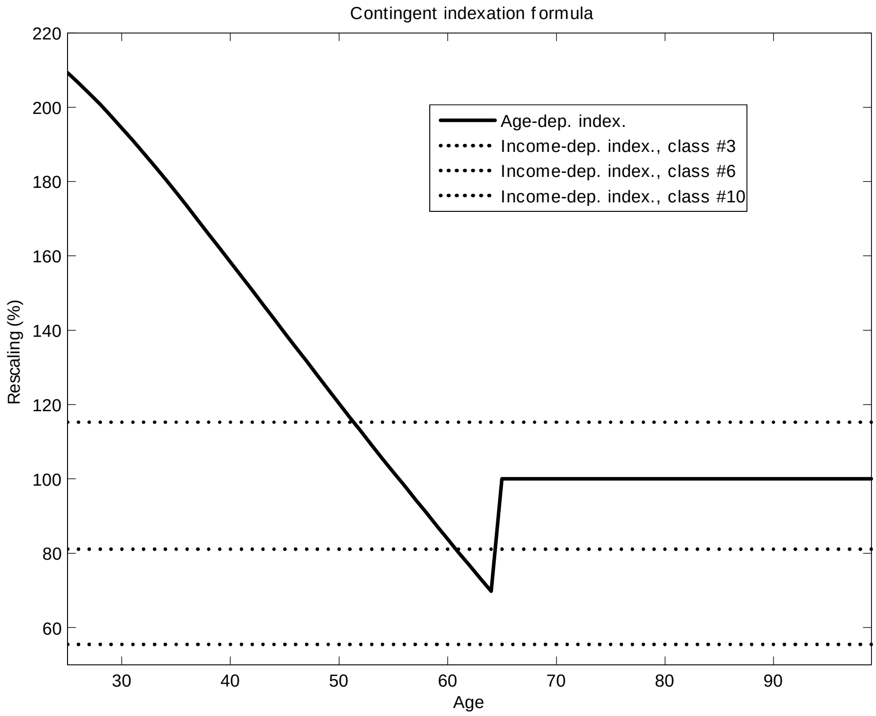


Figure 1. Contingent indexation

Finally, the starting contribution rate is set such that aggregate contributions at time 1 coincide with aggregate benefits in the absence of shocks. The rate that satisfies this condition is $\theta_1^S = 17.58\%$, which is close to the actual value in the Netherlands. We then choose initial assets A_0 that generate an initial funding ratio F_1 of 140% in the absence of shocks. Initial assets amount to roughly 1.9 times the initial level of income in the economy.[5] The contribution rate is capped at $\theta^{S,\max} = 25\%$.

[5] This is on the high side compared to the actual Dutch situation. However, in our model every worker participates in the pension fund, while in the Netherlands this is only part (though a majority) of those who are employed. A large fraction of the workers have their pension arranged through insurance companies, while the self-employed do not participate in pension funds either (they have the possibility to build up their pension through an insurance company, but the financial reserves of insurance companies are not considered part of the pension buffers).

Table 1. Calibration of the exogenous parameters

Symbol	Description	Calibration
General setting		
D	Number of cohorts (= maximum death age -25)	75
R	Number of working cohorts (= retirement age -25)	40
γ	relative risk aversion	2
β	Discount factor	0.98
q	Discount rate unborn generations	4%
$\left\{x_j^e, x_j^h\right\}_{j=1}^D$	Household portfolio composition	SCF (2009)
$\{e_i\}_{i=1}^I$	Efficiency index	WIID (2008)
$\{s_j\}_{j=1}^I$	Seniority index	Hansen (1993)
First-pillar parameters		
ρ^F	Benefit scale factor	0.17
$\left\{\delta^l, \delta^u\right\}$	Income thresholds in the contribution formula	$\{0.56\%, 1.10\%\}$
Second-pillar parameters		
μ	Accrual rate	1.75%
λ	Franchise share	0.33
$\left\{K^s, K^l\right\}$	Length of restoration periods	$\{5, 15\}$
$\left\{\xi^l, \xi^m, \xi^u\right\}$	Boundaries of pension fund buffer	$\{5\%, 25\%, 60\%\}$
$\left\{z^e, z^h\right\}$	Fund portfolio composition	$\{45\%, 5\%\}$
$\{\kappa_1, \iota_1\}$	Initial indexation	$\{100\%, 100\%\}$
$\{\alpha_1, \alpha_2\}$	Parameters for age-dependent indexation	$\{2.1183, 0.4209\}$
$\{\nu_1, \nu_2\}$	Parameters for income-dependent indexation	$\{1.1524, 0.0854\}$
θ_1^S	Initial contribution rate	17.58%
$\theta^{S,\max}$	Upperbound on contribution rate	25%

The deterministic growth rate of the newborn cohort, $\mu = 0.47362\%$, is the average growth from a regression using 20 observations on the annual variation in the number of births in the US between 1986 and 2005 (the source is the Human Mortality Database); details on the regression are in Appendix 6.2. This Appendix also describes our calibration of the survival probabilities based on the Lee-Carter model (Lee and Carter, 1992). The combination of survival probabilities and birth rates determines the size of each cohort. The starting value of the old-age dependency ratio (i.e., the ratio of retirees over workers) is 25.23%, in line with the OECD figures for 2005.

Crucial is the calibration of average price inflation, nominal income growth and the bond, equity and housing returns. We loosely follow the literature in this regard (see, e.g., Brennan and Xia, 2002, and van Ewijk et al., 2006) and set the average inflation rate at $\pi = 2\%$, the average nominal income growth rate at $g = 3\%$ (which corresponds to an average real productivity growth of 1% per year), the average one-year bond interest rate at $r_1^b = 3\%$, and the average housing return at $r^h = 4\%$. The average equity return is set at $r^e = 5.2\%$ to generate a funding ratio that is stable over time in the absence of shocks and policy parameter changes.[6] Innovations in these five variables follow the VAR(1) process described in Appendix 6.2.3. Appendixes 6.2.4 and 6.2.5 provide details on the calculation of the parameters of the process for the swap curve $\left\{r_{k,t}^s\right\}_{k=1}^D$ and the bond yield curve $\left\{r_{k,t}^b\right\}_{k=1}^D$.

[6] In this situation, the ratio is approximately constant for the first 20 years under uniform indexation, and still around 110% after 75 years. The first adjustment is made only after 44 years.

Table 2. Calibration of the averages of the random variables

Symbol	**Description**	**Calibration**
π	Inflation rate	2%
g	Nominal income growth rate	3%
r_1^b	Nominal bond return	3%
r^e	Nominal equity return	5.2%
r^h	Nominal housing return	4%

Note: see the Appendix for the stochastic component

To obtain the optimal consumption rules from equation (18) we solve the individual decision problem recursively by backward induction using the method of "endogenous gridpoints" (Carroll, 2006). To avoid the curse of dimensionality caused by having state variables for the shocks listed in Table 2, we determine the optimal consumption profile in year t under the assumption that the shocks in year $t-1$ are equal to their average, and in t there are innovations following the multivariate process (19). We approximate the random variable distributions by means of a Gauss-Legendre quadrature method (see Tauchen and Hussey, 1991) and discretise the state space using a grid of 100 points with triple exponential growth.[7] For points that lie outside the state space grid, we use linear extrapolation to derive the optimal rule.

We simulate $N = 1,000$ times a sequence of vectors of unexpected shocks over $2D - 1$ years, drawn from the joint distribution of all the shocks. The shocks are identical for all the cohorts that are alive in a given year. The number of years of one simulation run equals the time distance between the birth of the oldest cohort and the death of the youngest cohort. At each moment there are D overlapping generations. Our welfare analysis is however based on the economy as of the D^{th} year in the simulation. Hence, we track only the welfare of the cohorts that are alive in that year, implying that those that die earlier are ignored, and we track the welfare of cohorts born later, the latest one dying in the final period of the simulation. For the sake of simplicity, we label the relevant years for which we follow the economy and track welfare as $t = 1, ..., D$. The purpose of simulating the first $D - 1$ years is to simply generate a distribution of the assets held by each cohort at the end of $t = 0$.

In each simulation run, we assume that the ageing process stops after $t = 40$. That is, mortality rates at any given age no longer fall. This assumption is in line with the fact that some important ageing studies, such as those by the Economic Policy Committee and European Commission (2006) and the United Nations (2009), only project ageing (and its associated costs) up to 2050, hence roughly 40 years from now. Moreover, it is hard to imagine that mortality rates continue falling for many more decades at the same rate as they did in the past. In particular, many of the common mortal diseases have already been eradicated, while it will become more and more difficult to treat remaining lethal diseases. Effective treatment of those diseases will also surely be held back by the fact that the share of national income that can be spent on health care is bounded.

