

# Evaluating the Advanced Life Deferred Annuity – An Annuity People Might Actually Buy

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## ABSTRACT

Although annuities provide longevity insurance that should be attractive to households facing an uncertain lifespan, rates of voluntary annuitization remain extremely low. We evaluate the Advanced Life Deferred Annuity, an annuity purchased at retirement, providing an income commencing in advanced old age. Using numerical optimization, we show that it would provide a substantial proportion of the longevity insurance provided by an immediate annuity, at much lower cost. At plausible levels of actuarial unfairness, households should prefer it to both immediate and postponed annuitization and an optimal decumulation of unannuitized wealth. Few households would suffer significant losses were it used as a 401(k) plan default.

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## 1. Introduction

Immediate annuities provide insurance against outliving one's wealth. Previous research has shown that this insurance ought to be valuable to risk-averse households facing an uncertain lifespan. But rates of voluntary annuitization remain extremely low.

Many explanations have been offered for retired households' reluctance to annuitize.<sup>1</sup> Prominent is that annuities suffer from a considerable degree of actuarial unfairness. That is, for the average household, the expected value of the income, discounted by a rate of interest and annual survival probabilities, is considerably less than the premium paid. But it seems likely that actuarial unfairness cannot fully explain the low level of voluntary annuitization, and that households are also influenced by a possibly not wholly rational reluctance to give up access to their life savings.

In the past, low rates of voluntary annuitization were not a matter of great policy concern because most households held substantial proportions of their wealth in pre-annuitized form through Social Security and defined benefit pensions. However, the displacement of defined benefit plans by 401(k)s and projected reductions in Social Security replacement rates will increase the importance of a well-functioning and attractive annuity market.

This paper evaluates a proposal, first brought to the attention of the academic community by Milevsky (2005), for an innovative annuity product – which he named the Advanced Life Deferred Annuity (ALDA). Milevsky envisaged an inflation-protected annuity that would be purchased at retirement or even earlier. But in contrast to a traditional annuity, income payments would only start at some advanced age, (say) 85, providing insurance against the risk of living exceptionally long. Scott, Watson, and Hu

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<sup>1</sup> For a survey of possible explanations, see Brown and Warshawsky (2004).

(2007) and Scott (2008) makes the point that if, for some reason, households are only prepared to annuitize part of their wealth, it is best to use them to finance consumption at very advanced ages where the probability of survival is lowest, funding consumption at younger ages from unannuitized wealth.

Although a few insurance companies have very recently begun to offer ALDA type products with benefits fixed in nominal terms, no company has thus far launched the type of inflation-protected product proposed by Milevsky. In the absence of market data, we estimate the money's worths of ALDAs, from analyses of the money's worths of the ALDA type products currently available, and of nominal and inflation-protected immediate annuities. We test the sensitivity of our results to alternative mortality and interest rate assumptions.

We compare retirement wealth decumulation strategies based around the inflation-protected ALDA with the alternatives of the purchase of an inflation-protected annuity immediately on retirement, postponing the purchase of an annuity until some advanced age, and undertaking an optimal decumulation of unannuitized wealth. We show that strategies based around the ALDA have three important advantages. First, they enable households to preserve liquidity at least until the ALDA payments commence, because their purchase cost is a fraction of the cost of immediate annuities, thus overcoming a potentially important psychological barrier to annuitization. We calculate that a household planning to smooth consumption through its retirement would need to allocate only 15 percent of its age 60 wealth to an ALDA with payments commencing at age 85, holding the remainder of its wealth in unannuitized form to finance consumption from age 60 to 85. Second, although a risk-averse household facing an uncertain lifespan

would prefer the full longevity insurance provided by an actuarially fair annuity to the partial longevity insurance provided by an actuarially fair ALDA, at plausible projected levels of actuarial unfairness, the household would prefer the ALDA to full annuitization. The intuition is simply that the household is suffering much less actuarial unfairness, but getting almost as much longevity insurance. An ALDA also dominates an optimal decumulation of unannuitized wealth. Third, ALDAs have the potential to improve and simplify the process of retirement wealth decumulation. We show that simple rules-of-thumb that perform almost as well as the optimal can be applied to the management of wealth decumulation over a period ending on the date that the ALDA income commences. In contrast, widely advocated rules for managing the decumulation of unannuitized wealth over an entire lifetime are highly suboptimal.

Finally, we consider the extent to which government and employers should encourage the take-up of ALDAs by, for example, making them a default option in 401(k) plans. A potential concern is that defaulting retirees into an ALDA might harm those who would rationally choose not to purchase. In Gong and Webb (2008), we calculated subjective mortality tables for each individual in the Health and Retirement Study (HRS) based on their self-reported survival probabilities.<sup>2</sup> Using these tables, we show that in expected utility terms even high mortality HRS households would be better off purchasing an ALDA than undertaking an optimal decumulation of unannuitized wealth.

The remainder of the paper is organized as follows. Section two explains how the ALDA would work and presents analyses comparing the money's worths of ALDAs with those of immediate annuities. Section three calculates how much longevity insurance an ALDA would provide. Section four compares ALDAs, annuities, and optimal

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<sup>2</sup> The HRS is a panel of over 7,000 individuals born between 1931 and 1941, and their spouses of any age.

decumulations of unannuitized wealth in practice. Section five considers whether ALDAs can safely be used as a default in 401(k) plans, and section six concludes.

## **2. How Would an ALDA Work and How Would Its Money's Worth Compare?**

The concept was brought to our attention by Milevsky (2005) who envisaged an inflation-protected deferred annuity that would be purchased by installments over an individual's working life, but which would only come into payment at an advanced age, (say) 85 or older. One possible drawback to this idea is the likely reluctance of individuals to contribute during their working lives towards the cost of a product that would only provide benefits in advanced old age. Instead, such a product might be more attractive if purchased at or near retirement.<sup>3</sup> We therefore estimate the money's worth of an inflation-protected joint life and two thirds survivor ALDA purchased with a lump sum at either age 60 or 65.

Milevsky, page 118, reports that although several insurance companies have either already launched or are about to launch variants of the ALDA, their design features increase their cost and "detract from the ultimate objective, which is to *encourage annuitization at the lowest possible cost.*" The market has subsequently begun to develop, and we are aware of four insurance companies that offer ALDA-type products

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<sup>3</sup> Other drawbacks to selling ALDAs by installments over the individual's working life include: 1) The administrative costs involved in collecting small premiums over many years. 2) The exposure of insurance companies to additional mortality risk, because they would have to take a view on mortality improvements over extremely long time horizons, and 3) The lack of interest rate hedges over extremely long time horizons. Even ignoring the above factors, payment by installments is likely to result in, at most, only a small reduction in cost, evaluated in present value terms. Annuities are able to offer a higher return than similar unannuitized investments because their return is boosted by "mortality credits," the re-allocation of money in the annuity pool from those who die to those who survive. Mortality rates, and therefore mortality credits, are relatively low at younger ages, and as a result, the additional benefit from purchasing an ALDA before retirement would probably be correspondingly small.

designed with the sole purpose of insuring longevity risk, albeit none with inflation-protected benefits.

In section four, we evaluate the ALDA concept using numerical optimization techniques. Our calculations assume a single risk-free asset in which both the household and insurance company can invest. The insurance company sells both immediate annuities and ALDAs, the prices of which are calculated using the risk-free interest rate and projected annuitant mortality tables. We consider the robustness of our findings to alternative assumptions about variations in money's worths across purchase and commencement ages and policy types. All benefits and returns are expressed in real (inflation-adjusted) terms. The household, which has population mortality for the appropriate birth cohort, chooses between purchasing an annuity and an ALDA immediately on retirement, and postponing the annuitization decision, so as to maximize expected discounted utility. Households that do not annuitize optimally decumulate unannuitized wealth. Households that purchase an ALDA either undertake an optimal decumulation over the period ending with the date that the ALDA payments commence, or follow a rule-of-thumb of consuming an equal amount every year.

