Social Security Reform in a Dynastic Life-Cycle Model with Endogenous Fertility

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Abstract

This paper studies the effects of a fully funded social security reform with endogenous fertility in a detailed, general equilibrium life-cycle model with dynasties whose members differ in skills and life uncertainty. We find that as high skill households tend to save relatively more in assets than in children, models with exogenous fertility underestimate the aggregate capital stock in the PAYG steady state. These models also predict that the capital stock increases after the fully funded reform. However, because the high skill households respond to the reform by having more children and investing less in assets, the average fertility increases and the aggregate capital stock falls. The welfare gains from the elimination of social security seem to more than compensate the agents for the lost insurance against life-span and earnings risks. Finally, while in the fully funded system all parents rely on the old-age support from children during retirement, in the PAYG system only low skill parents receive transfers from their children.

J13 Fertility; H55 Social Security and Public Pensions; E62 Fiscal Policy; Public Expenditures, Investment, and Finance; Taxation

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1 Introduction

This paper studies the effects of a fully funded social security reform on welfare, efficiency and inequality in a dynastic, life-cycle general equilibrium model with endogenous fertility. We compare the steady states of the pay-as-you-go and the fully funded system when parents choose the number of children optimally. In our model of two-sided altruism, the motivation for fertility comes from a combination of 'parental altruism' and 'old age security' approaches. In the first one, parents' utility depends on their own consumption, the number of children and the utility of each child (see the seminal paper by Barro and Becker (1989)). In the 'old age security' models (also 'children as investment' in Boldrin and Jones (2002) or Nishimura and Zhang (1992)), aging parents expect to be cared for by their children.

These two approaches have very different qualitative and quantitative implications. In the Barro and Becker model, the motivation for having children is parental altruism. Getting utility from their own consumption as well as from the utility of their children, parents perceive their children's lives as a continuation of their own. In this type of models, the effect of government pensions on fertility is very small. An increase in social security benefits brings the same substitution effect as an increase in the cost of raising a child. When a more generous social security system is introduced, there is a transitional effect of lower fertility followed by a return to the same fertility level of the original steady state. This is in contradiction with empirical evidence. The 'old age security' models are based on Caldwell (1982) theory of the fertility transition in which transfers from children to parents are behind the high fertility choices. Institutions or government policies (such as social security) that eliminate the need for such transfers within a family bring about a reduction in fertility.

Boldrin et al. (2005) compare the effects of changes in social security system on fertility choice in the Barro and Becker (1989) and a Caldwell model based on Boldrin and Jones (2002). They start from an observation that in developed countries, fertility rates have constantly decreased between 1950 and 2000. Fertility in Europe has fallen from about 2.8 to about 1.5 and from about 3.0 to approximately 2.0 in the United States. That is, while in 1920 the average total fertility rates were almost equal in the United States and in Europe, today they are about 0.4-0.8 children different, depending on a country. From a cross-section data on 104 countries, Boldrin et al. (2005) find a strong negative relationship between the TFR and the size of a country's social security and pension system.

Boldrin and Jones (2002) find that in models based on parental altruism, social security generates at most only small (and usually positive) effects on fertility. On the other hand, the Caldwell-type models based on the old age security motive account for between 40% to 60% of the observed fertility differences between the United States and other developed countries or in the United States over time. In their computational experiment, an increase in the social security from 0 to 10% of GDP leads to an increase in the capital-output ratio from 2.2 to 2.4, a fall of consumption by 3%, and a reduction of TFR from 1.15 to 0.9.[1]

[1]Ehrlich and Kim (2005), in a model with altruistic parents, find that decreasing social security from 10% to 0% increases fertility by approximately 0.1 children per woman. To obtain simultaneous growth in per capita income and a fall in fertility, Fernandez-Villaverde (2001) studies a model with capital-specific

Another test of these two modeling approaches is their prediction on the capital-output ratio. In the data surveyed by Boldrin and Jones (2002), the U.S. capital-output ratio has either remained constant or increased in the 20th century. Among the European countries, the capital-output ratio and the social security sector are substantially higher than those in the Unites States. When social security increases, the Caldwell model predicts an increase in the capital-output ratio while the Barro and Becker model predicts its decrease.

These findings indicate that the 'old age security' motivation for fertility fits the empirical observations much better than the one of 'parental altruism'. In this paper, our specification of the dynastic utility function allows for both kinds of altruism and a test of their quantitative importance.

Another stream of the literature on social security reforms is represented by the pure life-cycle models with heterogenous agents and exogenous fertility: Conesa and Krueger (1999), De Nardi et al. (1999), or Imrohoroglu et al. (1999). These papers find that the elimination of social security brings large welfare, aggregate and distributional effects. They report important general equilibrium effects coming from a huge, around 30% increase of the capital stock in the fully funded (FF) steady state. In these models, agents are generally better off in the new steady state but the cost of a transition could be prohibitively high.[2]

Our paper builds on the last important contribution to the social security literature, the dynastic models based on Fuster (1999), Fuster, Imrohoroglu, and Imrohoroglu (2003) and Fuster, Imrohoroglu, and Imrohoroglu (2007). These papers study the welfare effects of the social security reform in a general equilibrium model with overlapping generations of altruistic agents but with exogenous fertility. In the dynastic framework with a two-sided altruism, in addition to the life-cycle and insurance motives, individuals also save for bequest motives. Therefore, old agents do not necessarily have a lower marginal propensity to save than young agents. Fuster, Imrohoroglu, and Imrohoroglu (2003) find that the current social security system with a 44% replacement rate crowds out only 6% of the capital stock. In other words, the PAYG system has much lower effect on the aggregate saving rate in a framework with altruistic dynasties. They also find that incorporating mortality risk and ability shocks is very important for quantitative results.

Our goal is to evaluate these results in a detailed, dynastic life-cycle model where fertility is endogenous. As in Fuster et al. (2003), we present a general equilibrium, overlapping generations model with two-sided altruism among individuals whose differences in skills (education) and life-time expectancy lead to heterogeneity in income, wealth, and therefore, fertility. Old age security and parental altruism, affected by the social security system, are the major forces behind fertility decisions. In the PAYG system, the social security benefits parents receive are independent of the number of children they have. The fully funded system internalizes the fertility decision: parents finance their retirement consumption only from savings or from the old age support provided by their own children. Thus children are perceived as an alternative investment good, costly in terms of time and goods. Finally, the fully funded reform eliminates the social security tax which is distortive and costly for borrowing constrained agents, possibly enabling them to have

technological change and capital-skill complementarity.

[2]In a related paper Conesa and Garriga (2008) propose to eliminate these transition costs by issuing bonds that will be repaid from the future efficiency gains.

more children.

Fertility choice in a dynastic model requires two theoretical contributions: first, we adapt the transformation in Alvarez (1999) to individual dynastic households composed of three overlapping generations. Second, contrary to models with exogenous fertility, dynasties that die off cannot be replaced by artificially created new families. Rather, it is the fertility choice of other households that more than replaces the deceased dynasties and leads to a constant population growth in the steady state.

We calibrate the benchmark model to the U.S. data. As in Fuster et al. (2003), we assume an exogenous labor supply, abstract from individual earnings uncertainty over the life cycle, and limit attention to steady states. We find that the effects of endogenous fertility are large and important in their direction. In the PAYG system, low skill (low education) agents invest relatively more in terms of children while the high skill (high education) agents invest relatively more in terms of assets. These savings-fertility differences lead to a 20% higher aggregate capital stock than in an otherwise identical PAYG steady state but with exogenous fertility. The fully funded social security reform increases fertility by 10.3% and decreases the capital stock by 8.3%. This is because high skill individuals shift from investment in capital to investment in the old-age support from children. Consequently, as in the data, the FF system decreases the capital-output ratio. In the absence of the social security system, the old-age support is an important motivation for the high fertility decisions: All parents receive transfers from their children that amount to around 40% of parental consumption during retirement. In the PAYG system, only the low skill parents receive transfers from children.