To allow for the cleanest possible comparison among the various indexation policies, we use the same shock series under all policies, while, moreover, during the initialisation phase of each simulation run no policy responses occur and all the cohorts receive uniform indexation. Hence, the starting situation at the start of $t = 1$ is identical in each run under the various policies. Because

[7] We create an equally-spaced grid of the function $log(1 + log(1 + log(1 + s)))$, where s is the state variable. The grid with "triple exponential growth" applies the transformation $exp(exp(exp(x) - 1) - 1) - 1$ to each point x of the equally-spaced grid. This transformation brings the grid back to the original scale of the state variable, but determines a higher concentration on the low end of possible values. A grid with triple exponential growth is more efficient than an equally-spaced grid as the consumption function is more sensitive to changes at small values of the state variable.

welfare depends on the size of the buffer after the initialisation period in the simulation run, we reset the stock of pension fund assets such that the buffer at the end of $t = 0$ equals 140%. Finally, the process z_t is re-normalised to unity at the end of $t = 0$ and the nominal pension claims of the various cohorts are rescaled accordingly. At the start of the preceding $D - 1$ dummy years, liabilities are set at the steady state values implied by the income level at that moment. They are computed using (10) under the assumption of no shocks (i.e. expectations are treated as if they are realised).

4 Results

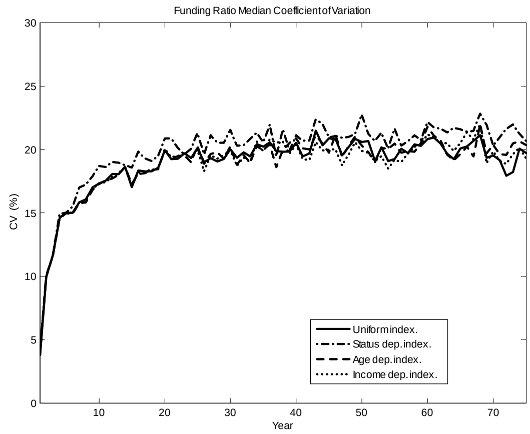
4.1 Benchmark

Figure 2 shows the median funding ratio resulting from our simulations.[8] Although all the four policies effectively keep the ratio within the $[1 + \xi^m, 1 + \xi^u]$ interval, the ratio under status-dependent policy is usually below the one under the other policies. The reason is that this policy is less effective at restoring the funding ratio, since it can modify the indexation of only part of the population to reach a desired target ratio.

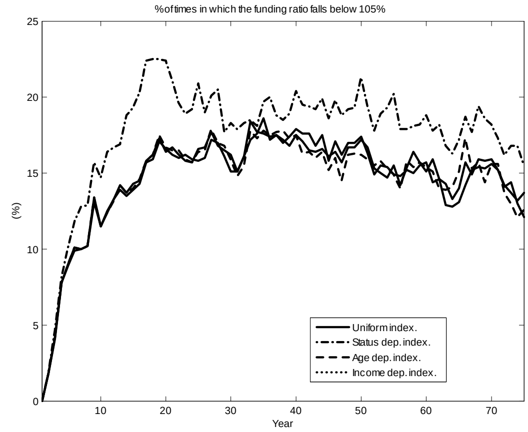
Figure 2. Median funding ratio

The funding ratio is slightly more volatile under status-dependent indexation policy (see Figure 3, panel a). With the same policy, furthermore, it is more likely that the funding ratio falls below the threshod $1 + \xi^l$ for underfunding (see Figure 3, panel b). In fact, under this policy the fund manager is more constrained in keeping the ratio stable, as it cannot change the indexation of the retirees.

[8]We report the median rather than the average, because the former is not affected by the few extreme outcomes in our simulations.



a. Coefficient of variation



b. Cases below 105%

Figure 3. Funding ratio volatility

Table 3 presents the summary statistics for the various indexation policies. Indeed, the status-dependent policy produces a more volatile funding ratio with a median coefficient of variation of about 20%, while in all other cases it is below 19%. For such policies the funding ratio lies below the threshold $1+\xi^m$ in about one-third of the cases and above $1+\xi^u$ also in about one-third of the cases. The two fractions indicate a rough overall balance between the number of times in which policy aims at keeping indexation low and contributions high if necessary and the number of times in which policy is aimed at the opposite configuration. Under status-contingent policy it is a little more likely to have low rather than high indexation.

Although the differences in volatility are rather small, there is a more pronounced difference in the probability of implementing a change in the policy parameters. The indexation parameters $\{\kappa_t, \iota_t\}$ are less frequently changed under the status-dependent policy (in 21.67% of the cases) than under the other policies (around 28% of the cases). When the parameters are changed more frequently, their standard deviation across simulations in a given year is smaller. In fact, under status-contingent policy, the adjustment in the parameters has to be larger because it affects a smaller part of the population (the workers only).

The final part of Table 3 reports several of welfare measures. Social welfare is highest under status-dependent policy when the future generations are neglected (C_1^A is highest) and under uniform indexation when they are taken into account (C_1^T is highest). Welfare differences are generally small to modest small. The largest difference is between a status-dependent policy able to generate for an average individual a certainty equivalent consumption increase of 0.60% relative to a uniform indexation policy. Despite this, we find a large majority support for a shift from uniform indexation to age-dependent indexation ($D_1 = 63.13\%$) and especially to status-dependent indexation ($D_1 = 97.09\%$). This suggests that the various cohorts and skill groups in the economy may experience very different welfare effects from a shift from uniform to alternative indexation policies.[9]

[9] It is worth pointing out that there is no exact correspondence between ΔC_1^A and D_1: the former measure converts into certain equivalent consumption the sum of all the individual value functions, while the latter first converts each individual value function. Since the conversion function is non-linear, Jensen's inequality applies and the two measures produce different outcomes.

Table 3. Benchmark comparison of the indexation variants

%	Uniform	Status-dep.	Age-dep.	Skill-dep.
		Ratio volatility		
Median coeff. of var.	18.8331	19.9060	18.9126	18.7851
Prob. of a ratio below $1+\xi^l$	14.6773	17.5773	14.5840	14.5747
Prob. of a ratio below $1+\xi^m$	31.7027	35.2293	31.6213	31.4480
Prob. of a ratio above $1+\xi^u$	33.1560	31.0733	33.4653	33.3507
Prob. of intervention	27.9275	21.6738	28.0148	27.7880
		Policy parameters		
κ_t, average	93.5513	90.1605	93.8798	93.6689
std. dev.	(200.7988)	(254.7742)	(203.3143)	(200.4585)
ι_t, average	83.0122	78.7285	83.8928	82.9427
std. dev.	(189.1137)	(226.6732)	(194.9137)	(190.3703)
θ_t^S, average	14.5508	15.4028	14.4780	14.5972
std. dev.	(10.2934)	(10.4342)	(10.2864)	(10.2852)
		Welfare		
C_1^A	75.2455	75.6932	75.1948	75.2756
ΔC_1^A in %	-	0.5950	-0.0674	0.0400
C_1^T	35.6420	34.0040	35.4488	35.6104
ΔC_1^T in %	-	-4.5956	-0.5419	-0.0885
D_1	-	97.0924	63.1346	42.0818

Note: 'Prob. of intervention' means that at least one of $\left\{\theta_t^S, \kappa_t, \iota_t\right\}$ *is changed.*

Figure 4 shows the inter-generational welfare comparison across the four policies, based on equation (32). A positive value of ΔCEC indicates that a given cohort is better-off under the alternative policy rather than under the uniform indexation policy.

The figure shows that almost all the cohorts are best off with a status-dependent indexation, while their welfare gains relative to uniform indexation policy expressed in CEC may be over 3% (for workers over 60 and "younger" retirees). Status-dependent indexation raises welfare as it eliminates uncertainty about indexation as of retirement. Cohorts in retirement years are clearly better-off with this policy, as they expect a certain degree of income indexation over all their remaining years. The welfare gain is larger the larger the potentially remaining years of life. The welfare increase as a result of a shift to status-dependent indexation is smaller the younger is worker. While workers will receive a safe level of indexation as of retirement, during the their working life they will face a more volatile degree of indexation (the "cost" of providing the elderly with safer indexation). Due to discounting, the higher degree of certainty at retirement will weigh less heavily the younger is the worker. Notice that cohorts aged 33 or younger at $t = 1$ are actually worse off under status-dependent indexation (although the welfare loss is small).