In the above model, the money's worths of the ALDAs and other annuities may affect the household's ranking of the alternative strategies. For example, if the money's worth of an immediate inflation-protected annuity is lower at older purchase ages, a household that delays annuitizing will not only forego mortality credits, but also face a greater degree of actuarial unfairness, making early annuitization relatively more attractive.<sup>4</sup> We expect that ALDAs will have lower money's worths than immediate

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<sup>4</sup> In practice, households that delay annuitization face the risk that annuity rates may have declined, either as a result of declines in interest rates, which the household can hedge against, albeit imperfectly, by

annuities to someone with population mortality because the relative survival rates of annuitants are much higher at older ages. But they may still be the most attractive option if they provide a lot of longevity insurance relative to the premium paid.

In the remainder of this section, we present our calculations of annuity and ALDA money's worths. We start our analysis by calculating the relationship between money's worth and age of purchase for the immediate inflation-protected annuities sold by the three companies currently offering such products.<sup>5</sup> If these companies also sold nominal ALDAs, we could infer the likely money's worth of inflation-protected ALDAs from the relationship between the money's worths of their nominal and inflation-protected immediate annuities.

Unfortunately, the above companies do not sell nominal ALDAs. To estimate the likely money's worths of inflation-protected ALDAs, we therefore proceed in three steps. We first compare the money's worths of inflation-protected annuities with those of nominal annuities sold by the same companies to obtain an estimate of the impact on money's worth of providing inflation-protected benefits, and whether that impact varies with policy duration. We then compare the money's worths of nominal immediate annuities and nominal ALDAs sold by two of the four companies that sell both types of products. Finally, we infer from the above relationships the likely relationships in a more developed market between the money's worths of inflation-protected immediate annuities purchased at various purchase ages, and inflation-protected ALDAs. Of necessity, the

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investing in bonds of appropriate duration, or as a result of revisions to the insurance company's mortality assumptions.

<sup>5</sup> During the course of our enquiries, we learned of companies offering products that provided CPI indexed income payments, but with accompanying cash and death benefits. We did not include these products in our analysis because the focus of our research is on products whose sole function is to provide longevity insurance. We also excluded a TIAA-CREF variable immediate annuity invested in TIPS. This product does not provide a fixed income indexed to inflation because the return on TIPS depends not only on inflation, but on movements in the real interest rate.

calculation is imprecise, and we therefore test the robustness of our ranking of the alternative strategies to alternative assumptions regarding the money's worths of the various annuitization options.

In particular, the interest rate chosen affects not only the money's worth of an annuity, but also the relative money's worths of immediate annuities and ALDAs, by reason of the latter's greater duration. The relevant interest rate for our purposes is not the unobserved interest rate used by the insurance company to price the annuity, but one that the household can earn on alternative investments with a similar degree of risk.<sup>6</sup>

When calculating the money's worth of inflation-protected annuities, the only available interest rate is that based on the current term structure of TIPS interest rates, there being markets in neither TIPS STRIPS, nor inflation indexed corporate bonds.

When calculating the money's worth of nominal annuities, Mitchell, Poterba, Warshawsky, and Brown (1999) included calculations based on the term structures of both Treasury and BAA corporate bond interest rates. One justification for using the Treasury rate is the existence of state level policyholder protection. But policyholders may nonetheless attach some weight to the risk of insurance company insolvency. If the alternative "safe" investment is a portfolio of high grade corporate bonds, this might argue for the use of a corporate bond interest rate.<sup>7</sup> To enable comparisons to be made with previous research, we present calculations using not only our preferred interest rate,

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<sup>6</sup> In practice, households can invest in variable immediate annuities that give them the benefit of both mortality credits and the equity premium. Including risky assets in the model would greatly add to its complexity because one would have to allow all the annuity options to be available with both variable and fixed payouts to avoid the investment allocation decision from distorting the annuitization decision. We believe that including risky assets would distract from the focus of the analysis which is the annuitization decision.

<sup>7</sup> Other complicating factors include investment management expenses, which are not separately identified in calculations of the returns on immediate annuities, and the treatment of default risk on corporate bonds. Dushi and Webb (2006) report that in 2003 corporate bond fund management expenses averaged 102 basis points, equivalent to 10.1 percent in present value terms at age 65.



that on AA grade corporate bonds, but also the Treasury and BAA corporate rates, notwithstanding our concerns that the BAA interest rate may understate and the Treasury STRIP rate correspondingly overstate the money's worth of annuities.<sup>89</sup>

We follow Mitchell, Poterba, Warshawsky, and Brown (1999) by using both population and annuitant survival probabilities. Population mortality is sourced from unpublished Social Security life tables for the appropriate birth cohort.<sup>10</sup> Annuitant mortality tables are period tables, reflecting mortality rates of people alive in a particular year. These tables need to be converted into cohort tables that estimate projected mortality rates of annuitants born in a particular year. The above authors did this by assuming that annuitant mortality declined at the same rate as Social Security Administration projections for the whole population.<sup>11</sup> We adopt a slightly different approach, projecting annuitant mortality using Projection Scale AA, believing that this better measures insurance companies' mortality expectations.<sup>12</sup>

A complication arises in that insurance companies' obligations often extend beyond the maturity date of the longest maturity Treasury bonds, exposing them to reinvestment risk. Although the U.S. Treasury has recommenced issuing non inflation-indexed thirty year bonds, thirty year TIPS are no longer available, and the longest dated

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<sup>8</sup> Life insurance companies typically have AA ratings or better – see [www.immediateannuities.com](http://www.immediateannuities.com)

<sup>9</sup> We follow previous research by estimating the term structure of interest rates on corporate bonds by adding the estimated corporate bond risk premium to the term structure of Treasury interest rates.

<sup>10</sup> We thank Felicitie Bell of the Social Security Administration for making them available to us. They are based on the intermediate mortality assumptions used in the 2002 Trustees' Report.

<sup>11</sup> More specifically, they multiplied annuitant mortality by the ratio of the mortality rate obtained from the Social Security mortality table for the relevant birth cohort to the mortality rate obtained from the Social Security 1995 period mortality table.

<sup>12</sup> We use the Annuity 2000 basic life table. This and other life tables can be downloaded from the Society of Actuaries website [www.soa.org](http://www.soa.org) and analyzed using the SOA's Table Manager software. A basic life table shows current period mortality rates without any conservative margin. Published life tables are period tables – they show mortality rates of people of various ages alive in a particular reference year. The SOA publishes projection scales that forecast the rate of decline in mortality rates by age, the most up-to-date of which is Scale AA. They are applied to period mortality tables to construct cohort tables forecasting mortality rates of people born in a particular reference year.

TIPS matures in 2032. Reinvestment risk is a more serious problem for ALDAs than for regular annuities because ALDA payments are concentrated at advanced ages. We assume that insurance companies are able to reinvest their assets at rates of return equal to that currently obtainable at the long portion of the yield curve.<sup>13</sup>

We start by calculating the money's worths of inflation-protected immediate joint life and 2/3 survivor annuities purchased at five year intervals from ages 60 to 85 from the three companies currently selling this product. We compare these with the money's worths of the same companies' nominal annuities purchased at the same ages.<sup>14</sup>

We report inflation protected results based the TIPS term structure (in columns one (using annuitant mortality rates) and five (using population mortality rates). Nominal annuity money's worths are reported using the Treasury STRIPS, AA, and BAA interest rates at columns two to four (annuitant mortality) and six to eight (population mortality).