Assumptions on agents' heterogeneity (survival probabilities and skill differences) are quantitatively important: fertility and allocation responses by different types of households significantly affect aggregate levels and equilibrium prices. To isolate these effects, we simulate four alternative cases that differ in their assumptions on survival and income uncertainty. We find that children are used relatively more for insurance against survival uncertainty while assets are used for insurance against skill risk in future generations. It seems that the PAYG system provides the high skill households with the means to insure against the latter risk. Namely, their bequests are much higher than in the fully funded system, contributing to greater wealth inequality.

We also find that all newborn households and the majority of the population are better off in the fully funded steady state. Unfortunately, the complexity of the model does not allow us to simulate a transition between the two steady states. In other papers with exogenous fertility, agents usually prefer the new steady state but the transition to reach it is too costly. The main reason is that agents need to accumulate capital during initial stages of the transition (see Conesa and Krueger (1999)). However, in our endogenous fertility model, the capital stock decreases in the fully funded steady state. This deaccumulation of capital could provide for additional consumption to households who would suffer from the transition in models with exogenous fertility.

These results indicate that models with exogenous fertility contain two errors: First, they undervalue the capital stock in the PAYG steady state by forcing the high skill agents to invest in children as much as the low skill agents do. Second, these models predict the opposite direction of changes (with a huge magnitude) in the capital stock after the fully funded reform. These errors might lead to misleading conclusions about the behavior of different groups of the population, aggregate outcomes, welfare gains, transition dynamics and political support for the social security reform.

The paper is organized as follows. The next Section describes the modeling issues important for endogenous fertility. Section 3 develops the main model and equilibrium. Section 4 presents the calibration of the benchmark model. Numerical results are shown in Section 5. Section 6 provides discusses extensions and directions for future work. The Appendix contains the details on intervivo transfers.

2 Modeling Endogenous Fertility

This section develops a life-cycle model with endogenous fertility in the dynastic framework. For a simple exposition of modeling issues, we present a reduced one-period version of the dynastic model with individual households as in Fuster et al. (2003).

2.1 A Simple Dynastic Model with Exogenous Fertility

The decision unit is a household, composed of one father $f = 1$ and a fixed number of sons $\bar{s}$. In this model of two-sided altruism, there is no strategic behavior between the father and the children as they pool resources and maximize the same utility from a consumption per household member, $u(c/(f + \bar{s}))$. A dynasty is a sequence of households that belong to the same family line. We abstract here from life uncertainty.

The father is retired without any income. His sons of the next generation work for a wage w at a stochastic productivity shock z. Effectively, the sons provide a transfer to the father as old age security support. At the end of the period, the father dies and an exogenous number of children $\bar{n} = \bar{s}$ is born to each son. In the following period, the sons establish $\bar{s}$ new households in which each son becomes a single father with his own $\bar{n}$ sons. Savings a' is divided equally among the sons.

Households are heterogeneous regarding their asset holdings, a, and skills z. The Bellman equation for a household with a state $(a, f = 1, \bar{s}, z)$ is

$$v(a, \bar{s}, z) = \max_{c,a'} \left\{ u\left(\frac{c}{1+\bar{s}}\right) + \bar{s}^{\eta}\beta E[v(a', \bar{n}, z')|z] \right\},$$

subject to a budget constraint,

$$c + \gamma\bar{s}\bar{n} + \bar{s}a' \leq (1+r)a + \bar{s}wz,$$

where a is assets, r is the interest rate, and γ is the cost of raising children.[3] Altruism in the sense of Barro and Becker (1989) is represented by a parameter η. The skill of sons in each household is partially correlated with the skill of their father through a Markov process.

The important point is that the discounted present value on the right hand side of the Bellman equation is, like in the Barro-Becker formulation, multiplied by $\bar{s}^{\eta}$. This multiplication incorporates the present discounted value of all the new $\bar{s}$ households into the

[3] In their model of exogenous fertility, Fuster, Imrohoroglu, and Imrohoroglu (2003) do not have the cost of children ($\gamma = 0$). Their altruism parameter $\eta = 1$.

dynastic value function. In this way the value function covers all households that have belonged, belong, and will belong to the dynasty. Note that while these new households have the same amount of assets and composition, they might differ in their realized household idiosyncratic shock z'.

2.2 A Simple Dynastic Model with Endogenous Fertility

However, it is not possible to simply multiply the future value by s^η when fertility is a choice. The dynamic programming problem would not be well defined. Alvarez (1999) derives an equivalent formulation of the problem,

$$V(a,s,z) = \max_{c,a',n\geq 0} \left\{ u\left(\frac{c}{1+s}\right)(1+s)^\eta + \beta E[V(a',n,z')|z] \right\}, \tag{1}$$

subject to a budget constraint

$$c + \gamma sn + sa' \leq (1+r)a + swz. \tag{2}$$

This formulation does not work when the decision-making unit is an individual household rather than the economy-wide dynasty: As the future value is not multiplied by s^η, the dynasty does not take into account its division into the s new households. All but one of the newly established households headed by former sons, who inherit the same amount of assets, are not valued. For example, imagine two households: one with a single son and the other with five sons. Assume just here that they consume the same amount per household member, both have the same number of children per son, n, and the same savings *per son*, a'. Then the formulation in equation (1) would correctly value only the household with the single son. The other household with five sons would be substantially undervalued as only one out of the five sons would be be considered in the continuation of the dynastic functional equation.

In order to study the behavior and allocations of individual heterogeneous households, we need to incorporate the splitting households back into the model up to a limit: If all these households were fully incorporated, the dimensionality of the state space would grow geometrically. The goal is to find an abstraction where 1) the budget constraint remains related to a single household, and at the same time 2) the *number* of relatives which a household considers in its decisions stays manageable and realistic.

For the latter condition we choose to only keep track of the newly established (i.e. separated) households and only for one generation when the direct relatives are alive. These households are headed by the new fathers who are brothers. Their number is b, equal to the number of sons s in the previous period. Being identical, these brothers start their own families with the same bequest a' and the same number of sons s. On the other hand, their children might draw different skills.

Thus we model a household whose head comes from a family that had b sons. Their number only multiplies the utility from consumption of the household members applying the same parameter of altruism η. Importantly, none of these separated households enters each other's period utility from consumption nor the budget constraint.

INSERT FIGURE 1 ABOUT HERE

A timeline for a household is in Figure 1. The state of an individual household is (a, b, s, z), where b is the number of sons in the previous period. The value function is

$$V(a, b, s, z) = \max_{c, a', n \geq 0} \left\{ u\left(\frac{c}{1+s}\right)(1+s)^{\eta} b^{\eta} + \beta E[V(a', s, n, z') | z] \right\},$$

subject to the above budget constraint in equation (2).

In other words, the economic unit is a single household. The existence of living, direct relatives has a positive externality. This externality is lost when the head of a household (father) dies. Everything else kept constant, large families have higher utility than smaller ones.

3 The Economy

This section describes the full overlapping generations model with endogenous fertility based on Fuster et al. (2003). The economy is populated by $2T$ overlapping generations. Each household consists of a father, $f \in F = \{0, 1\}$, sons $s \in S = \{0, 1, 2, 3, ...\}$, and the children each son decides to have, $n \in S$. For dynastic reasons discussed above, each household also values the fact that it comes from a family with $b \in S$ sons in the previous period. We will abbreviate the household composition as $h = (f, b, s, n)$. Because the full model has uncertain lifetimes, zeros indicate persons who are not alive. We assume that children share the mortality of sons and that when the father dies, the connection to his brothers is lost and the household ceases to value other households of the dynasty. All decisions are jointly taken by the $m = f + s$ adult members in a household.

The model period is 5 years. A household lasts $2T$ periods or until all its members have died. A timeline for a household is shown in Figure 2. The model age of the household is related to the age of sons. At $j = 1$ the sons are 20 when the father is 55 (model age $j = T + 1$). The father retires at real age 65 (model age j_R) and lives at most to age 90 which corresponds to model age $j = 2T$.

INSERT FIGURE 2 ABOUT HERE

In a life-cycle model with endogenous fertility we have to allow the sons to choose the number of children in the appropriate period of the life cycle. If the sons survive to model age $j_N - 1$ (real age 30), they choose the number of children born in the following period j_N. Children live in the same household in periods $j = j_N, \ldots, j_T$. After period T, the sons form s new households in which each of them becomes the single father in period $T + 1$. Each son takes his n children to his new household. Therefore, conditional on survival, during the first $j_N - 1$ periods an individual's life overlaps with the life of the parent and during the remaining $j = j_N, \ldots, 2T$ periods also with the lives of his own children. The fertility decisions of all households imply an endogenous population growth rate $\bar{n}$ that an individual household takes as given.