The welfare differences for the various cohorts between the other contingent indexation policies and the uniform policy are much smaller. They are very close to 0 for the retired, for whom there is no rescaling of the indexation correction ($f(i, j, \tau) = 1$ for $j > R$) and the policy parameters $\left\{\theta_t^S, \kappa_t, \iota_t\right\}$ are very close to their values under uniform indexation. As regards age-dependent indexation, we observe a small welfare gain for the cohorts near retirement (they face lower volatility in the indexation parameters) and for the youngest cohorts. The latter experience age-dependent indexation over their entire their lifespan. Hence, they face more volatility in the indexation parameters when they are young and smaller volatility when they are older. The latter effect dominates in the overall welfare effect as higher uncertainty when works on a low level of accumulated pension rights, while the higher degree of certainty when older applies to a larger stock of pension rights.

Unsurprisingly the welfare differences between the skill-dependent indexation policy and the uniform policy is small. After all, the curve is obtained through aggregation over all skill groups, while the idea of skill-dependent indexation is to make indexation less certain for some groups (who would lose out) and more certain for other groups (who would gain).

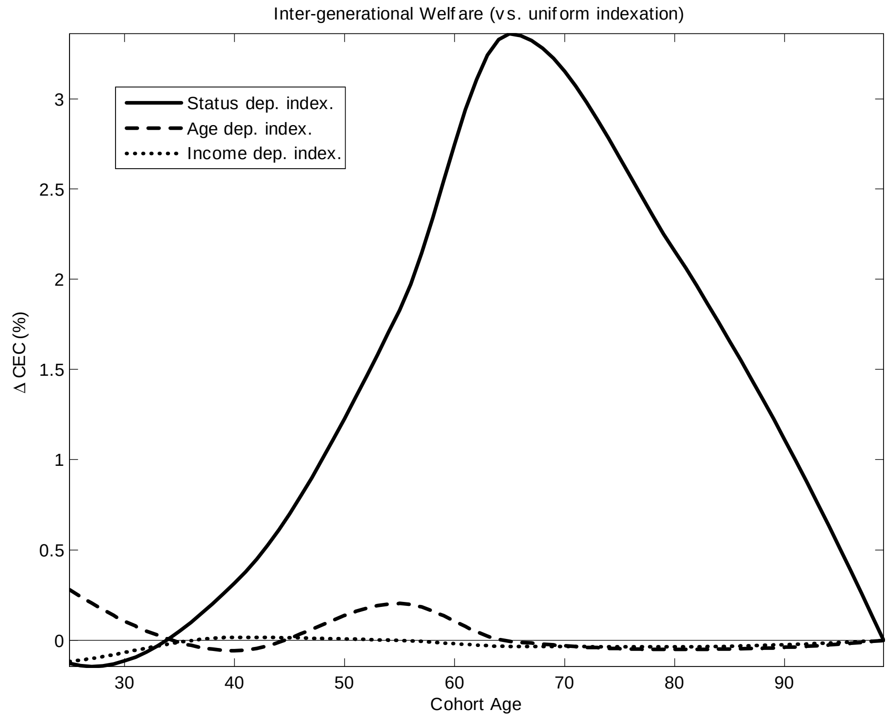


Figure 4. Intergenerational welfare comparison

$\Delta CEC > 0$: better-off with the contingent policy

Figure 5 considers the welfare difference between uniform and skill-dependent indexation (as a function of age) by skill group. It shows the graphs for skill groups 3, 6, and 10. It is clear from the figure that cohorts belonging to the lower skill groups benefit more from the policy, as they face smaller corrections of their indexation. Under this calibration, only the two highest skill groups actually face larger corrections. However, this is enough to neutralize the positive effects for the eight less-skilled groups when we compute the aggregate welfare across the groups.

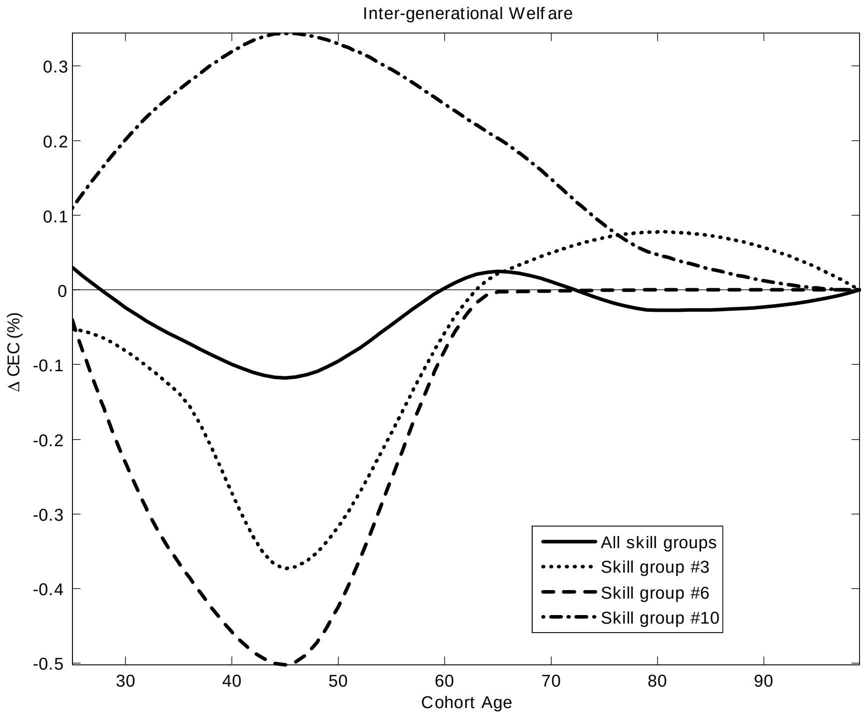


Figure 5. Welfare comparison, income-contingent indexation

4.2 Alternative policies

4.2.1 Fixed price indexation at retirement

We consider here a variant of status-dependent indexation, in which retirees always receive full indexation to price inflation (instead of to nominal income growth), but not more than that.

The second colum of Table 4 reports the key statistics from this variant. Compared to the benchmark status-dependent indexation policy of Table 3, this case generates a volatility of the funding ratio, average values and standard deviations of the policy parameters closer to those under uniform indexation and alternative contingent policies. Figure 6 (solid line) displays the inter-generational welfare comparison of this policy with the uniform indexation policy. Although the maximum attainable welfare gain is lower than under the benchmark status-dependent policy (it is at most 2.83% instead of 3.36%), younger generations now are also better-off, as the lower indexation to the retirees alleviates the burden of the parameter adjustment faced during the working years.

4.2.2 Age- and skill-dependent indexation combined

We combine here age-dependent indexation with skill-dependent indexation. Benefits are then computed using a rescaling function $f(i,j,\tau)$ defined as

$$f(i,j,\tau) = \left\{ \begin{array}{cc} 1 & \tau = 0 \\ g^a(i,j)\, g^s(i,j) & \tau = 1 \end{array} \right\}$$

The overall spread between the maximum correction (skill class 1, age R) and the minimum correction (skill class 10, age 1) is the product of the spread under each policy, that is, 900%.

The last column of Table 4 reports the summary statistics from the simulation of this case. There is no relevant difference compared to age- and income- dependent policies of Table 3. However, the fraction D_1 of individuals preferring this policy to a uniform indexation policy is now above 50% (it is 61.56%), between the fractions of those preferring an age-dependent indexation and those preferring an income-dependent indexation (respectively 63.13%, and 42.08%, see Table 3). Figure 6 shows in the dashed line the inter-generational welfare comparison between this policy and the uniform policy. We see from a comparison with Figure 4 that the curve mimics the one under the age-dependent indexation policy.

Table 4. Alternative policies

%	**Uniform**	**Fixed price index.**	**Age-skill dependent**
	Ratio volatility		
Median coeff. of var.	**18.8331**	19.5214	18.9150
Prob. of a ratio below $1+\xi^l$	**14.6773**	15.7067	14.6133
Prob. of a ratio below $1+\xi^m$	**31.7027**	33.1947	31.6693
Prob. of a ratio above $1+\xi^u$	**33.1560**	32.6653	33.4027
Prob. of intervention	**27.9275**	23.0790	27.9380
	Policy parameters		
κ_t, average	**93.5513**	92.4600	93.2584
std. dev.	**(200.7988)**	(237.7732)	(199.0640)
ι_t, average	**83.0122**	83.9604	82.7924
std. dev.	**(189.1137)**	(223.2408)	(189.2975)
θ_t^S, average	**14.5508**	14.0190	14.4703
std. dev.	**(10.2934)**	(10.6649)	(10.3039)
	Welfare		
C_1^A	**75.2455**	75.6064	75.1299
ΔC_1^A in percent	**-**	0.4796	-0.1536
C_1^T	**35.6420**	33.5776	35.4204
ΔC_1^T in percent	-	-5.7923	-0.6217
D_1	-	99.5267	61.5593

See note to Table 3.