The table has two striking features. The first is the fact that annuity money's worths to households with annuitant mortality consistently exceed 1.00 when evaluated at TIPS or STRIPS interest rates.<sup>15</sup> The second is the extent to which the relative competitiveness of the three companies varies with purchase age and annuity type. Clearly, it pays to shop around. In contrast, each company appears to offer similar

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<sup>13</sup> We do not consider aggregate mortality risk. Friedberg and Webb (2007) use the Lee-Carter (1992) model to evaluate the aggregate mortality risk faced by annuity providers. They show that aggregate mortality risk is essentially uncorrelated with the returns on the "market portfolio" as measured by the S&P500. Applying the Capital Asset Pricing Model, they argue that it should be possible, at least in theory, to transfer aggregate mortality risk to the financial markets at very low cost. Alternatively, ALDA purchasers could be required to participate in this risk.

<sup>14</sup> We thank Kelli Hueler and Kathleen Schillo of Hueler Companies for providing us with annuity price information. They supplied institutional prices that are somewhat more favorable than retail prices. The advantage of using their data is that all their quotes were obtained on the same day, thus enabling us to use a consistent set of interest rate assumptions across companies and products

<sup>15</sup> James and Song (2001) suggest that insurance companies may be able to offer annuitant money's worths greater than 1.00 because they invest at least part of the premiums in risky assets. They found that United Kingdom inflation-protected annuities generally had lower money's worths and conjectured that this was because insurance companies' were less able to earn a risk premium when their annuity obligations were price indexed. There is no evidence of a similar effect in the United States.

money's worths on its nominal and inflation protected annuities at any given age, when both are evaluated at Treasury rates and using annuitant mortality tables, indicating that the provision of inflation protection has little effect on annuitant money's worths.<sup>16</sup>

The final section of Table One shows the annuity money's worths that a purchaser could obtain at various ages if he "shopped around." Assuming annuitant mortality, the money's worths of nominal annuities vary little with purchase age when the payments are discounted at the AA corporate bond interest rate, and increase somewhat with age when the BAA rate is used. The inflation protected money's worth is broadly constant from age 60 to 75, and declines somewhat at older ages, reflecting the withdrawal of Company C from the market. In contrast, money's worths decline considerably at older ages when population mortality is used, reflecting the increasing divergence between annuitant and population survival rates at older ages.

So what inflation protected annuity money's worth should we assume in our numerical optimization calculations? Households seeking an unannuitized inflation-protected investment have no choice but to invest in TIPS. This would point to an annuitant money's worth of well above 1.00, possibly around 1.10. But households rarely invest much of their wealth in TIPS, and our preference is therefore for an annuitant money's worth of around 1.00, similar to that for nominal annuities when the AA bond rate is used. Even that implies very competitive pricing and we therefore test the robustness of our results to an alternative assumption of 0.90.

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<sup>16</sup> Relative to a nominal annuity, the payments on an inflation protected annuity are weighted towards older ages. Relative to the population as a whole, annuitants are disproportionately likely to survive to these older ages. In consequence, inflation protected annuities have lower money's worths when evaluated using population mortality tables. Our population mortality differences between nominal and inflation-protected annuity money's worths are similar to those calculated by Brown, Mitchell, and Poterba (2000).

We then calculate the money's worth of nominal annuities and nominal ALDAs sold by two of the four insurers selling nominal ALDAs. As the payments are heavily back-loaded, the money's worths are much more sensitive to both interest rates and mortality assumptions than those of immediate annuities. Company D sells a wide range of ALDAs. The first two rows of Table Two show the money's worths of their nominal joint life two thirds survivor annuities at purchase ages of 60 and 65. Rows three to ten show the money's worths of their joint life ALDAs at purchase ages of 60 and 65 and commencement ages of 70, 75, 80, and 85. At the Treasury interest rate, and using annuitant mortality tables, ALDA money's worths are higher than those of immediate annuities, and increase substantially as the deferral period lengthens. At the AA bond interest rate, they are slightly higher, and increase slightly as the deferral period lengthens. At the BAA interest rate, they are somewhat lower.

Company E only sells single life nominal ALDAs, deferred to age 85.<sup>17</sup> Rows eleven and twelve show the money's worth of their male life immediate annuities at purchase ages of 60 and 65. Rows thirteen and fourteen show the money's worths of male life ALDAs at purchase ages of 60 and 65 and a commencement age of 85. Rows fifteen to eighteen show corresponding money's worths for female life annuities and ALDAs.

At the Treasury interest rate, and using annuitant mortality tables, this company's ALDAs have higher money's worths than annuities. At the AA rate, their money's worths are slightly lower, and at the BAA rate, considerably lower.<sup>18</sup>

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<sup>17</sup> In correspondence, this insurance company stated that they believed this to be the most appropriate deferral period.

<sup>18</sup> An unusual feature of the data is that although male and female money's worths are similar when annuitant mortality tables are used, male money's worths are much lower when population tables are used.

From the perspective of households with population mortality, both companies' ALDAs have quite low money's worths – as low as 52.2 percent even at the AA corporate bond interest rate. These low money's worths are a poor guide to the value households might place on insuring consumption at advanced ages. This is because securing that same consumption stream by setting aside part of one's unannuitized wealth becomes increasingly inefficient at such ages, as the likelihood increases that the household will die before consuming its wealth.

Given our preference for the AA bond interest rate, we believe that joint life ALDAs, and especially those with long deferral periods, will have similar or perhaps modestly lower annuitant money's worths than immediate annuities.

### **3. How Much Longevity Insurance Would an ALDA Provide?**

The literature – for example Mitchell, Poterba, Warshawsky, and Brown (1999), Brown and Poterba (2000), and Dushi and Webb (2004) uses numerical optimization techniques to calculate the value to the household of the longevity insurance provided by annuities. This is usually expressed in terms of “annuity equivalent wealth,” the factor by which unannuitized wealth must be multiplied so that the household can enjoy the same expected utility through an optimal decumulation of its unannuitized wealth as it would enjoy were it to purchase an actuarially fair annuity with that wealth.

A similar measure can be constructed of the value of an ALDA by calculating the factor by which unannuitized wealth must be multiplied so that the household is indifferent between an optimal decumulation of unannuitized wealth and the purchase of

an actuarially fair ALDA.<sup>19</sup> The calculation is more complex than that of annuity equivalent wealth because we must jointly determine the optimal proportion of initial wealth to spend on the ALDA and the optimal decumulation of the household's remaining wealth from retirement until the age the ALDA payments commence.

We follow the literature by assuming a constant relative risk aversion utility function of the following form:

$$U_m(C_t^m, C_t^f) = \frac{(C_t^m + \lambda C_t^f)^{1-\gamma}}{1-\gamma}, U_f(C_t^f, C_t^m) = \frac{(C_t^f + \lambda C_t^m)^{1-\gamma}}{1-\gamma} \quad (1)$$

where  $\lambda$  measures the jointness of consumption,  $C_t^m, C_t^f$  denote the consumption of the husband and wife at time  $t$ , and  $\gamma$  is the coefficient of risk aversion. When  $\lambda$  equals one, all consumption is joint. When  $\lambda$  equals zero, none of the household's consumption is joint. The household's expected utility equals each period's utility, multiplied by population average survival probabilities for couples currently aged 60 or 65, as appropriate, and discounted by a rate of time preference that equals the interest rate. For simplicity and to facilitate comparison with previous research, we ignore pre-annuitized wealth, or alternatively assume that pre-annuitized wealth is used to finance basic consumption that does not contribute to the household's utility.