Households are heterogeneous regarding their asset holdings, age, composition and skills. The skill is revealed to each son in period $j = 1$ when he is aged 20 and enters the labor market. The skill is correlated with that of his father: it can be high or low, $z \in Z = \{H, L\}$, following a first-order Markov process

$$Q(z, z') = Prob(z' = j \mid z = i) \qquad i, j \in \{H, L\},$$

where z' and z are the labor abilities of the sons and their father, respectively. In other words, within a household, all offsprings have the same skill which might be different from that of the parent.

The skill is fixed for the whole life and determines an individual's age-efficiency profile, $\{\varepsilon_j(z)\}_{j=1}^{2T}$, as well as life expectancy through $\psi_j(z)$, the conditional probability of surviving to age $j+1$ for an individual with an ability z who is alive in period $j = 1, \ldots, 2T$. We impose that at the terminal age $\psi_{2T}(z) = 0$. If all household members die, this branch of the dynasty disappears and their assets are distributed to all living adult persons in the economy by the government as lump sum accidental bequests.

3.1 Preferences

Individuals are altruistic, that is they care about their predecessors and descendants. As in Jones and Schoonbroodt (2007), household members in a dynasty care about their own consumption in the period, the number of children, and the utility of their children. In particular, 1) the utility of adult household members is increasing and concave in their own consumption; 2) parents are altruistic (holding fixed the number of children and increasing their future utility increases (strictly) the utility of the parent); 3) holding children's utility constant, increasing the number of children increases (strictly) the utility of the parent; and 4) the increase described in 3) is subject to diminishing returns. In addition, as discussed above, the household values the number of father's brothers in the separated households.

The household jointly maximizes utility from a per-adult consumption,

$$U(c, h) = u\left(\frac{c}{f+s}\right)(f+s)^{\eta} b^{\eta},$$

where η is a parameter of altruism as in Barro and Becker (1989). The last term, b^{η}, represents the number of sons in the previous period.[4]

If the father is not alive, the link to other households in the dynasty is broken and the utility of the household with a state $h = (0, 0, s, n)$ is

$$U(c, h) = u\left(\frac{c}{s}\right) s^{\eta}.$$

The function u is a standard CES utility function, $u(\tilde{c}) = \frac{\tilde{c}^{1-\sigma}}{1-\sigma}$, for a per-adult consumption $\tilde{c}$. The preference parameters must satisfy the monotonicity and concavity requirements for optimization. We follow the standard assumption in the fertility literature where children and their utility are complements in the utility of parents (see also Lucas (2002)). Therefore, $u(c) \geq 0$ for all $c \geq 0$, u is strictly increasing and strictly concave and $0 < \eta < 1$. This implies $0 \leq \eta + \sigma - 1 < 1$ and $0 < 1 - \sigma < 1$. Jones and Schoonbroodt (2007) analyze these properties in detail.[5]

[4] For simplicity, we apply the same parameter of altruism. Observe that the brothers might not be actually alive because they draw their survival shocks independently.

[5] The other combination of parameters has different implications for the quantitative properties of the model. Children and their utility are substitutes if $u(c) \leq 0$ for all $c \geq 0$, u is strictly increasing and

In terms of Boldrin and Jones (2002), this model exhibits a cooperative care for parents by all siblings. This model would be classified as an A-efficiency model by Golosov, Jones, and Tertilt (2007), where an additional child's value is based exclusively on the extra utility brought about to parents, grandparents, siblings or other relatives.

3.2 Production

We assume that the aggregate technology is represented by the standard Cobb-Douglas production function,

$$F(K_t, L_t) = K_t^{\alpha}(A_t L_t)^{1-\alpha},$$

where K_t and L_t represent aggregate capital stock and labor (in efficiency units) in period t. A_t is the technology parameter that grows at a constant exogenous rate $g > 0$. The capital stock depreciates at a rate $\delta \in (0,1)$. Competitive firms maximize profits renting capital and hiring effective units of labor from households at competitive prices r_t and w_t, respectively.[6]

3.3 Government

The government in the economy finances its consumption G by levying taxes on labor income τ_l, capital income τ_k, and consumption τ_c. Social security benefits B are financed by a tax on labor income τ_{ss}. Finally, the government administers the redistribution of accidental bequests. All these activities are specified in detail below.

3.4 Budget Constraint

Households are heterogeneous regarding their asset holdings, age, abilities, and composition. Denote (a, h, z, z') as the individual state of an age-j household, where a represents assets, $h = (f, b, s, n)$ is the household's composition, and (z, z') are the father's and sons' skills, respectively.

The budget constraint of a household with $m = f + s$ adult members is

$$(1+\tau_c)(c + \gamma_j^g(h; z, z')) + (1+g)a' = [1 + r(1-\tau_k)]a + e_j(h; z, z') + m\xi, \tag{3}$$

where c is the total household consumption, a' is savings of the whole household, ξ is the lump-sum transfer of accidental bequests, and τ_c and τ_k are the consumption and capital tax rates, respectively. In the calibration section we explain in detail the expenditures on children, $\gamma_j^g(h; z, z')$, a function of household income and the number of children n in period j.

The after tax earnings of the adult members is given by

$$e_j(h; z, z') = \begin{cases} fB_{j+T}(z) + s(1-\gamma_j^w(n))\varepsilon_j(z')(1-\tau_{ss}-\tau_l)w & \text{if } j \geq j_R - T, \\ [f\varepsilon_{j+T}(z) + s(1-\gamma_j^w(n))\varepsilon_j(z')](1-\tau_{ss}-\tau_l)w & \text{otherwise,} \end{cases} \tag{4}$$

strictly concave and $\eta < 0$. A comparison of these two parameterizations will be the subject of our future work.

[6] In what follows, we drop the time subscripts.

where $\gamma_j^w(n)$ represents a fraction of the sons' working time devoted to n children in period j, τ_{ss} and τ_l are the social security and labor income tax rates, respectively. $B_{j+T}(z)$ are social security benefits, which depend on the father's average life-time earnings and wage in the retirement period.[7]

In all optimization problems below we impose a no-borrowing constraint, $a' \geq 0$.

3.5 Value Function at Age $j = 1, 2, ..., j_N - 2$

Let $V_j(a, h, z, z')$ be a value function of an age-j household with a assets, h members, and (z, z') skills. In periods $j = 1, 2, ..., j_N - 2$ there are no children yet so $h = (f, b, s, 0)$. The maximization problem is

$$V_j(a, h, z, z') = \max_{c, a'} \left\{ U(c, h) + \beta(1+g)^{1-\sigma} \widetilde{V}_{j+1}(a', h', z, z') \right\},$$

subject to the budget constraint (3) and the after-tax earnings defined in (4).

The transition process for the value function is, due to life uncertainty,

$$\widetilde{V}_{j+1}(a', h', z, z') = \begin{cases} \psi_{j+T}(z)\psi_j(z')V_{j+1}(a', (1, b, s, n'), z, z') & \\ \quad + \psi_{j+T}(z)(1 - \psi_j(z'))V_{j+1}(a', (1, b, 0, 0), z, z') & \\ \quad + (1 - \psi_{j+T}(z))\psi_j(z')V_{j+1}(a', (0, 0, s, n'), z, z') & \\ & \text{if } f = 1, s > 0, \\ \psi_{j+T}(z)V_{j+1}(a', (1, b, 0, 0), z, z') & \text{if } f = 1, s = 0, \\ \psi_j(z')V_{j+1}(a', (0, 0, s, n'), z, z') & \text{if } f = 0, s > 0. \end{cases} \tag{5}$$

While this transition is specified for all possible ages $j < T$, with no children in periods $j = 1, 2, ..., j_N - 2$, we impose $n' = n = 0$. Note again that sons share their survival uncertainty with their children and that the household stops remembering other relatives b when the father dies. Finally, if all members of the household die this branch of the dynasty disappears.