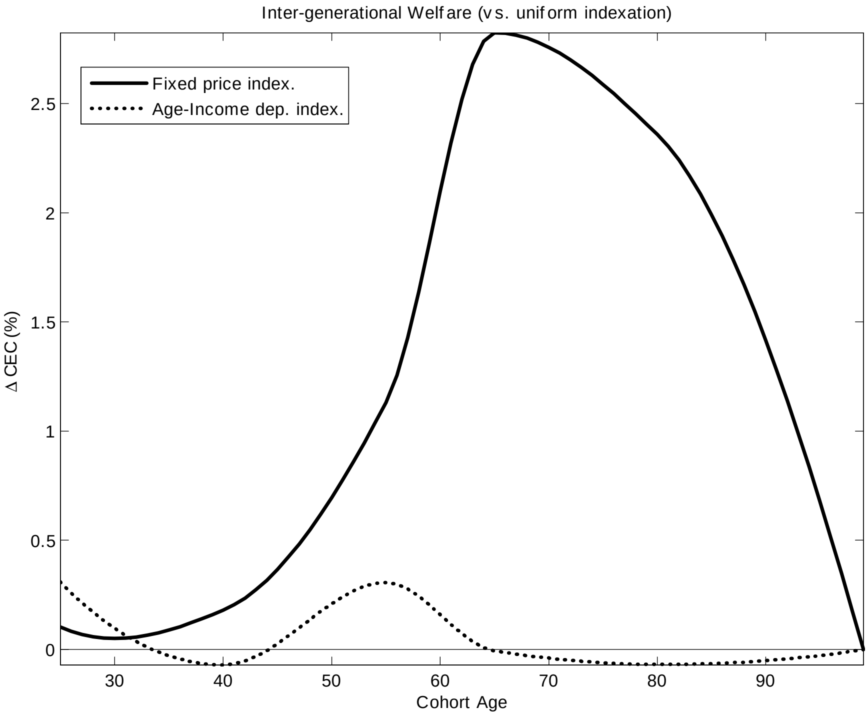


Figure 6. Welfare comparison, alternative policies

4.3 Robustness check

4.3.1 Spread between maximum and minimum indexation

We replicate the benchmark analysis using a different calibration of the parameters $\{\alpha_1, \alpha_2\}$ and $\{\nu_1, \nu_2\}$. The parameters are still chosen to generate the same level of liabilities as under uniform indexation, but this time they give rise to a different distribution of indexation across and within cohorts. In the benchmark case of Section 4.1 the spread between maximum and minimum correction was set at 300%; here we set it to lower (150%) and higher (500%) levels. The resulting parameter combinations are $\{\alpha_1 = 1.3616, \alpha_2 = 0.1361\}$, $\{\nu_1 = 1.0708, \nu_2 = 0.0397\}$ and $\{\alpha_1 = 2.7172, \alpha_2 = 0.6464\}$, $\{\nu_1 = 1.1886, \nu_2 = 0.1057\}$ respectively.

Table 5 reports the summary statistics from this analysis. The values in the table are in line with those for the corresponding policies in Table 3. A remarkable difference is that, when the spread is 150%, the funding ratio more often lies between $1+\xi^l$ and $1+\xi^m$ under the age-dependent policy (this happens in $40.50 - 14.72 = 25.78\%$ of the times, instead of around 17% in most other simulation) rather than between $1 + \xi^m$ and $1 + \xi^u$.

Social welfare including future generations, C_1^T, deteriorates under both policies as the maximum spread becomes larger. This effect is absent in the measure excluding future generations, C_1^A, as some groups of individuals (the youngest, the poorest) have larger welfare gains that offset the larger losses of the other groups. Indeed, the fraction of individuals supporting age-dependent rather than uniform indexation is virtually unaffected by the spread size (it is around 68%), and the fraction of those supporting skill-dependent indexation slightly falls with the spread size (from 45% to 38%). Overall, we still find a majority support for age-dependent policy but not for skill-dependent policy.

Table 5. Varying the indexation spread

%	**Spread:** 150%		**Spread:** 500%	
	Age-dep.	**Skill-dep.**	**Age-dep.**	**Skill-dep.**
	Ratio volatility			
Median coeff. of var.	19.0578	18.8788	19.0742	18.8818
Prob. of a ratio below $1+\xi^l$	14.7240	14.5560	14.8480	14.5240
Prob. of a ratio below $1+\xi^m$	40.5013	31.5987	31.9347	31.5440
Prob. of a ratio above $1+\xi^u$	33.6200	33.3640	33.3600	33.4147
Prob. of intervention	27.7914	27.9167	27.8228	27.9537
	Policy parameters			
κ_t, average	93.6865	93.5186	94.0686	93.3732
std. dev.	(202.4731)	(199.0098)	(204.2607)	(198.4291)
ι_t, average	82.6277	82.9550	82.7534	82.6977
std. dev.	(190.2551)	(191.3603)	(191.0686)	(187.8790)
θ_t^S, average	14.4370	14.4439	14.4595	14.5560
std. dev.	(10.3284)	(10.3198)	(10.3058)	(10.2983)
	Welfare			
C_1^A	75.2561	75.2647	75.2440	75.1645
ΔC_1^A in percent	0.0141	0.0256	-0.0020	-0.1077
C_1^T	35.6206	35.6468	35.3828	35.5025
ΔC_1^T in percent	-0.0599	0.0135	-0.7272	-0.3913
D_1	67.7309	44.6790	67.5347	38.4413

See note to Table 3.

Figure 7 reports the intergenerational welfare comparison between uniform indexation and the two contingent policies for the two spreads. Although the curves resemble those of Figure 4, we notice more pronounced gains and losses under age-dependent policies when the spread is larger.

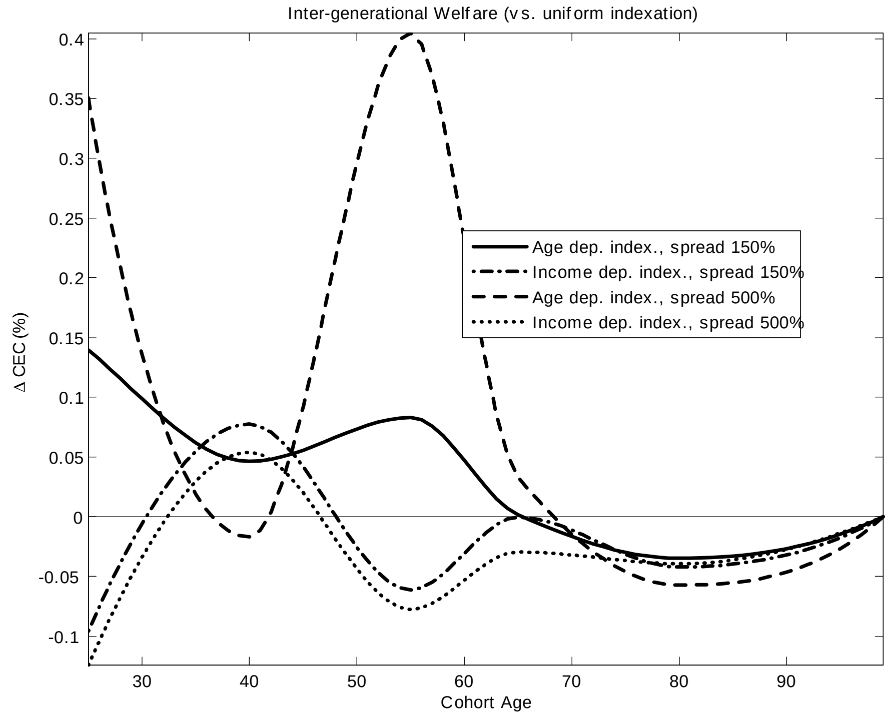


Figure 7. Welfare comparison, indexation spread

4.3.2 Fund portfolio composition

In the benchmark analysis the fund's portfolio contains bonds, equity and real estate assets in constant proportions. Here we allow the portfolio composition to vary with the size of the funding ratio, a situation closer to the reality. Specifically, we consider the following portfolio strategy:

$$\left\{z_t^e, z_t^h\right\} = \left\{ \begin{array}{ll} \left\{\underline{z}^e, \underline{z}^h\right\} & \text{if } F_{t-1} < 1+\xi^l \\ \left\{z^e, z^h\right\} & \text{if } F_{t-1} \in \left[1+\xi^l, 1+\xi^m\right] \\ \left\{\overline{z}^e, \overline{z}^h\right\} & \text{if } F_{t-1} > 1+\xi^m \end{array} \right\}$$

In particular we study two variants. In one variant $\left\{\underline{z}^e, \underline{z}^h\right\} = \frac{1}{2}\left\{z^e, z^h\right\}$ and $\left\{\overline{z}^e, \overline{z}^h\right\} = \frac{3}{2}\left\{z^e, z^h\right\}$, that is, the proportion of bonds is increased when the funding ratio is low, in an effort to reduce the portfolio risk, and reduced in when the funding ratio is high, to benefit from the higher (expected) returns on equity and real estate assets. In the second variant $\left\{\underline{z}^e, \underline{z}^h\right\} = \frac{3}{2}\left\{z^e, z^h\right\}$ and $\left\{\overline{z}^e, \overline{z}^h\right\} = \frac{1}{2}\left\{z^e, z^h\right\}$, that is, holdings of bonds are reduced in the case of a low funding ratio, in an attempt to reach a quick restoration of the fund assets through the higher expected returns on equity and real estate assets, and raised in the case of a high funding ratio, to lock in this high ratio with a safer portfolio. We develop the analysis concentrating only on the uniform indexation policy (the one currently adopted in the Netherlands) and the age-dependent indexation policy.