To calculate annuity and ALDA equivalent wealth, we proceed as follows. We first calculate the household's expected utility if it buys an actuarially fair annuity at retirement. We then close the annuity market. We use numerical optimization techniques to calculate an optimal decumulation of the household's wealth and the

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<sup>19</sup> In contrast, Scott (2008) calculates the percentage increases in consumption that a single individual can achieve if he invests various proportions of his wealth in annuities or ALDAs, under the simplifying assumption that he requires equal consumption at every age from 65 to 100, regardless of survival probability. This would result in a far from expected utility maximizing age profile of consumption, as for plausible preference parameters, and for periods prior to the ALDA income commencing, individuals will prefer greater consumption from unannuitized wealth at ages when they are most likely to be alive.

expected utility of that decumulation plan. We then calculate the amount by which the household's wealth must be increased so that its expected utility equals that obtainable when it annuitizes. This increased amount is divided by the household's original wealth to obtain the household's annuity equivalent wealth. We assume that the household and the insurance company are both able to invest in a single risk-free asset yielding 2.35 percent, the average yield on long dated TIPS in February 2007, the month we started running the programs, and that this also equals the household's rate of time preference.<sup>20</sup>

The calculation of ALDA equivalent wealth is analogous. For ALDAs that commence payment at ages 70, 75, 80, 85, and 90, we determine the optimal proportion of initial wealth to spend on the ALDA, and the optimal decumulation strategy over the period ending on the date that the ALDA income commences.<sup>21</sup> We then close the ALDA market and calculate the amount by which the household's wealth must be increased so that its expected utility equals that obtainable when it purchases an ALDA.

The first line of each of the two panels of Table Three shows our calculations of annuity equivalent wealth at ages 60 and 65, assuming coefficients of risk aversion of two to five. At age 60, annuity equivalent wealth varies from 1.216 at a coefficient of risk aversion of two, to 1.291 at a coefficient of five.<sup>22</sup> At age 60, the household would be indifferent between an annuity with an expected present value of \$100.00, and

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<sup>20</sup> Previous research assumed a three percent interest rate and rate of time preference. A complication arises if the rate of interest differs from the rate of time preference in that households will then prefer annuities with increasing or decreasing real income streams.

<sup>21</sup> A particular issue is whether we should impose the constraint that the household consumes all of its unannuitized wealth by the time the ALDA payments commence. It can sometimes be optimal not to do so – for example, if a member of the household dies shortly before that date. We think it is unreasonable to expect a household to solve a decumulation problem of such complexity, and assume that all wealth is consumed by that time.

<sup>22</sup> Our results differ slightly from those of Brown and Poterba (2000). We obtain almost identical results when we calculate AEW for their somewhat earlier birth cohort, using their assumed values for  $\lambda$  and the rate of time preference; the very small remaining difference likely reflects differences in the assumed timing of income and consumption.

\$129.10 of unannuitized wealth. Annuity equivalent wealth is higher at older commencement ages reflecting higher annual mortality rates and an increasing advantage to be obtained from reallocating wealth from those who die to those who survive.

The following lines of each of the two panels show ALDA equivalent wealth for ALDAs with payments commencing at ages 70 to 90, assuming that the household follows an optimal decumulation strategy prior to the ALDA income commencing. Immediately below the results for each commencement age, we show the percentage of the value of the full longevity insurance provided by the annuity that is provided by the ALDA. Below the results and percentages, we show the proportions of initial wealth that the household should optimally allocate to the purchase of the ALDA.

ALDA equivalent wealth is, of course, less than annuity equivalent wealth, and is lower at older commencement ages. But even at age 85, the ALDA provides more than half the longevity insurance provided by the annuity, at a fraction of the cost in terms of foregone liquidity.<sup>23</sup> Even at a commencement age of 90, an ALDA purchased at age 60 yields ALDA equivalent wealth equals 1.138, or 47.5 percent of annuity equivalent wealth, assuming a coefficient of risk aversion of five. But the household will optimally spend only 4.3 percent of its initial wealth on purchasing the ALDA.

#### **4. ALDAs and Annuities in the Presence of Actuarial Unfairness**

Annuities are actuarially unfair, reflecting both adverse selection and expense loads. In this section, we recalculate annuity and ALDA equivalent wealth for each of

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<sup>23</sup> The additional liquidity provided by the ALDA is largely illusory. The household only enjoys additional liquidity until it has exhausted its financial wealth and it is at advanced ages when medical costs are both large and uncertain that liquidity will have the greatest value.



the strategies described in the previous section, taking account of projected levels of actuarial unfairness.

In practice, households that choose not to annuitize do not calculate decumulation strategies using numerical optimization techniques. Little is known about how households make asset decumulation decisions in retirement, but it seems plausible that households use rules-of-thumb, to the extent that they plan at all.<sup>24</sup> Some of these are likely to be highly sub-optimal. For example, some retirement planning tools suggest that households should accumulate sufficient wealth by retirement to finance consumption over their life expectancy. Such a strategy offers a 50-percent chance of destitution in old age. It is sometimes asserted that annual consumption in retirement should be no more than four percent of initial wealth, because Monte-Carlo simulations show that households decumulating at that rate have only a small chance of outliving their wealth.<sup>25</sup> In the absence of a bequest motive or a desire to retain liquidity, this strategy is clearly sub-optimal because the household can obtain a higher income with zero probability of outliving its wealth by buying an inflation-protected annuity.

One advantage of the ALDA over a decumulation of unannuitized wealth is that it transforms the complex task of decumulating one's wealth over an uncertain lifespan into the much simpler task of decumulating over a fixed period ending on the date that the ALDA payments commence. In this section, we show that a household does very nearly as well consuming an equal amount each period prior to the date the ALDA commences as it would do if it attempted to consume the optimal amount each period, taking account

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<sup>24</sup> DeNardi, French, and Jones (2006) analyze the wealth decumulation paths of Health and Retirement Study households and show that health and longevity risks and social insurance programs have substantial effects.

<sup>25</sup> See, for example, Bengen (1994).

of the annual survival probabilities of each spouse. Of course, the household that attempts an optimal decumulation can end up a great deal worse off if it gets its calculations wrong.

As explained in Section Two, our base case assumption is that annuities have money's worths of 1.00 to households with annuitant mortality. This is considerably higher than the values of 0.925 (Treasury rate) and 0.85 (BAA corporate bond rate) calculated by Mitchell, Poterba, Warshawsky, and Brown (1999) using 1995 data. One reason for our higher money's worths is that we assume that households are able to obtain institutional prices, whereas the above authors used an all-company average of retail quotes.<sup>26</sup> But there also seems to be trend towards increased money's worths, identified by the above authors who compared 1985 with 1995 data, and more recently by James and Song (2001).

Table Four shows the equivalent wealth of various strategies relative to a base case of undertaking an optimal or rule-of-thumb decumulation of unannuitized wealth, taking account of actuarial unfairness. The results in panel A are calculated under the assumption that annuities and ALDAs have a money's worth of 100 percent to a household with annuitant mortality (equivalent to a money's worth of 89.6 percent for a 60 year old couple with population mortality). To illustrate the extent to which our results are sensitive to alternative assumptions regarding annuity and ALDA money's worths, we show, in panel B, equivalent wealth under an alternative assumption of 90 percent annuitant money's worth (equivalent to a money's worth of 80.6 percent to a 60 year old couple with population mortality). Strategies with higher equivalent wealth are preferred, and those with values exceeding one are preferred to the base case. The table

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<sup>26</sup> See footnote 14.

reports results for coefficients of risk aversion of two, three, four, and five, and assumes no pre-annuitized wealth, retirement ages of 60, and 65, and population mortality for the 1947 (age 60) and 1942 (age 65) birth cohorts.

The first row of panels A and B reports the value of full annuitization immediately on retirement. At annuitant money's worth of 100 percent, households are 8.9 to 15.6 better off annuitizing immediately on retirement at age 60 than undertaking an optimal decumulation of unannuitized wealth, depending on the assumed level of risk-aversion. At annuitant money's worth of 90 percent, households would be 2.0 percent worse off annuitizing at a coefficient of risk aversion of two, but 4.0 percent better off at a coefficient of five.