3.6 Value Function at Age $j_N - 1$

At the age $j_N - 1$, each son chooses the number of children that will be born in the following period. Being identical, all sons choose the same number of children, $n' \geq 0$,

$$V_{j_{N-1}}(a, h, z, z') = \max_{c, a', n' \geq 0} \left\{ U(c, h) + \beta(1+g)^{1-\sigma} \widetilde{V}_{j_N}(a', h', z, z') \right\}.$$

While the budget constraint (3) and after-tax earnings (4) are unchanged, the transition for the value function (5) now has $n' \geq 0$.

[7] For a detailed definition see Fuster et al. (2003) and the calibration section. Variables are transformed to eliminate the effects of labor augmenting productivity growth.

3.7 Value Function at Age $j = j_N, \ldots, T-1$

Children are born in period j_N and become a state variable in $h = (f, b, s, n)$ in periods $j_N, \ldots, T-1$. The value function is,

$$V_j(a, h, z, z') = \max_{c,a'} \left\{ U(c, h) + \beta(1+g)^{1-\sigma} \widetilde{V}_{j+1}(a', h', z, z') \right\},$$

subject to (3), (4), and (5). The number of children per son in the next period is $n' = n$ or $n' = 0$ if the sons die. Having children is costly in terms of goods, $\gamma_j^g(h; z, z')$, and working time, $\gamma_j^w(n)$.

3.8 Value Function at Age $j = T$

At the end of period T, a household transforms itself into s new households of the next dynastic generation in period $j = 1$. The father reaches the end of his life, each of the s sons becomes a single father and the children become n sons in each of the s newly established households (conditional on survival). Therefore, at $j = T$, the value function of a household $h = (f, b, s, n)$ with assets a and skills (z, z') is

$$V_T(a, h, z, z') = \max_{c,a'} \left\{ U(c, h) + \beta(1+g)^{1-\sigma} \sum_{z''} \psi_T(z') V_1(a', h', z', z'') Q(z', z'') \right\},$$

subject to a special period-T budget constraint,

$$(1 + \tau_c)(c + \gamma_T^g(h; z, z')) + (1+g)sa' = [1 + r(1 - \tau_k)]a + e_T(h; z, z') + m\xi,$$

the after-tax earnings (4), and a transformation to s new households with a composition

$$h' = (1, s, n, 0),$$

provided that the sons survive to form their own households. Otherwise, this branch of the dynasty dies off.

Note that the sons equally divide the household's assets. The skill of the sons in each of the new households is z'', correlated with the ability of the father, z'. Finally, there are no children in period $j = 1$ so $n' = 0$.

3.9 Stationary Recursive Competitive Equilibrium

Let $x = (a, f, b, s, n, z, z') \in X = (A \times F \times S \times S \times S \times Z \times Z)$ be an individual household's state. Denote $\{\lambda_j\}_{j=1}^T$ as age-dependent measures of households over x. Its law of motion for each $(a', f', b', s', n', z, z') \in X$ in periods $j = 1, ..., T-1$, is

$$\lambda_{j+1}(a', f', b', s', n', z, z') = \sum_{\{x:\, a' = a_j(x), n' = n_j(x)\}} \Psi_j(f', s'; f, s)\, \lambda_j(x),$$

where $\Psi_j(f', s'; f, s)$ is the probability that an age j household of type (f, s) becomes a type (f', s') in the next period. The number of sons from the last period is remembered

$b' = b$ if the father survives and zero otherwise.[8] For a simpler exposition, $n_j(x) = 0$ for $j < j_N - 1$ and $n_j(x) = n$ for $j \geq j_N$.

The law of motion for the measure of age $j = 1$ households

$$\lambda_1(a', f', b', s', 0, z', z'') = \sum_{\{x: a'_j = a'_T(x)\}} s\, \Psi_T(f', s'; f, s)\, Q(z', z'')\, \lambda_T(x)$$

is now adjusted for the newly formed sons' households. Now $b' = s$ if the sons survive and zero otherwise.[9]

Importantly, dynasties whose members die disappear from the economy. They are not artificially replaced by new households with zero assets and some arbitrary composition. In an equilibrium with endogenous fertility, new households established by the sons are so many that they not only replace the deceased dynasties but also deliver the desired population growth. Therefore, in the following definition there is no condition on new dynasties.

Definition 1 *Given fiscal policies $(G, B, \tau_l, \tau_k, \tau_c, \tau_{ss})$, a stationary recursive competitive equilibrium is a set of value functions $\{V_j(\cdot)\}_{j=1}^T$, policy functions $\{c_j(\cdot), a'_j(\cdot)\}_{j=1}^T$ and $n'_{j_N-1}(\cdot)$, factor prices (w, r), aggregate levels (K, L, C), lump-sum distribution of accidental bequests ξ, cost of children (γ_j^g, γ_j^w), measures $\{\lambda_j\}_{j=1}^T$, and a population growth rate $\overline{n}$, such that:*

1. *given fiscal policies, prices and lump-sum transfers, the policy functions solve each household's optimization problem;*

2. *the prices (w, r) satisfy*

$$r = F_K(K, L) - \delta \text{ and } w = F_L(K, L);$$

[8]The transition probability matrix $\Psi_j(f', s'; f, s)$ for periods $j = 1, ..., T-1$ is

	$f' = 0, s > 0$	$f' = 1, s' = 0$	$f' = 1, s' > 0$	$f' = 0, s' = 0$
$f = 0, s > 0$	$\psi_j(z')$	0	0	$1 - \psi_j(z')$
$f = 1, s = 0$	0	$\psi_{j+T}(z)$	0	$1 - \psi_{j+T}(z)$
$f = 1, s > 0$	$(1 - \psi_{j+T}(z))\psi_j(z')$	$\psi_{j+T}(z)(1 - \psi_j(z'))$	$\psi_{j+T}(z)\psi_j(z')$	$(1 - \psi_{j+T}(z))(1 - \psi_j(z'))$
$f = 0, s = 0$	0	0	0	1

.

[9]For $j = T$ the probability matrix, $\Psi_T(f', s'; f, s)$, is

	$f' = 0, s' = 0$	$f' = 1, s' = n$
$s = 0, n = 0$	1	0
$s > 0, n \geq 0$	$1 - \psi_T(z')$	$\psi_T(z')$

.

3. *markets clear:*

$$K = \sum_{j,x} a_j(x)\,\lambda_j(x)(1+\overline{n})^{1-j},$$
$$L = \sum_{j,x} [f\,\varepsilon_{j+T}(z) + s(1-\gamma_j^w(n))\,\varepsilon_j(z')]\,\lambda_j(x)(1+\overline{n})^{1-j},$$
$$C = \sum_{j,x} c_j(x)\,\lambda_j(x)(1+\overline{n})^{1-j};$$

4. *the measures* $\{\lambda_j\}_{j=1}^T$ *grow at the constant population growth rate,* $\overline{n}$*;*

5. *the lump-sum distribution of accidental bequests satisfies*

$$\xi = (1+r)\sum_{j,x} a'_j(x)\Psi_j(0,0;f,s)\,\lambda_j(x)(1+\overline{n})^{1-j};$$

6. *the government's budget is balanced*

$$G = \tau_k\,r\Big(K - \frac{\xi}{1+r}\Big) + \tau_l\,wL + \tau_c(C+C^g),$$

where C^g *is the aggregate cost of children in terms of goods;*

7. *the social security tax is such that the budget of the social security system is balanced*

$$\sum_{j=j_R}^{2T}\sum_x f\,B_j(z)\,\lambda_j(x)(1+\overline{n})^{1-j} = \tau_{ss}wL;$$

8. *and the aggregate feasibility constraint holds,*

$$C + C^g + (1+\overline{n})(1+g)K + G = F(K,L) + (1-\delta)K.$$

4 Calibration of the Benchmark Economy

In order to obtain results comparable to those in the previous papers on social security reforms, we use the same parameters as Boldrin and Jones (2002) and Fuster et al. (2003). In particular, we set the intertemporal elasticity of substitution to $\sigma = 0.95$ (as in Boldrin and Jones (2002)) and the annual discount factor $\beta = 0.988$ (as in Fuster et al. (2003)). We find that a parameter of altruism $\eta = 0.055$ leads to the same population growth in the steady state of the benchmark economy with the PAYG social security replacement rate $\theta = 0.44$ as in the U.S. data ($\bar{n} = 0.012$). These parameter values also satisfy the optimization restrictions (see Jones and Schoonbroodt (2007) for details). The resulting capital-output ratio is 2.83, similar to that in Boldrin et al. (2005). All parameters are presented in Table 1.