These policies determine two opposite effects on the funding ratio. Compared to the benchmark situation of Table 3, only the second variant is able to reduce the volatility of the ratio, while the first variant actually increases it. Notice in particular that under the second variant the funding ratio lies more frequently in the interval $\left(1+\xi^l, 1+\xi^m\right)$ than under the benchmark (around $35-14 = 21\%$ of the time rather than 18%), and it is less frequently above $1+\xi^u$ (27% rather than 33%). As a consequence, on average under the second variant the indexation parameters are lower, and the contribution rates higher. The implication for welfare is that many cohorts are worse off under the second variant and the D_1 statistic is substantially above 50% in this case. In contrast, many cohorts prefer the first variant, as its more frequent policy interventions are able to generate on average larger indexations and lower contribution rates.

Table 6. Portfolio composition

%	**Bond holding if low ratio**			
	High (variant 1)		**Low (variant 2)**	
	Uniform	**Age-dep.**	**Uniform**	**Age-dep.**
	Ratio volatility			
Median coeff. of var.	19.4095	20.0300	18.0159	17.7808
Prob. of a ratio below $1+\xi^l$	14.5560	14.5240	14.5920	14.5200
Prob. of a ratio below $1+\xi^m$	31.2034	31.8513	34.7162	35.0124
Prob. of a ratio above $1+\xi^u$	33.5120	33.0200	27.7280	27.1213
Prob. of intervention	28.5676	28.3988	27.1179	26.9772
	Policy parameters			
κ_t, average	93.9160	93.5964	90.2125	90.9142
std. dev.	(207.0244)	(206.4927)	(202.8326)	(203.0081)
ι_t, average	84.8801	84.6584	68.9498	68.0742
std. dev.	(205.4961)	(209.2161)	(178.3491)	(174.6744)
θ_t^S, average	13.5575	13.3905	16.3461	16.5005
std. dev.	(10.8141)	(10.7980)	(8.3208)	(8.1831)
	Welfare			
C_1^A	75.431	75.3655	75.1243	75.1507
ΔC_1^A	0.2465	0.1595	-0.1611	-0.1260
C_1^T	35.5094	35.7258	35.4310	35.4643
ΔC_1^T	-0.3720	0.2351	-0.5920	-0.4986
D_1	73.6935	80.3179	34.7224	49.7111

See note to Table 3.

Figure 8 shows the inter-generational welfare comparison of the uniform indexation policy under the two variants with the benchmark uniform indexation policy; a similar picture emerges from the comparison of age-dependent policies. We see that the youngest generations prefer variant 1 that increases the bond holding when the ratio is low, as they pay contribution rates 1% lower on average. Furthermore, the retirees prefer variant 2 to the benchmark policy, as it produces a lower volatility of the indexation parameters.

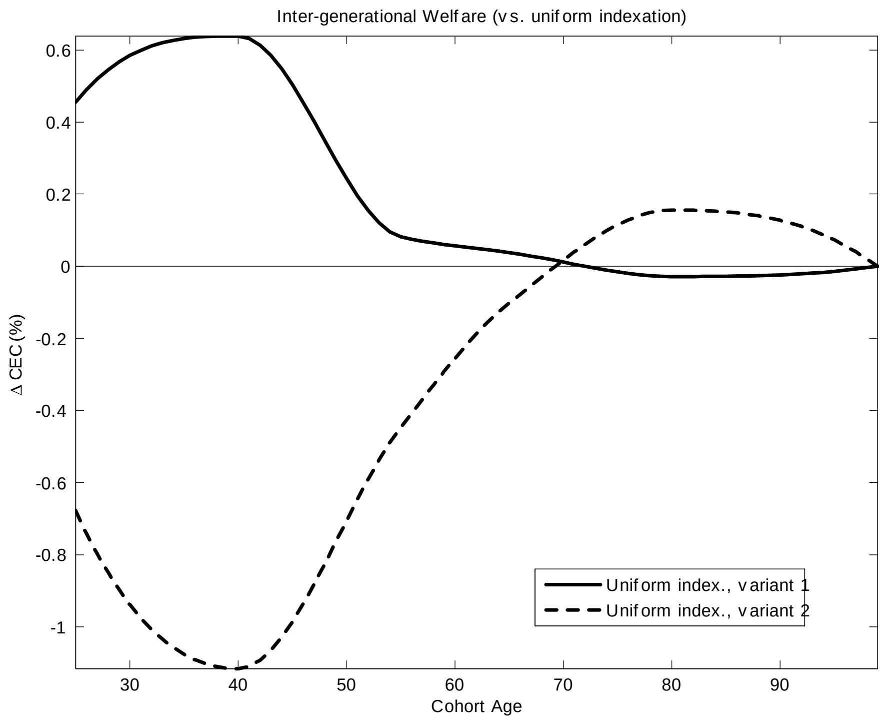


Figure 8. Welfare comparison, portfolio composition

5 Conclusions

In this paper we have investigated the welfare implications of different ways of indexing second pillar funded pensions. We have also explored the consequences of those different policies for the pension buffers. We described the economy with an OLG model of a small-open economy, featuring a two-pillar pension system similar to the one in the Netherlands. The economy was subject to demographic, economic and financial shocks that we calibrated from US data. We compared uniform indexation to all the individuals (the method currently followed in the Netherlands), status-dependent indexation that provides a fixed indexation to the retirees, age-dependent and skill-dependent indexation policies. We find that not all the policies are equally effective in responding to exogenous shocks. Indeed the status-dependent policy, leaving to the workers all the burden of the fund's restoration, on average produces lower and more volatile funding ratios. Except for the youngest cohorts all the other cohorts benefit from status-dependent indexation when indexation for the retired always follows nominal income growth. When indexation for the retired always follows price inflation, all cohorts can be made to benefit. We find some welfare gain when indexation is linked to skill or (especially) age. There are however large differences among generations. To benefit more from age-dependent policy are middle-aged cohorts, who have more nominal rights and enjoy the highest indexation. In contrast, the generations of young workers and retirees prefer the skill-dependent policy, respectively because they pay lower contribution rates and profit from from a lower volatility in the indexation of their benefits. Combining age- and skill-dependent policies the welfare improvement is generalised, although each cohort receives a lower gain than under its preferred policy.

6 Appendix

6.1 Detailed rules for the adjustment of the policy parameters

The adjustment policy works as follows. In case *no restoration plan* from an earlier period is still active in t:

1. If $F_t < 1 + \xi^l$, a *short-term restoration plan* is started that after K^s years in the absence of shocks brings back along a linear growth path the funding ratio at $1+\xi^l$. Hence, the sequence of policy parameter combinations $\left(\theta^S_{t+1}, \kappa_{t+1}, \iota_{t+1}\right)$, ..., $\left(\theta^S_{t+K^s}, \kappa_{t+K^s}, \iota_{t+K^s}\right)$ is set at period t such that the funding ratios $\widetilde{F}_{t+1}$, $\widetilde{F}_{t+2}$, ..., $\widetilde{F}_{t+K^s}$ projected from F_t in the absence of further shocks hit the target funding ratios $\bar{F}_{t+\tau} = F_t + \left[\left(1+\xi^l\right) - F_t\right] \frac{\tau}{K^s}$ for years $\tau = 1, ..., K^s$. For every period $t+\tau$ along the restoration path, we first reduce productivity indexation $\iota_{t+\tau}$ up to a minimum level of zero. If this is not enough, we reduce parameter $\kappa_{t+\tau}$ up to a minimum level of zero. If this is still not enough, we raise the contribution rate $\theta^S_{t+\tau}$ up to a maximum of $\theta^{S,\max}$. If after applying all these measures the funding ratio still falls short of its target $\bar{F}_{t+\tau}$, we set $\theta^S_{t+\tau} = \theta^{S,\max}$, $\kappa_{t+\tau} = \iota_{t+\tau} = 0$ and apply a reduction in nominal rights $m_{t+\tau} > 0$ such that $\widetilde{F}_{t+\tau} = \bar{F}_{t+\tau}$.