An alternative to full annuitization at retirement is to undertake an optimal partial decumulation of unannuitized wealth, and then purchase an annuity at some older age. Dushi and Webb (2004) showed that at plausible constant levels of actuarial unfairness it was optimal to delay. The expense saving more than compensated for the loss of mortality credits.<sup>27</sup> At lower money's worths, and at smaller coefficients of risk aversion, it was optimal to extend the period of delay, and eventually not to annuitize at all.

The second row of the tables reports results for annuitization at the optimal age. At 100 percent annuitant money's worth, the household is better off annuitizing at age 60 (or 65) than delaying, regardless of coefficient of risk aversion, so the entries on the first and second rows are identical. At 90 percent annuitant money's worth, the household is better off delaying than either purchasing annuity immediately on retirement, or undertaking an optimal decumulation of unannuitized wealth regardless of degree of risk

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<sup>27</sup> In practice, a household pursuing this strategy runs the risk that annuity rates may decline as a result of adverse movements in interest rates and mortality assumptions.

aversion. The optimal period of delay is greater at lower coefficients of risk aversion. We calculate that for a household aged 65, the optimal delay ranges from thirteen years when CRRA equals two, to five years when CRRA equals five.

Rows three to seven show results when the household purchases an ALDA with commencement ages of 70, 75, 80, 85, or 90. We assume that the household allocates the optimal proportion of its wealth to the ALDA, and consumes the optimal amount every period from retirement until the age the ALDA income commences. As with the results for immediate and deferred annuitization, more risk-averse households place a higher value on the longevity insurance provided by ALDAs.

At both assumed levels of money's worth, immediate and deferred annuitization and the purchase of ALDAs with any deferral period are all preferable, with one minor exception, to an optimal decumulation of unannuitized wealth. But ALDAs with optimal deferral periods dominate both immediate and deferred annuitization. At higher money's worths than those assumed in our table, immediate annuitization comes to dominate the ALDA. At lower than our assumed money's worths, unannuitized decumulation comes to dominate immediate annuitization. At lower assumed levels of money's worth, the optimal ALDA deferral period increases. But at any money's worth other than zero, an ALDA with an optimal deferral period will dominate an optimal decumulation of unannuitized wealth.

The impact of variations in money's worths across product types can be gauged by comparing money's worths across the two tables. For example, at age 60 and at a coefficient of risk aversion of five, the purchase of an immediate annuity with a 100 percent annuitant money's worth dominates any of the 90 percent money's worth ALDA

deferral periods. In calculations that are not reported, we find that at lower assumed money's worths, the optimal strategy is less sensitive to variations in money's worth across product type.

The above calculations assume that the household undertakes an "optimal" decumulation of unannuitized wealth, trading off the benefit of higher consumption early in retirement when it is more likely to be alive, against the risk of low consumption at older ages. Rows eight to twelve show annuity equivalent wealth when the household adopts an alternative rule-of-thumb strategy of equal consumption in all periods prior to the age at which the ALDA payments commence. By comparing the entry for the naïve strategy with the corresponding entry for the optimal strategy, one can calculate how much worse off the household is as a result of behaving sub-optimally. Depending on coefficient of risk aversion, and assuming 100 percent annuitant money's worth, a household aged 60 purchasing an ALDA with payments commencing at age 85 would be only 1.2 (1.083-1.071) percent to 0.8 (1.132-1.124) percent worse off. The household does almost as well following this simple rule-of-thumb as it would were it to carefully calculate an optimal strategy.<sup>28</sup>

Finally, rows thirteen to seventeen (sophisticated strategy) and eighteen to twenty two (naïve strategy) show the percent of initial wealth that a household should optimally spend on an ALDA, taking account of actuarial unfairness. Assuming the ALDA has a 100 percent money's worth to a household with annuitant mortality, a household aged 60

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<sup>28</sup> Under CRRA utility, the coefficient of risk aversion ( $\gamma$ ) equals the inverse of the intertemporal elasticity of substitution. If the rate of interest equals the rate of time preference the optimal annual percentage decline in consumption for a single individual equals the annual mortality risk, divided by the coefficient of risk aversion. The optimal decline for married couples is complicated by the effects of the risk of changes in household composition, but is broadly similar. At younger ages, annual mortality risk is quite low, and there is little to be gained from reallocating consumption from older to younger ages.

with population mortality, purchasing an ALDA with income payments commencing at age 85, and following a naïve decumulation strategy in the interim, should spend between 12.5 and 14.3 percent of its wealth on an ALDA, setting aside the remainder of its wealth for consumption between age 60 and 85. For any given commencement age, the optimal proportion of current wealth that should be invested in the ALDA increases with both purchase age and degree of risk aversion, the latter effect resulting from the CRRA risk aversion parameter doubling as the inverse of the intertemporal elasticity of substitution. But most of the variation comes from the choice of commencement age.

The optimal ALDA deferral period depends on its money's worth and the household's coefficient of risk-aversion and is highly sensitive to the assumed money's worth.<sup>29</sup> At 100 percent annuitant money's worth, a household purchasing an ALDA at age 60 should choose deferral to age 72 at a coefficient of risk aversion of two but only 61 at a coefficient of risk aversion of five. At 90 percent money's worth, the corresponding ages are 80 and 78. But the losses that flow from choosing an inappropriate commencement age are relatively minor, provided the proportion of wealth invested in the ALDA is appropriate for the commencement age.

The above calculations all assume a single risk free asset in which both the household and the insurance company invest. Blake, Cairns, and Dowd (2003) and Horneff, Mitchell, Maurer, and Stamos (2007) analyze the portfolio allocation and annuitization decision when variable immediate annuities are available.<sup>30</sup> They show that

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<sup>29</sup> The optimal deferral period for singles is shorter than that for couples.

<sup>30</sup> We contrast variable immediate annuities with variable deferred annuities. Deferred annuities lack the essential characteristic of an immediate annuity, namely the transfer of wealth from those who die young to those who are unlucky enough to live unusually long. The mortality credits resulting from this transfer enable annuities to offer a higher return than similar unannuitized investments, particularly at older ages. Variable immediate annuities provide a lifetime income that increases or decreases if the return on the underlying investments exceeds or falls short of a specified rate.

variable immediate annuities are attractive because their return is enhanced by both the equity premium and mortality credits. One option might therefore be to offer both inflation-protected and equity linked ALDAs. One problem that immediately arises when households invest in risky assets is that households following a plan of consuming a fixed amount every period prior to the ALDA payments commencing cannot be certain when they will exhaust their financial assets. One solution, proposed by Huang, Milevsky, and Salisbury (2007) is for an ALDA that combines longevity with investment portfolio insurance. They envisage an ALDA that would commence payment when a hypothetical investment in some market index that had been subject to a periodic withdrawal of some pre-specified amount had been exhausted. But payments would be conditional on one or both members of the household being alive at that time.

Our results complement those of Scott, Watson, and Hu (2007). They assume that, for a variety of reasons, households might only want to annuitize part of their wealth. They show that a single individual who only wants to annuitize, five, ten, or twenty percent of his wealth will be better off choosing an ALDA than regular or deferred annuitization or an optimal decumulation of unannuitized wealth and that spending only a small proportion of the household's wealth on an ALDA yields almost as great ALDA or annuity equivalent wealth as full annuitization.<sup>31</sup> We do not constrain the proportion of wealth that is spent on ALDAs or other annuity products, and calculate the optimal proportion of wealth to spend on annuities, and the optimal ALDA commencement age.

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<sup>31</sup> Their annuity equivalent wealth numbers are not comparable because they calculate values for single males, and it is well-established that longevity risk pooling within marriage reduces the value of annuitization.