INSERT TABLE 1 ABOUT HERE

The production function is Cobb-Douglas with a capital share $\alpha = 0.34$ and an annual depreciation rate $\delta = 0.044$ as in Fuster et al. (2003).

The demographic structure is the same as in Fuster et al. (2003), who follow Elo and Preston (1996) and set the survival probabilities ψ such that the life expectancy at real age 20 is 5 years longer for high skill individuals (college graduates) than that of low skill.

4.1 Earnings, Social Security and Taxation

The efficiency profiles for low and high skills as well as their transition probabilities are the same as in Fuster et al. (2003). In their model with exogenous fertility, the proportion of high-skill agents equals the share of college graduates (28%) and the correlation between the father's and sons' wages is the same as in the data (0.4).

Retirement benefits in the benchmark economy with a replacement rate 44% of the average earnings are calibrated according to Fuster et al. (2003). The marginal replacement rate equals 20% for earnings below the average, 33% for earnings above the average and below 125%, and 15% for earnings above 125% and below 246% of the average earnings. The benefits are further adjusted for low and high skill individuals. The social security tax $\tau_{ss} = 0.115\%$ clears the social security budget at 7.6% of GDP. In the steady state of the fully funded economy the replacement rate is set to zero.

The fiscal parameters are standard, taking values of 35% for the capital income tax and 5.5% for the consumption tax. The labor income tax clears the government budget constraint. Government consumption is set at 22.5% of the total output. As the latter does not change much across all steady states we model, tax on labor income τ_l around 0.16 clears the government budget constraint in all these steady states.

Fuster et al. (2003) find important differences depending on whether the FF reform is neutral with respect to government consuming the same *percentage* of GDP or the same *amount* of real goods. In our model with endogenous fertility, it turns out that when the government in the FF reform consumes the same percentage of GDP it also consumes almost the same amount of goods as in the PAYG steady state (the outputs in both steady states are very close). Thus in our paper the comparison of these two scenarios of government consumption neutrality is redundant.

4.2 Cost of Children

The *Report on the American Workforce* by the U.S. Department of Labor (1999) shows the average combined annual and weekly hours at work for married couples by presence and age of the youngest child. In 1997, the combined annual (weekly) hours were 3,686.6 (74.8) for couples with no children under age 18, 3,442.7 (70.4) with children aged 6 to 17, 3,545.0 (72.2) with children aged 3 to 5, and 3,316.5 (68.3) with children under 3 years. The labor force participation increases with time elapsed since the last birth, age of mother, education, and annual family income. It decreases only with the number of children. Table 2 presents these time costs, γ^w, adapted to this model's period structure as a fraction of a son's working time. We take the weekly measure as it is close to the estimates in Boldrin and Jones (2002).

INSERT TABLE 2 ABOUT HERE

Expenditures on children are taken from the 1990-92 Consumer Expenditure Survey of the Bureau of Labor Statistics. Estimates of the major budgetary components are for 12,850 husband-wife families with 1998 before-tax incomes between \$36,000 and \$60,600 (average \$47,900), controlling for income level, family size, and age of the younger child. Compared with expenditures on each child in a family with two children, households with one child spend on average 24 percent more on the single child, and those with three or more children spend on average 23 percent less on each child. Therefore, the USDA adjusts the expenditures by multiples of 1.24 and 0.77, respectively.[10] The cost in the model γ^g is shown in the last column as a fraction of the average household before-tax income.

5 Results

The pay-as-you-go social security system provides two important insurance roles. First, it partially substitutes for missing annuity markets during retirement. Second, it partially insures individuals against their permanent labor productivity shock. However, the social security tax is distortive on the consumption-savings margin and costly for borrowing constrained agents. Finally, as any social security system it affects fertility decisions and influences the timing, direction, and amount of intergenerational transfers. Because the social security benefits are independent of the number of children inside the household, the PAYG system does not internalize the fertility decision.

To understand the importance of these effects, we compare the benchmark steady state of the PAYG social security system with a replacement rate $\theta = 0.44$, calibrated to the U.S. data, to that of the fully funded system (FF) with a zero replacement rate.

We follow Fuster et al. (2003) and present four additional cases of the PAYG and FF steady states in order to isolate individual forces in the model. In the first case, we impose certain lifetimes ($\psi = 1$) for all household types. In the second case, we keep uncertain lifetimes but impose the same survival probabilities for both high and low skill individuals, i.e., $\psi_H = \psi_L$. The third case has no skill differences ($\varepsilon_H = \varepsilon_L$) and, therefore, the same but uncertain lifetimes ($\psi_H = \psi_L$). Finally, in the fourth case, we compute the two steady states with exogenous fertility.

In all these cases we hold all parameters constant. When comparing our results to those in Fuster et al. (2003), it must be kept in mind that endogenous fertility requires different dynastic utility function, preference parameters (σ, η) as well as the calibrated cost of children in terms of working time and goods.

[10]Expenditures include housing and education. These numbers are comparable to the findings of Deaton and Muellbauer (1986) and Rothbarth (1943). They find that in an average husband-wife family 25-33% of household expenditures are attributable to one child, 35-50% to two children, and 39-60% to three children. In Boldrin and Jones (2002), children cost 3% of family time (6% of mother's) and 4.5% of goods in per capita GDP. The latter was used to match their benchmark TFR and is rather low.

5.1 The Benchmark PAYG Steady State and its Fully Funded Reform

Table 3 shows the results for the benchmark calibration of the U.S. economy. The aggregate allocations and prices are on the top of the table. In the middle, there are fertility, savings, welfare and demographic outcomes for different household types. At the bottom, we show the political support for the FF reform.

INSERT TABLE 3 ABOUT HERE

In the benchmark PAYG steady state, the average fertility is 1.67 children per son, matching the U.S. population growth rate 1.24%. Among the complete households, the LL type of households have the highest fertility (1.83 children per son) and the HH type the lowest (1.11). When the sons live alone, the L types have on average 1.85 children. The FF reform eliminates the social security tax ($\tau_{ss} = .115$) and the social security benefits. It internalizes the old-age support inside the households. Consequently, the overall fertility increases by 10.2 percent.[11] All households increase fertility except for the lonely L sons. The reform substantially increases the fertility of households with high skill fathers (21.2% for HL and 57.3% for HH households, respectively). These households are shifting from saving in assets to saving in children.

In the benchmark PAYG steady state, the relation of fertility by household type is $LL > LH > HL > HH$. The higher fertility in households headed by low skill fathers coincides with the old-age support the fathers receive from their children in the last retirement periods. On the other hand, the high skill fathers support their children in all periods (these transfers will be discussed in detail below). The low fertility in these households is also due to a lower replacement ratio and a high opportunity cost of children for the high skill sons. Thus fertility decreases in skills (education) of the father and then of the sons. In the 2004 Population Survey, low skill individuals have higher fertility than high skill individuals. In their survey of U.S. demographic history, Jones and Tertilt (2006) find that fertility is declining in education and income. The Children Ever Born measure (CEB, per woman per cohort among 40-44 year old women) with a high school diploma is 1.943, and for women with a college degree 1.672. That is, high skill individuals have 13.9% lower CEB than those with low skill (when measured by the educational attainment of the husband, it is 12%). In our model, the difference for L and H lonely sons in the benchmark PAYG steady state is 13.0%. Among the complete households, fertility in LH households is 9.9% lower than in LL households, and 28.8% lower in HH than in HL households.

After the FF reform, the relation of fertility becomes $HL > LL > LH > HH$. Although the highest gain in fertility is in the HH households, the change in this ordering is brought by the increased fertility of the HL households. Fertility differences by skill decline to 2.3% (H vs. L), 5.3% (LH vs. LL), and 8.5% (HH vs. HL). Thus the FF reform reduces fertility differences across different household types.[12]

[11] In the Caldwell model in Boldrin and Jones (2002), the increase in fertility is 20-25%.