2. If $1+\xi^l \leq F_t < 1+\xi^m$, a *long-term restoration plan* is started that after K^l years in the absence of shocks brings back along a linear growth path the funding ratio at $1+\xi^m$. Hence, the sequence of policy parameter combinations $\left(\theta^S_{t+1}, \kappa_{t+1}, \iota_{t+1}\right)$, ..., $\left(\theta^S_{t+K^l}, \kappa_{t+K^l}, \iota_{t+K^l}\right)$ is set at period t such that the funding ratios $\widetilde{F}_{t+1}$, $\widetilde{F}_{t+2}$, ..., $\widetilde{F}_{t+K^l}$ projected from F_t in the absence of further shocks hit the target funding ratios $\bar{F}_{t+\tau} = F_t + \left[(1+\xi^m) - F_t\right] \frac{\tau}{K^l}$ for years $\tau = 1, ..., K^l$. For every period $t+\tau$ along the restoration path, we first reduce productivity indexation $\iota_{t+\tau}$ up to a minimum level of zero. If this is not enough, we reduce price indexation $\kappa_{t+\tau}$ up to a minimum level of zero. If this is still not enough, we raise $\theta^S_{t+\tau}$ up to a maximum of $\theta^{S,\max}$. If after applying all these measures the funding ratio still falls short of $\bar{F}_{t+\tau}$, we set $\theta^S_{t+\tau} = \theta^{S,\max}$, $\kappa_{t+\tau} = \iota_{t+\tau} = 0$, but we apply no reduction in nominal rights.

3. If $1+\xi^m \leq F_t < 1+\xi^u$, there are two cases:

   (a) In the absence of any missed nomimal rights (see below), the next-year policy parameters are set to $\theta^S_{t+1} = \theta^S_t$ and $\kappa_{t+1} = \iota_{t+1} = 1$.

   (b) In the presence of missed (unrestored) nominal rights, the next-year policy parameters are set to $\theta^S_{t+1} = \theta^S_t$ and $\kappa_{t+1} = \iota_{t+1} = 0$.

4. If $F_t \geq 1+\xi^u$, m_{t+1} is set to restore any missed nominal rights (as described below) to the extent that the funding ratio does not fall below the target ratio $1+\xi^u$.[10] If after restoring possible missed nominal rights still $\widetilde{F}_{t+1} > 1+\xi^u$, then further adjustment to the policy parameters is made. First, we restore possible missed price indexation (see below). Then, we restore possible missed productivity indexation and, finally, we reduce the contribution rate θ^S_{t+1} up to a minimum of 0. If after applying all these measures the funding ratio in the absence of shocks still exceeds $1+\xi^u$, we raise price indexation by an *extra* amount $\hat{\kappa}_{t+1} > 0$

[10] Dutch pension law says that a pension fund is not allowed to reduce contribution rates until any earlier reduction in nominal rights is undone.

such that over a period of three years along a linear path in the absence of shocks the funding ratio is back at $1+\xi^u$.

In case a *long-term restoration plan* from an earlier period is still active in t:

1. If $F_t < 1+\xi^l$, the long-term restoration plan is cancelled and the policymaker follows the above policy under "no restoration plan" from an earlier period still active in t. That is, it sets up a short-run restoration plan as determined above.
2. If $1+\xi^l \leq F_t < 1+\xi^m$, there are two cases:
   (a) If $F_t < \bar{F}_t$, we reduce productivity indexation up to a minimum of $\iota_{t+1}=0$ to produce a projected ratio $\widetilde{F}_{t+1}=\bar{F}_{t+1}$in the absence of shocks. If this is not enough, we reduce price indexation up to a minimum of $\kappa_{t+1}=0$. If this is still not enough, we increase the contribution rate up to a maximum of $\theta^S_{t+1}=\theta^{S,\max}$. If after applying these measures next period's funding ratio still falls below $\overline{F}_{t+1}$, we set $\theta^S_{t+1}=\theta^{S,\max}$, $\kappa_{t+1}=\iota_{t+1}=0$ and undertake no further action.
   (b) If $\bar{F}_t \leq F_t < 1{+}\xi^m$, the policy parameters are those prescribed by the existing long-term restoration plan.
3. If $1+\xi^m \leq F_t < 1+\xi^u$, then the above policy under "no restoration plan" from an earlier period still active in t is followed.
4. If $F_t \geq 1+\xi^u$, then the above policy under "no restoration plan" from an earlier period still active in t is followed.

In case a *short-term restoration plan* from an earlier period is still active in t:

1. If $F_t < 1+\xi^l$, there are two cases:
   (a) If $F_t < \bar{F}_t$, we reduce productivity indexation up to a minimum of $\iota_{t+1}=0$ to produce a projected ratio $\widetilde{F}_{t+1}=\bar{F}_{t+1}$in the absence of shocks. If this is not enough, we reduce price indexation up to a minimum of $\kappa_{t+1}=0$. If this is still not enough, we increase the contribution rate up to a maximum of $\theta^S_{t+1}=\theta^{S,\max}$. If after applying these measures next period's funding ratio still falls below $\bar{F}_{t+1}$, we set $\theta^S_{t+1}=\theta^{S,\max}$, $\kappa_{t+1}=\iota_{t+1}=0$ and $m_{t+1}>0$ such that in the absence of shocks $\widetilde{F}_{t+1}=\bar{F}_{t+1}$.
   (b) If $\overline{F}_t \leq F_t < 1{+}\xi^l$, the policy parameters are those prescribed by the existing short-term restoration plan.
2. If $1+\xi^l \leq F_t < 1+\xi^m$, then the above policy under no restoration plan from an earlier period still active in t is followed. That is, a long-term restoration plan is set up in the way described above.
3. If $1+\xi^m \leq F_t < 1+\xi^u$, then the above policy under no restoration plan from an earlier period still active in t is followed.
4. If $F_t \geq 1+\xi^u$, then the above policy under no restoration plan from an earlier period still active in t is followed.

We restore missed price and productivity indexation and missed nominal rights as follows. Let us take the case of price indexation. For this case, we define two processes, an "actual" process (tracking the actual indexation that has been given, where π is long-run average inflation),

$$p_t^{\kappa,a} = (1 + \kappa_t \pi)\, p_{t-1}^{\kappa,a}, \tag{35}$$

and a "shadow" process that corresponds to always having full indexation:

$$p_t^{\kappa,s} = (1 + \pi)\, p_{t-1}^{\kappa,s}. \tag{36}$$

We set the processes equal to unity at $t = 1$ (D periods into the simulation run): $p_1^{\kappa,a} = p_1^{\kappa,s} = 1$.

Suppose that in period t, the funding ratio exceeds $1+\xi^u$. Then, indexation for the next period will at least be equal to full indexation: $\kappa_{t+1} \geq 1$. In case $p_t^{\kappa,a} < p_t^{\kappa,s}$, the indexation in the next period will be set at most so high that the missed indexation is restored in expected terms. That is, κ_{t+1} will be set at most such that $p_{t+1}^{\kappa,a} = p_{t+1}^{\kappa,s}$, which is equivalent to $(1 + \kappa_{t+1}\pi)\, p_t^{\kappa,a} = (1 + \pi)\, p_t^{\kappa,s}$, which in turn is solved as:

$$\kappa_{t+1}^{restore} = \frac{1}{\pi}\left(\frac{p_t^{\kappa,s}}{p_t^{\kappa,a}} - 1\right) + \frac{p_t^{\kappa,s}}{p_t^{\kappa,a}}.$$

Finally, we define κ_{t+1}^u as the indexation rate that brings the funding ratio to $1 + \xi^u$ next year in the absence of further shocks. Actual indexation κ_{t+1} will be set at:

$$\kappa_{t+1} = \min\left\{\max\left\{1, \kappa_{t+1}^u\right\}, \kappa_{t+1}^{restore}\right\}.$$

The processes (35) and (36) continue further until the end of the simulation run.

For missed productivity indexation, we similarly define the "actual", respectively "shadow", processes:

$$\begin{aligned} p_t^{\iota,a} &= \left(1 + \iota_t\left(\frac{1+g}{1+\pi} - 1\right)\right) p_{t-1}^{\iota,a}, \\ p_t^{\iota,s} &= \left(1 + \left(\frac{1+g}{1+\pi} - 1\right)\right) p_{t-1}^{\iota,s}, \end{aligned}$$

where $p_1^{\iota,a} = p_1^{\iota,s} = 1$. Restoration of indexation is completely similar to that in the case of price indexation.