## 5. Using ALDAs as a Default in 401(k) Plans

Previous research has demonstrated the power of defaults to influence savings decisions, most notably the 401(k) participation decision (Beshears, Choi, Laibson, and Madrian (2006)), and the choice between a single and a joint life annuity in defined benefit pension plans (Johnson, Uccello, and Goldwyn (2003)). If it is believed that households are making inappropriate annuitization decisions, then one solution might be to default them into an ALDA at retirement. But the decision to purchase an ALDA may not be in the best interest of high mortality households and, unlike the 401(k) participation decision, is irrevocable. In this section we review previous research on the distributional consequences of mandatory annuitization. We then calculate the distributional consequences, in both money's worth and expected utility terms, of defaulting households into ALDAs.

Brown (2003) calculated the distributional consequences for single individuals with no pre-annuitized wealth of mandatory annuitization on uniform and actuarially fair terms. He found that for the average individual in high mortality groups, for example black males with less than a high school education, annuity money's worth would be substantially less than the premium paid. But the average individual in all groups would be better off in expected utility terms.

As Brown points out, group averages may conceal considerable within-group heterogeneity. Using the methodology developed in Gan, Hurd, and McFadden (2005), Gong and Webb (2008) constructed subjective mortality tables for each HRS individual, based on the individual's estimate of his or her probability of surviving to age 75.<sup>32</sup> They

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<sup>32</sup> To summarize, individuals in the HRS were asked to assess their probabilities of surviving to ages 75 and 85, on a scale of one to ten in wave one, and a scale of one to 100 in subsequent waves.



showed that these subjective life tables varied appropriately with known determinants of mortality, and aggregated closely to published mortality tables. They then used these tables to calculate annuity equivalent wealth for each HRS married couple household turning 65 between 1994 and 2000, taking account of longevity risk sharing within marriage and each household's proportion of pre-annuitized wealth. They showed that 16.5 percent of all households and 36.5 percent of those with less than a high school education would be worse off in expected utility terms as a result of mandatory annuitization on uniform and actuarially fair terms.

Using the same methodology, we calculate the whole distribution of ALDA and annuity money's worth for above sample, and then calculate ALDA equivalent wealth for a prototypical high mortality household. We first assume actual (or in the case of ALDAs, projected) levels of actuarial unfairness.<sup>33</sup> But defaulting high mortality households into annuities or ALDAs may reduce the equilibrium level of actuarial unfairness, and mandating annuitization or the purchase of ALDAs might reduce it still further.<sup>34</sup> To illustrate the distribution of money's worths under a program of mandatory

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The data suffers from serious focal response problems, with some individuals giving responses of 0.0 and 1.0. These focal responses cannot be used directly as the measure of true subjective survival probabilities, because the distribution of true responses should be continuous and the true probabilities cannot be literally zero or one.

Gan, Hurd, and McFadden (2005) proposed a Bayesian updating method for recovering subjective annual survival probabilities from the AHEAD panel of somewhat older individuals born before 1924. More specifically, they assumed that an individual's true belief regarding his or her survival probability is unknown to the econometrician. However, the econometrician does know the distribution of those beliefs - the Bayesian "prior." The individual reports a survival probability based on, but not necessarily equal to, his true beliefs. The difference between his true and his reported beliefs represents measurement error. GHM use the self-reported survival probabilities to update the prior distribution and to obtain the posterior distribution. GHM then apply the mean of the posterior distribution as an individual's estimated subjective survival probability to the observed mortality data among the panel to estimate parameter values that best characterize each individual's belief as to his annual survival probabilities.

<sup>33</sup> We cannot use the ALDA prices reported in section two because they relate to the 1942 and 1947 birth cohorts, and the HRS households are on average somewhat older. Each household's money's worth is calculated using annuity and ALDA prices appropriate to its particular birth year.

<sup>34</sup> The actual money's worth would depend not only on insurance company expenses and sales loads, but also on program design. Women, who on average live longer than men, have lower 401(k) plan balances

annuitization of 401(k) plan balances or Social Security Individual Accounts, we alternatively assume that annuities and ALDAs are actuarially fair to households with population average mortality for the appropriate birth cohort.

Figure One shows the distribution of gains and losses, in money's worth terms, and as a percentage of annuitizable wealth, were couples in the HRS required to purchase joint life and two thirds survivor ALDAs and annuities at actual and projected levels of actuarial unfairness. The average household would perceive itself suffering a loss of 21.2 percent of its initial wealth as a result of annuitization, but only 7.3 percent as a result of the purchase of the ALDA. At the 5<sup>th</sup> percentile of the distribution of money's worth, the annuity and ALDA losses amount to 31.7 and 12.9 percent respectively. Purchase of an ALDA inflicts a much smaller loss on both average and high mortality households than the purchase of an annuity.

Figure Two shows the results of the same calculations when the annuity and ALDA are both actuarially fair. As one might expect, both the annuity and ALDA are perceived to be approximately actuarially fair by the average household – it would perceive itself as gaining 1.1 percent in money's worth terms from the annuity, and 1.4 percent from the ALDA.<sup>35</sup> At the 5<sup>th</sup> percentile, the 5.2 percent loss from the ALDA is much smaller than the 12.4 percent loss from the annuity.

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and would also have lower Social Security Individual Account balances because of their lower lifetime earnings. But within each gender, there is a positive correlation between wealth and longevity. The relative impact of these two factors would depend on the detailed design of the program, and in particular whether annuities were on joint or single lives, and whether individuals were required to annuitize a proportion of their wealth, or only up to a fixed dollar amount.

<sup>35</sup> Survival probability data is missing for proxy interviewees who likely had higher than average mortality. We imputed missing data, but were probably not wholly successful in correcting for this source of bias. We are therefore not surprised to find that average perceived money's worth slightly exceeds the premium paid.

But the above money's worth calculations are a poor guide as to the value high mortality households might place on ALDAs because they fail to take account of the longevity insurance they provide. Brown (2003) showed that variations in mortality had relatively little impact on the value households placed on annuities, the intuition behind his finding being that even high mortality households needed to set aside wealth for consumption in the event that they lived unusually long. We calculate ALDA equivalent wealth to the household whose subjective mortality beliefs correspond to the 5<sup>th</sup> percentile of the distribution of annuity money's worth. Assuming an ALDA money's worth of 90 percent to a household with annuitant mortality, we find that if the household purchased an ALDA with a commencement age of 85, it would have ALDA equivalent wealth in excess of one, irrespective of purchase age or coefficient of risk aversion.

The above calculations are contingent on a utility function that does not appear to be very predictive of current behavior. Although annuity equivalent wealth calculations indicate that immediate annuitization would increase the welfare of at least a substantial proportion of households, only a very small minority voluntarily annuitizes any of their wealth. This may reflect both ignorance and behavioral biases, but whatever the case, care needs to be taken when estimating the distribution of welfare gains with an expected utility framework that has substantive predictions so at odds with observed behavior.

A potential issue with defaulting households into any annuity product is the fiduciary risk of purchasing an irrevocable annuity for the employee without his consent. One partial solution might be to allow the purchase to be reversible for a period of years.<sup>36</sup>

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<sup>36</sup> It is difficult to cost this proposal because it is unclear to what extent annuity providers would suffer adverse selection among employees withdrawing from ALDA purchases.

## **6. Conclusion**

The ALDA provides a lot of longevity insurance at a relatively low cost. It also makes decumulation much simpler during the period before the ALDA payments commence.

It remains to be seen whether such a product would overcome annuity aversion. One possible solution might be to make the purchase of an ALDA the default in 401(k) plans. But this has the potential to harm high mortality households that would rationally choose to hold their wealth in unannuitized form and undertake a rapid decumulation of that wealth over their relatively short life expectancy. Our calculations indicate that, even when evaluated in money's worth terms, defaulting high mortality households into ALDAs would cause relatively little harm. This is because although ALDAs have a low money's worth to such households, they would only invest a small proportion of their total wealth in them. But even high mortality households might be better off in expected utility terms purchasing an ALDA than undertaking an optimal decumulation of unannuitized wealth.