[12] This result is not consistent with available data for cohorts born before the 1920s: While this inequality seems to be stable since then, Jones and Tertilt (2006) document that over the last 150 years of the U.S. demographic history, there has also been a decrease in fertility inequality, especially with respect to

Importantly, in the FF steady state the capital stock falls by 8.3% and the capital-output ratio falls from 2.839 to 2.582. The other Caldwell-type models with endogenous fertility (Boldrin and Jones (2002) and Boldrin et al. (2005)) show a similar fall in the capital-output ratio when social security is eliminated. Again, these results fit the observed capital-output pattern in the data.

The reason for the reduction of the capital stock is that in our model children and assets are substitutes. This is apparent from the next part of Table 3 which shows the change in assets held by different types of households. Households with a high skill father decrease their asset holdings by around 20 percent. These are the households whose fertility increases the most. On the other hand, households with a low skill father slightly increase their savings. The only exception is again the lonely sons who lower their fertility as well as assets. Another and related phenomenon of the FF reform is the change in the old-age support: In all household types, transfers during retirement go from sons to fathers (see more below).

Finally, the substantial jump of fertility in HH and HL households leads to an increase in the efficiency labor supply by 5.8%. These changes in labor and capital inputs offset each other for only a small 0.8% increase in output. Thus the general equilibrium effects seem to be smaller than those found in related models: the increase in after-tax wage is lower than the eliminated social security tax rate. The average consumption decreases as higher fertility implies higher cost of children.[13]

The changes in the capital stock are opposite to models where fertility is exogenous. Similar dynastic models by Fuster et al. (2003) and Fuster et al. (2007) report an increase of the capital stock by 6.1% and 12.1%, respectively. Pure life-cycle models of Conesa and Krueger (1999), De Nardi et al. (1999), or Auerbach and Kotlikoff (1987), all suggest an increase of around 30%. Imrohoroglu et al. (1999) report that capital stock increases by 26% and Storesletten et al. (1999) by 10% to 25%. These large changes in the capital stock are driven by the forced savings imposed on high skill households in assets rather than children as well as by the underestimated capital stock in the original PAYG steady state. Below, we will show in our fourth additional case that a PAYG steady state with exogenous fertility has around 20% lower capital stock than the benchmark PAYG steady state with endogenous fertility.

5.1.1 Life-Cycle Savings

In order to document the forces important for these savings-fertility decisions, Figure 4 shows the life-cycle accumulation of assets by complete households for the PAYG and FF steady states. The accumulation of wealth culminates in period $j = 4$ when children are born. In the consequent periods, the cost of children and father's retirement drive down the average wealth for all household types. Notice that households with a low skill father save less and leave lower bequests than those with a high skill father.

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income. The difference from the top to the bottom of the income distribution of fertility has been falling, from around 1.6 CEB for the 1863 cohort to a quarter of a child by the 1923 birth cohort, where it has stabilized.

[13]In the PAYG steady state, the cost of raising children is 29.6% of GDP with the consumption-to-output ratio 0.634. In the FF steady state, the cost is 33.0% of GDP and the ratio is 0.645.

INSERT FIGURE 4 ABOUT HERE

In the PAYG regime and households where the father has high skills, assets are mostly transferred to sons' new households. Especially in the HH households assets are almost fully bequest: social security benefits allow these households not to dissave at the end of the life cycle. On the contrary in the FF reform, the HH and HL households use their assets for consumption in the retirement periods. While savings in households with a low skill father remains basically the same after the FF reform, savings in households with a high skill father dramatically decline after period $j = 4$. As these types also substantially increase fertility, their bequest per son is even lower.

This means that the lower capital stock in the FF steady state comes not so much from the ability to accumulate capital during the pre-retirement periods but rather from a different usage of the capital stock during retirement. The PAYG benefits allow households with a high skill father to transfer wealth across generations. In the FF system, savings are used for retirement consumption and much less for bequests. Therefore, even sons of a high skill father now start their own households with a low stock of assets. Although the after-tax income is higher, this could be costly for those who draw a low ability shock.

Consequently in the FF steady state, assets are not so persistently accumulated across generations and wealth inequality decreases. The Gini coefficient of wealth inequality is 0.48 in the PAYG benchmark steady state while in the FF benchmark steady state it is 0.45. In earlier papers, De Nardi et al. (1999) and Fuster (1999) find similar changes.

These results suggest that in the PAYG system, if a household budget constraint permits, assets are used as a partial insurance against a low realization of skill in future generations.

5.1.2 Intervivo Transfers

Related to savings decisions are intervivo transfers that allow households to smooth consumption also within households.[14] The definition of how these transfers are computed follows Fuster et al. (2003) and is described in the Appendix. A person in a household needs a transfer from other members of the household if he cannot cover his consumption expenditures from his own income and savings. As in Fuster et al. (2003), it is assumed that the father holds all the assets of a household in period $j = 1$.

INSERT FIGURE 5 ABOUT HERE

In Figure 5, a positive number is the net transfer from the father to one son while a negative number is the net transfer from one son to the father. The full line represents the average amount of transfers in the PAYG social security system and the dotted line in the FF system.

Four main results stand out: First, in the PAYG system the high skill fathers support their sons while the opposite is true for the low skill fathers. In the LL households, transfers from sons represent around 50% and in the LH households around 75% of an adult's consumption. In the HL and HH households, contributions by the father to his sons are much smaller (23% and 11%, respectively).

[14] For an interesting survey of intergenerational transfers in Europe, see SHARE (2005).

Second, in the FF system the sons always support their father during his retirement. In all types of households the contribution is always larger than 40% of father's consumption (most in the LH households, 49%). Thus in the FF system all fathers rely on the old-age support from their sons. The PAYG breaks this system for high skill fathers. In the FF system, the old-age support for high skill fathers coincides with the largest response of savings and fertility: These households reduce savings by more than 20% while increasing fertility by 21% and 57% (in the HL and HH types, respectively).

Third, the transfers are small in the first three or four periods: both fathers and sons are almost self-sufficient in their total incomes. The fathers receive most of their incomes from savings, the sons from earnings. The father supports his sons at least in the initial periods because as the head of a household he holds all the assets in period $j = 1$ and is, at the same time, at the peak of his life-cycle earnings. The PAYG system contributes to these positive transfers as the father compensates the sons for social security taxes the latter pay. Importantly, the persistence of asset accumulation across high-skill generations in the PAYG leads to greater wealth inequality.

Fourth, the public transfer system of the PAYG system is replaced by the FF private transfers: the amount of intervivo transfers increases by 109%. The transfers are particularly large during father's retirement and child rearing.

In Fuster et al. (2003), transfers in the FF system go in the same direction. In their PAYG system, fathers support their sons in the first three periods while sons support the fathers in the last four periods (only in the HL case the transfers are always positive). This is likely because the sons have higher incomes as children are free (there is no cost of raising children in terms of time or goods) and all household types have the same number of sons.

Data from The Survey of Consumer Finances show that about 75% of transfers go from parents to children (see Gale and Scholz (1994)). In our PAYG steady state, only 56% of intervivo transfers is from fathers to sons (if we add bequests, the amount of transfers increases to 74%). However, the FF reform reduces the parental transfers to 46% of the total: on average, children now support parents.

5.1.3 Demographics

Table 3 also shows the distribution of households across skill types, together with the fraction of high skill households and the dependency ratio. Complete households with both father and sons alive constitute around 77% of all households, those with only sons alive 21% and those with only the father alive 2%. Given the transition function for the intergenerational transmission of skills, the most numerous households are of the LL type with around 45%.

As the FF reform mostly increases the fertility of high skilled agents, their fraction increases by 2.3%. Simultaneously, the aggregate labor productivity grows by 5.8% also due to the increased population growth. The increase in fertility also leads to a lower dependency ratio of the retired to working population from 18.7% to 17.5% (despite the increased longevity in the population).

Importantly, the FF reform improves the dependency ratio as well as the productivity of the economy through a better demographic and human capital composition.

5.1.4 Welfare Gains and Political Support

The middle part of Table 3 shows the percentage of consumption in the PAYG steady state that a particular household type would have to receive in order to be as well off as in the FF steady state. If the number is positive, the FF steady state is preferred to that with the PAYG system. We compare all newly established (newborn) households of sons at age $j = 1$ who survived their separation from their father's household as well as the average welfare for all households.