Finally, for reductions in nominal rights (captured by $m_t > 0$), we define the "actual", respectively "shadow", processes

$$\begin{aligned} p_t^{m,a} &= (1 - m_t)\, p_{t-1}^{m,a}, \\ p_t^{m,s} &= p_{t-1}^{m,s}, \end{aligned}$$

where $p_1^{m,a} = p_1^{m,s} = 1$. Again, if at some moment t, we have $p_t^{m,a} < p_t^{m,s}$ and the funding ratio exceeds $1 + \xi^u$, missed nominal rights can be given back up to a maximum level such that $p_{t+1}^{m,a} = p_{t+1}^{m,s}$. The exact formula for the restoration of missed nominal rights is

$$m_{t+1} = \max\left\{\min\left\{0, 1 - \frac{\widetilde{F}_{t+1}}{1+\xi^u}\right\}, \min\left\{0, 1 - \frac{p_t^{m,s}}{p_t^{m,a}}\right\}\right\}$$

where $\widetilde{F}_{t+1}$ is the projection at time $t+1$ of the funding ratio in the absence of further shocks. To see the first argument of this expression, notice that if $m_{t+1} = 1 - \frac{\widetilde{F}_{t+1}}{1+\xi^u}$, all nominal rights are multiplied by the factor $\frac{\widetilde{F}_{t+1}}{1+\xi^u}$. Hence, all future pension benefits are multiplied by this same factor and, then, total liabilities are multiplied by this same factor, implying that the funding ratio becomes $1+\xi^u$.

6.2 Details on the calibration

6.2.1 Growth rate of the newborn cohort

For the number of births in the US between 1985 and 2005 (source: HMD, 2009), we estimate the model:

$$n_t = n + \epsilon_t^n,$$
$$\epsilon_t^n = \varphi \epsilon_{t-1}^n + \eta_t^n, \quad \eta_t^n \tilde{} N\left(0, \sigma_n^2\right).$$

This yields $n = 0.0047362$, $\varphi = 0.4543931$ (standard error 0.2223041) and $\sigma_n = 0.0132662$ (standard error 0.0017105).

6.2.2 Survival probabilities

Our simulations require cohort life tables, which are incomplete for recent cohorts. Using easily available period life tables, however, leads to an over-estimate of mortality because of the well documented downward trend in mortality. To correctly estimate mortality, we follow the Lee-Carter model (Lee and Carter, 1992) and collect from HMD (2009) US period life tables from 1950 to 2005. These contain the total population on a year-by-year basis from ages 0 to 110. We call $\psi_{j,t}^p$ the probability of being alive in year t for individuals aged j, conditional on having been alive at age $j-1$. To distinguish the trend from fluctuations, we estimate with singular value decomposition the parameters of the Lee-Carter model:

$$\ln\left(1-\psi_{j,t}^p\right) = \alpha_j + \tau_j \chi_t + \eta_t^\psi,$$

where α_j and τ_j are age-varying parameters, χ_t is a time-varying vector and η_t^ψ is a random disturbance distributed as $N\left(0, \widetilde{\sigma}_\psi^2\right)$. Lee and Carter (1992) point out that the parameterisation is not unique. Therefore, we choose the one fulfilling their suggested restrictions:

$$\left\{ \begin{array}{c} \sum_{t=1}^{T} \chi_t = 0 \\ \sum_{j=1}^{D} \tau_j = 1 \end{array} \right\},$$

where $t = 1, .., T$ indicates the sample period. With these restrictions the estimated value for α_j will be the average probability over the sample that someone dies at age j , when having survived up to age $j-1$.[11] Consistently with the existing literature we assume that the mortality index χ_t evolves as a random walk with drift χ:

$$\chi_t = \chi_{t-1} + \chi + \epsilon_t^\psi,$$

with $\epsilon_t^\psi \tilde{} N\left(0, \sigma_\psi^2\right)$. With our data we estimate $\hat{\chi} = -1.2595$ and $\hat{\sigma}_\psi = 0.0266$, thereby implying a trend fall in the probability of dying at any age j, conditional on having survived up to age $j-1$.

[11] Notice that $\frac{1}{T}\sum_{t=1}^{T} \ln\left(1-\psi_{j,t}^p\right) = \frac{1}{T}\sum_{t=1}^{T}\left(\alpha_j + \tau_j \chi_t + \eta_t^\psi\right) = \alpha_j + \tau_j\left(\frac{1}{T}\sum_{t=1}^{T}\chi_t\right) + \left(\frac{1}{T}\sum_{t=1}^{T}\eta_t^\psi\right) = \alpha_j + \left(\frac{1}{T}\sum_{t=1}^{T}\eta_t^\psi\right)$ $= \hat{\alpha}_j + \left(\frac{1}{T}\sum_{t=1}^{T}\hat{\eta}_t^\psi\right) = \hat{\alpha}_j$, where $\hat{\alpha}_j$ is the estimate of α_j and $\hat{\eta}_t^\psi$ is the regression residual. The last equality is obtained by using that the sum of the residuals is zero.

In the simulations we assume that $\hat{\chi} = 0$ after year $t = 40$, that is, there is no further population ageing after 40 years. We make this assumption to avoid dealing with very large contribution rates in the first- and second-pillar systems and on the assumption that the ageing process cannot continue forever.

From the period life table estimates and the trend of the mortality index we calculate the cohort life tables as follows:

$$\begin{aligned}\ln\left(1 - \psi_{j,t-j+1}\right) &= \hat{\alpha}_j + \hat{\tau}_j\left(\hat{\chi}_{t-j+1} + j\hat{\chi}\right) \\ &= \hat{\alpha}_j + \hat{\tau}_j\hat{\chi}_{t+1},\end{aligned}$$

where $t-j+1$ is the year of birth of the cohort. Thus $\psi_{j,t-j+1}$ indicates the (estimated) probability of being alive at age j (end of period t) for the cohort of individuals born at the beginning of year $t - j + 1$, conditional on them being alive at age $j - 1$. In our model, the survival probabilities $\left\{\psi_{j,D}\right\}_{j=1}^{D}$ of the cohort born in year $t = 0$ are set equal to those of the actual cohort of individuals born in 1950.

The survival probability for the cohort born in the following year $t-j+2$ evolves according to:

$$\begin{aligned}\ln\left(1 - \psi_{j,t-j+2}\right) &= \hat{\alpha}_j + \hat{\tau}_j\left(\hat{\chi}_{t-j+2} + j\hat{\chi}\right) \\ &= \alpha_j + \hat{\tau}_j\left(\hat{\chi}_{t-j+1} + j\hat{\chi} + \hat{\chi}\right) \\ &= \alpha_j + \hat{\tau}_j\left(\hat{\chi}_{t+1} + \hat{\chi}\right) \\ &= \ln\left(1 - \psi_{j,t-j+1}\right) + \hat{\tau}_j\hat{\chi}.\end{aligned}$$

6.2.3 Economic shocks

We assume that the shocks to our five economic and financial variables (the inflation rate, the nominal wage growth rate, the one-year bond return, the equity return and the housing return) evolve according to a VAR(1) process. The underlying data are the following time series: for the inflation rate, the US Consumer Price Index; for the nominal income growth rate, the US hourly wage (source for both series: OECD, 2009); for the one-year bond return, the US end-of-year public debt yield at maturity one year (source: Federal Reserve, 2009); for the equity return, the MSCI US equity index (source: Datastream, 2009); for the housing return, the OFHEO house price index (now FHFA index, source: FHFA, 2009). All the series are annual over the period 1976-2005 (30 observations). For each series we take the deviations from the historical average.

Our shocks consist of a deterministic component, which is a linear combination of previous-year shocks, and a purely random component, given by realisations from i.i.d. innovations. The estimation of the deterministic component is shown in panel a of Table 7. It is worth pointing out that no variable in the specification of the equity return is significantly different from zero; indeed, a Wald chi-squared test does not reject the hypothesis that equity returns follow a purely random (white noise) process.

Table 7. VAR(1) regression

a. Deterministic coefficient estimates (matrix **B** in (19))

Variable	Inflation	Wage	Bond	Equity	Housing
Inflation (-1)	0.7864***	0.3060**	0.3694**	-1.5158	-0.8204***
	(0.1747)	(0.1192)	(0.1840)	(2.1683)	(0.2660)
Wage (-1)	0.0185	0.6609***	-0.0786	0.3825	1.0658***
	(0.1930)	(0.1317)	(0.2033)	(2.3953)	(0.2938)
Bond (-1)	-0.0555	-0.1661**	0.6857***	1.3535	-0.2609
	(0.1104)	(0.0753)	(0.1163)	(1.3700)	(0.1681)
Equity (-1)	0.0094	0.0125	0.0252	-0.0247	0.0119
	(0.0148)	(0.0101)	(0.01554)	(0.1831)	(0.0225)
Housing (-1)	0.2903***	0.0957*	0.1533**	-1.0446	0.6839***
	(0.0779)	(0.0531)	(0.0821)	(0.9669)	(0.1186)
Wald chi-squared	149.1552	233.2539	171.2329	3.9514	93.5409
p-value	0.0000	0.0000	0.0000	0.5564	0.0000

Notes: standard deviations in parentheses.