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<b>Table One: Money's Worths of Inflation Indexed and Nominal Annuities</b>									
		Annuitant Mortality				Population Mortality			
Interest rate		Treasury	AA Corporate	BAA Corporate	Treasury	AA Corporate	BAA Corporate		
		Inflation Indexed	Nominal		Inflation Indexed	Nominal			
Company A									
Purchase Age	60	0.951	1.100	1.000	0.891	0.850	1.017	0.932	0.837
	65	0.962	1.095	1.006	0.905	0.846	0.997	0.922	0.838
	70	0.973	1.088	1.009	0.919	0.845	0.978	0.914	0.839
	75	0.989	1.086	1.018	0.939	0.844	0.958	0.905	0.841
	80	1.012	1.093	1.034	0.965	0.841	0.937	0.893	0.840
	85	1.043	1.123	1.073	1.013	0.813	0.907	0.873	0.832
Company B									
Purchase Age	60	1.093	1.129	1.027	0.914	0.976	1.044	0.956	0.859
	65	1.086	1.122	1.030	0.927	0.955	1.021	0.945	0.858
	70	1.041	1.063	0.986	0.898	0.905	0.955	0.893	0.820
	75	1.043	1.066	0.999	0.921	0.890	0.941	0.888	0.825
	80	0.925	0.956	0.905	0.845	0.768	0.820	0.782	0.736
	85	0.836	0.880	0.840	0.793	0.651	0.710	0.684	0.652
Company C									
Purchase Age	60	1.114	1.132	1.029	0.916	0.995	1.047	0.959	0.861
	65	1.115	1.131	1.039	0.935	0.981	1.030	0.953	0.865
	70	1.110	1.126	1.045	0.951	0.965	1.012	0.946	0.869
	75	1.113	1.128	1.058	0.975	0.950	0.995	0.940	0.874
Best buy									
Purchase Age	60	1.114	1.132	1.029	0.916	0.995	1.047	0.959	0.861
	65	1.115	1.131	1.039	0.935	0.981	1.030	0.953	0.865
	70	1.110	1.126	1.045	0.951	0.965	1.012	0.946	0.869
	75	1.113	1.128	1.058	0.975	0.950	0.995	0.940	0.874
	80	1.012	1.093	1.034	0.965	0.841	0.937	0.893	0.840
	85	1.043	1.123	1.073	1.013	0.813	0.907	0.873	0.832

Notes: All annuities are joint life, 2/3 survivor, payable monthly, no guarantees



**Table Two: Money's Worths of Nominal Annuities and ALDAs**

Interest rate		Annuitant Mortality			Population Mortality		
		Treasury	AA Corporate	BAA Corporate	Treasury	AA Corporate	BAA Corporate
Company D							
Annuity							
Purchased at age	60	1.073	0.978	0.873	0.986	0.905	0.815
	65	1.062	0.977	0.882	0.961	0.891	0.811
ALDA							
Purchased at age 60							
Commencing at age	70	1.153	0.985	0.805	0.978	0.841	0.694
	75	1.216	1.006	0.788	0.974	0.811	0.640
	80	1.311	1.051	0.790	0.977	0.788	0.596
	85	1.392	1.082	0.779	0.940	0.734	0.531
Purchased at age 65							
Commencing at age	70	1.092	0.975	0.844	0.946	0.850	0.742
	75	1.149	0.993	0.823	0.938	0.815	0.681
	80	1.220	1.020	0.810	0.921	0.775	0.619
	85	1.337	1.084	0.824	0.909	0.740	0.566
Company E							
Single Males							
Annuity							
Purchased at age	60	1.143	1.048	0.941	1.012	0.936	0.851
	65	1.136	1.051	0.955	0.986	0.921	0.845
ALDA							
Purchased at age 60, commencing at age 85		1.252	0.975	0.704	0.687	0.539	0.392
Purchased at age 65, commencing at age 85		1.142	0.927	0.707	0.638	0.522	0.401
Single Females							
Annuity							
Purchased at age	60	1.122	1.024	0.917	1.039	0.954	0.860
	65	1.111	1.025	0.927	1.014	0.941	0.857
ALDA							
Purchased at age 60, commencing at age 85		1.226	0.953	0.688	0.903	0.704	0.509
Purchased at age 65, commencing at age 85		1.136	0.922	0.702	0.837	0.681	0.520

Both companies' annuities pay monthly benefits with no guarantees. Company D is joint life 2/3 survivor. Company E is single life - it does not offer a joint life ALDA

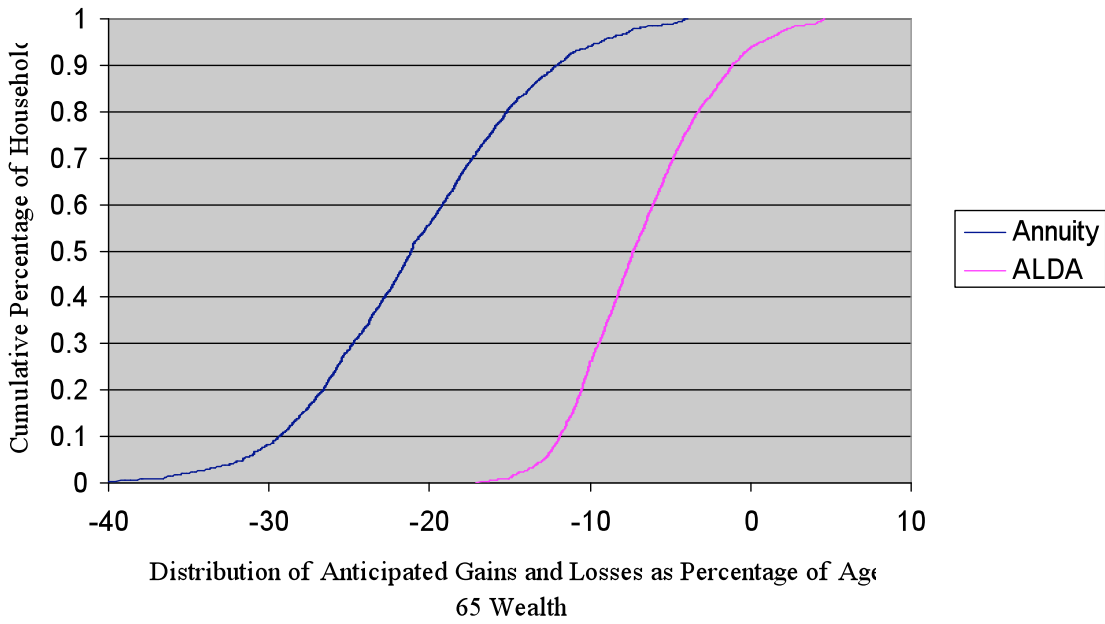
**Table Three: Comparison of ALDA Equivalent Wealth With Annuity Equivalent Wealth -**