The FF reform brings large welfare gains to all types of newborn households. However, when we compute the average welfare across all cohorts, households with a high skill father (HL and HH) are worse off. These households sacrifice high pensions while facing the increased cost of raising more children they have chosen for providing the old-age support. On the other hand, households with a low skill father (LL and LH) are always better off: they do not increase fertility much and their numerous sons bring home more income after the elimination of the social security tax. LH households are those who benefit the most: low skilled fathers lose small pensions while high skilled sons bring home higher incomes. Naturally, the lonely sons always prefer the FF system and the lonely fathers the PAYG benefits.

The bottom of Table 3 shows the political support for the reform, i.e., the percentage of households in each cohort and overall that are better off in the FF steady state. All newly established households ($j = 1$) as well as the majority of households are better off. On the other hand, households aged $j = 3, 4$, and 5 are worse off as they raise more (costly) children and support the retired father at the same time.[15] In later periods, as both sons' earnings and the likelihood of the father's death increase, the majority of households again prefers the reforms.

5.1.5 Return on PAYG Social Security

Another way to understand these welfare results is to examine the rate of return on social security in the PAYG system with a 44% replacement rate for households where both the father and the sons are alive.[16]

INSERT TABLE 4 ABOUT HERE

In Table 4, the rate of return on social security increases with the age of the father, who is collecting social security benefits for a longer time. Also, this rate is higher the lower the contributions made and the higher the benefits received. Hence the returns on PAYG are highest for the HL households and negative for the LH households no matter how long the father can collect the benefits (in Fuster et al. (2003) the return is positive for death at age 85). Of course, these LH households benefit the most from the reform. Households where only sons are alive have a negative return on social security. Also, the return is higher for HH and HL households because they have fewer sons and hence pay lower taxes.

When the life expectancy of the father is taken into account, only an HL household has a social security return higher than an after-tax return on capital. This confirms our

[15] These life-cycle effects are not present in Fuster et al. (2003) who do not consider the cost of children.

[16] See Fuster et al. (2003) for a definition and computation of the return.

findings above that HL households prefer the PAYG system while LH households with a negative return prefer the PAYG system the least.

In our paper the relation of returns is $r^{LH} < r^{LL} < r^{HH} < r^{HL}$, the same as for household wealth. In Fuster et al. (2003), the positions of HH and LL types are switched. This is because the number of children each type has is very different. For high skill fathers PAYG is important: They have few sons and the PAYG pensions represent a larger part of their old-age income. As discussed above, in the PAYG system the high high skill fathers support their children while the low skill fathers receive transfers from their sons.

5.2 Alternative Model Cases

As in Fuster et al. (2003) we present four additional cases of the PAYG and FF steady states that differ in lifetime uncertainty and/or skill differentials. In all these cases we keep the parameters from the benchmark calibration (namely η, the parameter of altruism).

5.2.1 Case 1: Certain Lifetimes ($\psi = 1$)

The first of our alternative calibrations is presented in Table 5. At the cost of increased longevity (all agents live till age 90), certain lifetimes eliminate the risk of losing the old-age support because of sons' death. Note that there are only complete households.

INSERT TABLE 5 ABOUT HERE

Compared to the benchmark steady states, both assets and fertility fall. Compared to the benchmark PAYG steady states, fertility declines by 14.9% and assets by 8.6%. The removal of survival uncertainty reduces the need for the buffer stock of children (their future incomes) and savings.[17] Prolonging expected lifetimes increases the population growth rate, but fertility falls. A smaller productive population contributes to lower output.

The FF reform increases fertility by 14.1% while the capital stock falls by 6.9%. Changes in fertility depend mostly on the father's skill. Fertility differences between parents of different skills are higher than in the benchmark specification but very similar: in the PAYG steady state, they are 11.2% (LH vs. LL) and 19.7% (HH vs. HL). After the FF reform, these numbers are 11.1% (LH vs. LL) and 19.9% (HH vs. HL). LH is the only type of household that increases its savings.

Welfare gains have the same signs as in the benchmark calibration (observe how HL households are worse off and newborns are better off). Naturally, the higher longevity increases the dependency ratio to more than 25%. The longer lives of low skill individuals substantially increase the fraction of LL types (61%). Finally, the social security tax rises to 16.9% in order to finance the retirement benefits.

As there is a larger fraction of retired agents who lose their social security benefits, the FF reform has the lowest political support (57%, still a majority). The support among households with a recently retired father is very small.

[17] Kalemli-Ozcan (2002) stresses the role of survival uncertainty for the insurance strategy, or hoarding of children, where parents bear more children than their optimal number of survivors.

5.2.2 Case 2: Equal Survival Probability ($\psi_H = \psi_L$)

Table 6 shows that when lifetimes are uncertain but the same for both skills, fertility decreases for all household types in both PAYG and FF steady states (relative to their benchmarks).[18]

INSERT TABLE 6 ABOUT HERE

The main reason is that the survival probabilities increase on average (i.e., for the most numerous low skill individuals) while skill uncertainty remains. The children of low skill parents are now more likely to survive and support the parents in their old age. This drives the fertility of low skill agents down in the PAYG system. Overall, fertility declines by 5%. On the other hand, high skill households now face a higher mortality risk and they do not dissave as much as in the certain lifetime case. The capital stock is similar to that in the benchmark steady states.

The FF reform increases fertility by 12% and reduces the capital stock by 3.1%. Again, fertility differences between different skills are higher than in the benchmark specification and similar across steady states. Especially the low skill agents increase their fertility as their survival probability increases.[19] Welfare gains and political support are not much different from the benchmark and certain lifetimes cases.

Compared to the benchmark specification, this and the certain lifetimes cases exhibit higher average survival probabilities. The reduced fertility suggests that children are used as insurance against survival uncertainty.

5.2.3 Case 3: Limited Heterogeneity ($\psi_H = \psi_L$, $\varepsilon_H = \varepsilon_L$)

In the limited heterogeneity case in Table 7, all households are the same in their survival probability and skills. The only risk they face is the equal survival uncertainty.

INSERT TABLE 7 ABOUT HERE

Fertility is the same for all agents, close to that in the benchmark PAYG and FF steady states. As there are no high skill agents and all households have the same fertility, the capital stock decreases by 22%: there is no need for the buffer stock except for life uncertainty. Correspondingly, the output declines as well.

The FF reform increases fertility by 7.8% while not changing much the capital stock. Welfare gains from the FF reform are big (for the average household) and the limited heterogeneity case obtains almost 100% support from all generations.

The large decline in savings in this case suggests that assets are primarily used to insure against a low future realization of skills among children.

[18] The survival probability is the weighted average of ψ_H and ψ_L.

[19] The increasein the PAYG steady state is 13.8% (H vs. L), 11.0% (LH vs. LL), and 25.7% (HH vs. HL), and in the FF steady state 9.5% (H vs. L), 10.7% (LH vs. LL), and 19.4% (HH vs. HL).

5.2.4 Case 4: Exogenous Fertility

Finally, Table 8 shows the case of exogenous fertility. Here, the fertility of all agents in the benchmark PAYG steady state is set to match the U.S. population growth rate.[20] We keep the same fertility rates in the FF steady state. Again, our results do not directly compare to those in Fuster et al. (2003) due to differences in the dynastic utility function, preference parameters and cost of children.

INSERT TABLE 8 ABOUT HERE

Exogenous and equal fertility across different household types implies that agents who would otherwise choose a low fertility are now forced to save in children rather than assets, and vice versa. As the high skill agents are those who have to lower their savings the most, the aggregate capital stock falls relative to the benchmark PAYG steady state by 20.6% (correspondingly, output falls, too).

Importantly, the FF reform with exogenous fertility has the opposite effect on aggregate levels. Only in this case the stock of capital increases by 6.7%, together with consumption, output, and the capital-output ratio.[21] The interest rate falls while the after-tax wage increases by 18.1%. Note that in this case it is households with a low skill father that increase savings after the FF reform while assets of households with a high skill father remain almost constant. As the high skill agents are forced to have more children, the fraction of high skill agents increases to 29.7%.