****: significant at 1%; **: significant at 5%; *: significant at 10%*

Wald chi-squared: test on the joint significance of the coefficients in each column.

The test follows a chi-squared distribution with 5 degrees of freedom.

b. Residual covariances and correlations (%)

Variable	Inflation	Wage	Bond	Equity	Housing
Inflation	0.0136	*50.2306*	*54.9103*	*20.8439*	*-15.2365*
Wage	0.0047	0.0063	*48.3280*	*-25.8828*	*-0.6701*
Bond	0.0079	0.0047	0.0151	*7.0268*	*4.7483*
Equity	0.0353	-0.0299	0.0125	2.1005	*0.2007*
Housing	-0.0032	-0.0001	0.0010	0.0005	0.0316

Note: correlations in italic; (co-)variances are in non-italic.

6.2.4 The swap curve

Deviations from the average swap returns follow the VADL(1) process of equation (21), in which each deviation is a function of all the deviations and other exogenous variables observed one month earlier. The exogenous variables are the innovations to the inflation rate, wage growth and the bond, equity and housing returns. Our dataset is a time series of US swap interest rates at any annual maturity from 1 to 10, plus maturities 12, 15, 20, 25 and 30 (source: Datastream, 2009). Many of these time series are not available before 1997. To obtain a reasonable number of observations, we therefore collect annual returns at monthly frequency to cover the period from 1997 to 2006 (120 observations).[12]

The VADL specification explains 15 variables observed in a given month (the swap return deviations) with an intercept and 20 variables observed one month earlier (the 15 swap return deviations, and the innovations to the 5 economic variables). The regression output is available upon request. For each dependent variable we reject the hypothesis that it follows a white noise

[12] We ignore observations in later periods to satisfy the assumption of stationarity. After 2006 one enters the highly unusual situation of the current crisis.

process, and the R-squared statistic lies between 0.9480 and 0.9967. The shocks are assumed to follow a multivariate normal distribution, with mean 0 and covariance matrix given by the covariance among the residuals of the regression. The volatility of the shock at maturity one (standard deviation 0.0111) is close to that for the shock to one-year bond returns (standard deviation 0.0123). Shocks at near maturities have very high correlations around 98%; the lowest correlation we observe – between shocks at maturities 1 and 30 – is however still pretty high (42%).

We use the regression output to generate random swap returns at the observed maturities. The time period in the model is one year, but the regression is conducted on monthly data. Therefore, for each period in the simulation we generate a sequence of 12 subsequent swap curves using the estimated VADL(1) process. Each draw requires as input the monthly innovations to the inflation rate, the nominal wage rate and the returns to the one-year bond, equity and housing. However, only annual innovations are known through the process (19). Thefore, we construct monthly shocks from annual shocks after noticing that equation (19) coincides with

$$\begin{pmatrix} \epsilon^{\pi}_{t'+12} \\ \epsilon^{g}_{t'+12} \\ \epsilon^{b}_{t'+12} \\ \epsilon^{e}_{t'+12} \\ \epsilon^{h}_{t'+12} \end{pmatrix} = \mathbf{A}^{12} \begin{pmatrix} \epsilon^{\pi}_{t'} \\ \epsilon^{g}_{t'} \\ \epsilon^{b}_{t'} \\ \epsilon^{e}_{t'} \\ \epsilon^{h}_{t'} \end{pmatrix} + \sum_{j=1}^{12} A^{12-j} \begin{pmatrix} \eta^{\pi}_{t'+j} \\ \eta^{g}_{t'+j} \\ \eta^{b}_{t'+j} \\ \eta^{e}_{t'+j} \\ \eta^{h}_{t'+j} \end{pmatrix}$$

where t' indicates the month, shocks at $t' = 0$ are set to 0, $A = B^{\frac{1}{12}}$ is obtained with single value decomposition and the monthly i.i.d. shocks arise from the (observed) annual i.i.d. shocks,

$$\begin{pmatrix} \eta^{\pi}_{t'+j} \\ \eta^{g}_{t'+j} \\ \eta^{b}_{t'+j} \\ \eta^{e}_{t'+j} \\ \eta^{h}_{t'+j} \end{pmatrix} = \left(\sum_{j=1}^{12} A^{12-j} \right)^{-1} \begin{pmatrix} \eta^{\pi}_{t-1} \\ \eta^{g}_{t-1} \\ \eta^{b}_{t-1} \\ \eta^{e}_{t-1} \\ \eta^{h}_{t-1} \end{pmatrix}$$

under the assumption that the shocks in months $t' + j, \quad j = 1, ..., 12$ are identical. We use these shocks to compute for each month the shocks to the yields at maturity 1-10, 12, 15, 20, 25 and 30 of the swap curve, according to equation (21).

From this sequence of 12 swap curve yields we consider the last one (say, the December one) in the simulations. We then adopt a linear interpolation over the available swap rates to obtain swap rates at any discrete maturity between 1 and 30. Rates at maturity longer than 30 are set equal to the rate at maturity 30. Swap returns are then built as the sum of the VADL(1) realisations and a vector of constants, derived from

$$\begin{pmatrix} r^s_1 \\ r^s_2 \\ \vdots \\ r^s_D \end{pmatrix} = \begin{pmatrix} r^b_1 \\ r^b_1 \\ \vdots \\ r^b_1 \end{pmatrix} + \begin{pmatrix} \overline{r^s_{1,t} - r^b_{1,t}} \\ \overline{r^s_{2,t} - r^b_{1,t}} \\ \vdots \\ \overline{r^s_{D,t} - r^b_{1,t}} \end{pmatrix}, \tag{37}$$

where r^b_1 is the calibrated average one-year bond return (see Table 2), and the difference $\overline{r^s_{k,t} - r^b_{1,t}}$ is the sample average of the swap return at maturity k in excess of the sample average of the one year bond return. We use this formula to make swap returns in magnitude comparable to the calibrated average one-year bond return.

6.2.5 The bond yield curve

We assume that the one-year interest rate coincides with the one-year bond return, while the interest rates for other maturities follow a similar process as for the swap curve. Our dataset is a time series of US yield returns at maturities 2, 3, 5, 7, 10, 20 and 30 (the only observed maturities – source is Federal Reserve, 2009). To make the comparison with the swap curve consistent, we take the same sample period (from 1997 to 2006) and frequency (monthly), even though we could use a longer series in this case. In the sample there are occasionally missing values for the yields at maturities 20 and 30. We impute the missing values using a linear interpolation method.

The regression output is available upon request. As for the swap curve, we obtain large R-squared statistics (between 0.9104 and 0.9958) and always reject the hypothesis that the interest rates follow a white noise process. Shocks at near maturities are highly correlated (usually above 95%, and never below 46%); they exhibit lower volatility than the corresponding shocks to the swap curve, especially at longer maturities. For instance, the standard deviation of the yield return at maturity 30 is only 53% of the standard deviation of the swap return at the same maturity. Both volatilities are, however, small compared to those of one-year bond returns.

We use the regression output to generate random yield returns at the observed maturities. As for the swap curve, for each year of the simulation we generate a sequence of 12 random yield returns, and make use of only the last realisation. We then adopt a linear interpolation over these yields to obtain the interest rates at any discrete maturity between 1 and 30. Interest rates at maturities longer than 30 are set equal to the interest rate at maturity 30. Yield returns at maturity $k \geq 2$ are then built as the sum of the VADL(1) realisations and a vector of constants, derived from

$$\begin{pmatrix} r_2^b \\ r_3^b \\ \vdots \\ r_D^b \end{pmatrix} = \begin{pmatrix} r_1^b \\ r_1^b \\ \vdots \\ r_1^b \end{pmatrix} + \begin{pmatrix} \overline{r_{2,t}^b - r_{1,t}^b} \\ \overline{r_{3,t}^b - r_{1,t}^b} \\ \vdots \\ \overline{r_{D,t}^b - r_{1,t}^b} \end{pmatrix},$$

where r_1^b is the average one-year bond return, and the difference $\overline{r_{k,t}^b - r_{1,t}^b}$ is the sample average of the yield return at maturity k in excess of the sample average of the one-year bond return. We use this formula to make the bond yield curve at the one-year maturity comparable to the one-year bond return calibrated earlier. The average bond yield curve shows a quadratic-looking profile similar to that of the average swap curve, although at each maturity the return is around 0.5% points lower than the corresponding return of the average swap curve.

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