Risk Aversion		2	3	4	5
Annuity or ALDA purchased at age 60					
Annuity equivalent wealth		1.216	1.251	1.274	1.291
ALDA commencing at age	70	1.206	1.240	1.265	1.291
		95.4%	95.7%	96.6%	100.0%
	75	1.191	1.224	1.246	1.269
		88.3%	89.0%	89.8%	92.4%
	80	1.167	1.197	1.218	1.234
		77.3%	78.6%	79.4%	80.6%
	85	1.134	1.161	1.178	1.190
		61.9%	64.2%	65.0%	65.2%
	90	1.093	1.116	1.130	1.138
		43.0%	46.1%	47.3%	47.5%
Proportion of initial wealth spent on ALDA	70	0.522	0.527	0.531	0.538
	75	0.343	0.348	0.351	0.354
	80	0.204	0.207	0.209	0.210
	85	0.103	0.105	0.106	0.107
	90	0.042	0.042	0.043	0.043
Annuity or ALDA purchased at age 65					
Annuity equivalent wealth		1.264	1.307	1.336	1.356
ALDA commencing at age	70	1.260	1.303	1.331	1.351
		98.5%	98.8%	98.5%	98.6%
	75	1.244	1.286	1.317	1.332
		92.7%	93.3%	94.6%	93.3%
	80	1.216	1.254	1.280	1.315
		82.0%	82.8%	83.4%	88.3%
	85	1.174	1.209	1.233	1.246
		66.0%	68.1%	69.6%	69.2%
	90	1.120	1.149	1.167	1.180
		45.4%	48.5%	49.8%	50.5%
Proportion of initial wealth spent on ALDA	70	0.703	0.707	0.709	0.711
	75	0.459	0.465	0.468	0.468
	80	0.270	0.273	0.276	0.280
	85	0.135	0.137	0.138	0.138
	90	0.053	0.054	0.054	0.054
Notes: Rate of time preference and real rate of interest both equal 2.35%. Husband and wife both aged 60 (65) with 1947 (1942) birth cohort mortality. Complementarity of consumption (l) = 0.5. Annuity has 2/3 survivor benefit					

<b>Table Four A - Alternative Strategies Relative to a Base Case of Unannuitized Decumulation - 100% Annuitant Money's Worth</b>						
Risk aversion			2	3	4	5
Wealth decumulation commencing at age 60						
	Row					
Annuitization immediately on retirement	1		1.089	1.121	1.141	1.156
Annuitization - optimal age	2		1.089	1.121	1.141	1.156
Sophisticated strategy - Purchasing an ALDA commencing at	3	70	1.093	1.122	1.141	1.160
	4	75	1.094	1.121	1.139	1.152
	5	80	1.091	1.117	1.134	1.146
	6	85	1.083	1.106	1.121	1.132
	7	90	1.065	1.086	1.099	1.108
Naïve strategy - Purchasing an ALDA commencing at	8	70	1.093	1.122	1.141	1.155
	9	75	1.093	1.120	1.138	1.152
	10	80	1.087	1.113	1.130	1.143
	11	85	1.071	1.096	1.112	1.124
	12	90	1.039	1.063	1.078	1.089
Sophisticated strategy - Proportion of initial wealth allocated to ALDA commencing at age	13	70	0.546	0.559	0.565	0.571
	14	75	0.372	0.386	0.393	0.397
	15	80	0.231	0.244	0.251	0.255
	16	85	0.125	0.135	0.140	0.143
	17	90	0.054	0.060	0.063	0.065
Naive strategy - Purchasing an ALDA commencing at	18	70	0.546	0.559	0.565	0.569
	19	75	0.371	0.386	0.393	0.397
	20	80	0.230	0.244	0.250	0.254
	21	85	0.124	0.134	0.139	0.142
	22	90	0.054	0.059	0.062	0.064
Wealth decumulation commencing at age 65						
Annuitization immediately on retirement	1		1.115	1.153	1.178	1.196
Annuitization - optimal age	2		1.115	1.153	1.178	1.196
Sophisticated strategy - Purchasing an ALDA commencing at	3	70	1.118	1.155	1.179	1.196
	4	75	1.119	1.154	1.177	1.194
	5	80	1.117	1.149	1.171	1.186
	6	85	1.106	1.136	1.155	1.169
	7	90	1.083	1.109	1.126	1.137
Naïve strategy - Purchasing an ALDA commencing at	8	70	1.118	1.154	1.179	1.196
	9	75	1.119	1.154	1.177	1.194
	10	80	1.113	1.146	1.168	1.184
	11	85	1.095	1.126	1.146	1.161
	12	90	1.054	1.084	1.103	1.116
Sophisticated strategy - Proportion of initial wealth allocated to ALDA commencing at age	13	70	0.721	0.730	0.735	0.738
	14	75	0.488	0.503	0.510	0.514
	15	80	0.301	0.316	0.324	0.328
	16	85	0.161	0.173	0.179	0.183
	17	90	0.069	0.076	0.080	0.082
Naive strategy - Proportion of initial wealth allocated to ALDA commencing at age	18	70	0.721	0.730	0.735	0.738
	19	75	0.488	0.503	0.510	0.514
	20	80	0.301	0.316	0.323	0.328
	21	85	0.160	0.172	0.178	0.182
	22	90	0.068	0.075	0.079	0.081
Notes: See Table Three						

<b>Table Four B - Alternative Strategies Relative to a Base Case of Unannuitized Decumulation - 90% Annuitant Money's Worth</b>						
Risk aversion			2	3	4	5
Wealth decumulation commencing at age 60						
	Row					
Annuitization immediately on retirement	1		0.980	1.009	1.027	1.040
Annuitization - optimal age	2		1.057	1.068	1.076	1.077
Sophisticated strategy - Purchasing an ALDA commencing at	3	70	1.031	1.057	1.074	1.091
	4	75	1.051	1.076	1.092	1.104
	5	80	1.065	1.088	1.103	1.115
	6	85	1.068	1.090	1.104	1.115
	7	90	1.059	1.079	1.092	1.100
Naïve strategy - Purchasing an ALDA commencing at	8	70	1.031	1.057	1.074	1.087
	9	75	1.050	1.075	1.091	1.103
	10	80	1.060	1.084	1.100	1.112
	11	85	1.057	1.080	1.096	1.107
	12	90	1.033	1.056	1.071	1.081
Sophisticated strategy - Proportion of initial wealth allocated to ALDA commencing at age	13	70	0.559	0.576	0.584	0.591
	14	75	0.384	0.403	0.412	0.418
	15	80	0.240	0.257	0.266	0.271
	16	85	0.131	0.143	0.150	0.154
	17	90	0.057	0.064	0.068	0.071
Naive strategy - Purchasing an ALDA commencing at	18	70	0.559	0.576	0.584	0.589
	19	75	0.384	0.402	0.412	0.417
	20	80	0.240	0.257	0.265	0.271
	21	85	0.130	0.142	0.149	0.153
	22	90	0.056	0.064	0.067	0.070
Wealth decumulation commencing at age 65						
Annuitization immediately on retirement	1		1.003	1.038	1.061	1.077
Annuitization - optimal age	2		1.073	1.087	1.094	1.110
Sophisticated strategy - Purchasing an ALDA commencing at	3	70	1.071	1.104	1.126	1.147
	4	75	1.092	1.124	1.145	1.160
	5	80	1.106	1.137	1.157	1.171
	6	85	1.110	1.139	1.158	1.171
	7	90	1.101	1.127	1.144	1.156
Naïve strategy - Purchasing an ALDA commencing at	8	70	1.071	1.104	1.126	1.142
	9	75	1.091	1.123	1.144	1.159
	10	80	1.102	1.133	1.153	1.168
	11	85	1.098	1.129	1.149	1.163
	12	90	1.073	1.103	1.122	1.136
Sophisticated strategy - Proportion of initial wealth allocated to ALDA commencing at age	13	70	0.731	0.744	0.750	0.754
	14	75	0.501	0.520	0.530	0.535
	15	80	0.313	0.332	0.341	0.347
	16	85	0.168	0.183	0.191	0.195
	17	90	0.073	0.081	0.086	0.089
Naive strategy - Proportion of initial wealth allocated to ALDA commencing at age	18	70	0.731	0.744	0.750	0.754
	19	75	0.501	0.520	0.529	0.535
	20	80	0.317	0.331	0.341	0.346
	21	85	0.167	0.182	0.190	0.194
	22	90	0.072	0.080	0.085	0.087
Notes: See Table Three						

**Figure 1: Distribution of Annuity and ALDA Money's Worth  
 – When Priced Using Annuitant Mortality Tables**



**Figure 2: Distribution of Annuity and ALDA Money's Worth  
 – When Priced Using Population Mortality Tables**