Further, only in this specification all types of households and cohorts are better off in the FF steady state, and by big percentages.[22] The highest gain is to households with low-skilled fathers (13% for LL, 3.8% for L).[23] Finally, exogenous fertility has the highest political support of all the cases we have studied.

Overall, the assumption of exogenous fertility dramatically affects the predictions of the model, especially those related to the capital stock.

6 Conclusions

Social security reform is one of the most important economic and political issues in the United States and other developed countries. This paper analyzes a social security reform

[20]In Fuster et al. (2003), the average number of children is 1.52. Our number is higher because we do not replace dynasties that die off.

[21]This is similar to Fuster et al. (2003), where the FF reform increases capital stock by 6.1%, output by 1.8%, consumption by 1.1%, and the capital-output ratio falls from 2.48 to 2.59.

[22]With the exception of lonely fathers.

[23]In Fuster et al. (2003), only the LH households prefer to be born into the FF steady state (gain +1.39%). All other household types suffer a welfare loss (most the HL types, -1.71%). However, large gains to L and H lonely sons lead to a positive average welfare gain of +0.43% of the steady state consumption. On the other hand, in Fuster et al. (2007) all newborn households prefer the FF reform. However, it is not clear that these gains come from endogenous labor or from a different assumption on government consumption.

in a general equilibrium model with altruistic dynasties and endogenous fertility. In turns out that assumptions on agents' heterogeneity (survival probabilities and skill differences) are quantitatively important: the fertility and allocation responses by different types of households lead to very different aggregate levels and equilibrium prices. A reduction in any type of uncertainty removes the need for precautionary savings in terms of assets and/or children.

The main actors of this model are the high skill agents. In the PAYG system, these agents save relatively more in assets than in children. Consequently, models with exogenous fertility underestimate the aggregate capital stock in the PAYG steady state. Further, the high skill households' responses to FF reform are much higher than those of the low skill households. As high skill agents switch to investing in the old-age support from children rather than in assets, fertility increases while the capital stock falls. Thus the FF reform with endogenous fertility leads to opposite aggregate outcomes than the same reform with exogenous fertility. Because the high skill agents have more children, the aggregate productivity increases together with the fraction of high skill (educated) individuals. The welfare gains from the elimination of social security tax seem to more than compensate the agents for the lost insurance provided by social security system against life-span and earnings risks.

These results indicate that models assuming exogenous fertility might be misleading with respect to the behavior of different groups of the population, aggregate outcomes, welfare gains and political support for the reform. Finally, endogenous fertility is also important for the transition analysis. In the literature with exogenous fertility, agents usually prefer the FF steady state but the transition to it is too costly as they need to invest a lot during the transition. However, in our endogenous fertility model, the capital-output ratio and the capital stock are already high in the PAYG system and both fall after the FF reform. The high initial stock of capital provides an additional consumption source for households who would otherwise suffer from the transition. This is important for theoretical purposes as well as for policy recommendations. Transition costs could be lower or could even turn into gains as there is no need to accumulate a higher capital stock for the new steady state.

This life-cycle dynastic model with endogenous fertility is open to many extensions such as the incorporation of endogenous labor, postponing of the retirement age or different fiscal arrangements. In many countries government policies support fertility by child allowances and maternity support, or try to limit fertility by restricting the number of children. Finally, an analysis of the transition between the PAYG initial steady state and the reformed steady state would evaluate the true cost of the reform. It would also enable us to study the elimination of social security benefits that is gradual or occurs only after a certain period of time.

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Appendix: Life-Cycle Intervivo Transfers

This Appendix presents the computation of intervivo transfers. Recall that all adult members of a household have the same objective function, they pool their resources and solve a joint maximization problem. As in Fuster, Imrohoroglu, and Imrohoroglu (2003), we assume that the father owns all assets of the household in period $j = 1$. Denote assets owned by the father at period j as $a_f(j)$ and those owned by each son as $a_s(j)$. The laws of motion of these asset holdings follow the individual budget constraints,

$$\begin{aligned}(1+g)a_f(j+1) &= [1+r(1-\tau_k)]a_f(j) + e_f(j) + \xi(j) - s \cdot nt(j) - (1+\tau_c)(c(j) + \gamma^g(j)),\\ (1+g)a_s(j+1) &= [1+r(1-\tau_k)]a_s(j) + e_s(j) + \xi(j) + nt(j) - (1+\tau_c)(c(j) + \gamma^g(j)),\end{aligned}$$

where $e_f(j)$ and $e_s(j)$ are after-tax earnings of the father and the son, $\xi(j)$ is the accidental bequest transfer from the government, $c(j)$ is the consumption of each adult member and $\gamma^g(j)$ are expenditures on children in the household. The intervivo transfer from the father to each son in period j is denoted by $nt(j)$. A negative transfer is a support from each son to the father.

A person living in the household needs a transfer if his individual consumption is larger than his total income in a particular period. For example, if a son's individual budget constraint implies that

$$[1+r(1-\tau_k)]a_s(j) + e_s(j) + \xi(j) - (1+\tau_c)\gamma^g(j) < (1+\tau_c)c(j),$$

then he needs to receive a transfer of

$$nt(j) = (1+\tau_c)c(j) + (1+\tau_c)\gamma^g(j) - [1+r(1-\tau_k)]a_s(j) - e_s(j) - \xi(j).$$

Naturally, the son's assets in the next period will be set to zero, $a_s(j+1) = 0$, and all assets of the household will be owned by the father, $a_f(j+1) = a(j+1)$.

If total incomes of the father and of the son are sufficient to finance their consumption, then the transfer is zero, $nt(j) = 0$, and the members distribute the next period assets according to their relative income contribution.

Compared to Fuster, Imrohoroglu, and Imrohoroglu (2003) and in Fuster, Imrohoroglu, and Imrohoroglu (2007), our model with endogenous fertility requires an assumption which members of the household bear the cost of raising the children $\gamma^g(j)$. Here we assume that it is shared by all adult members.

Parameters		
	Population	
j_{2T}	$= 14$	Maximum lifetime (90 years)
j_R	$= 10$	Retirement age (65 years)
j_N	$= 4$	Fertility age (35 years)
$\bar{n}_{USA}$	$= 0.012$	Population growth rate U.S.
ψ		Survival probabilities (Fuster)
	Utility	
β	$= 0.988$	Annual discount factor
σ	$= 0.95$	Relative risk aversion
η	$= 0.055$	Altruism
	Production	
g	$= 0.014$	Annual technology growth
δ	$= 0.044$	Annual depreciation rate
α	$= 0.34$	Capital share
$\pi_{LL} = 0.83$	$\pi_{HH} = 0.57$	Transition matrix for skills
e		Earnings profiles (Fuster)
	Fiscal Policy	
τ_k	$= 0.35$	Capital income tax rate
τ_c	$= 0.055$	Consumption tax rate
G	$= 0.225$	Government purchases (% GDP)

Table 1: Parameters

Cost of Children: Working Time		
Age of	Fraction of Combined Hours (γ_w)	
(Younger) Child	Weekly	Annual
0-4	0.0441	0.0755
5-9	0.0392	0.0662
10-14	0.0392	0.0662
15-19	0.0392	0.0662

Cost of Children: Expenditures		
Age of (Younger) Child	Annual Expenditure	Fraction of Income (γ_g)
One-Child Household		
0-4	\$10,354 = \$8,350 $\cdot$ 1.24	0.22
5-9	10,540 = 8,500 $\cdot$ 1.24	0.22
10-14	11,226 = 9,054 $\cdot$ 1.24	0.24
15-19	11,408 = 9,200 $\cdot$ 1.24	0.25
Two-Child Household		
0-4	\$17,690 = \$8,350 + \$9,340	0.36
5-9	17,840 = 8,500 + 9,340	0.37
10-14	18,394 = 9,054 + 9,340	0.38
15-19	18,540 = 9,200 + 9,340	0.39
n-Child Household ($n > 2$)		($n = 3$)
0-4	(\$8,350 + \$9,340 + (n-2)$\cdot$ \$9,200) $\cdot$ 0.77	0.43
5-9	(8,500 + 9,340 + (n-2)$\cdot$ 9,200) $\cdot$ 0.77	0.43
10-14	(9,054 + 9,340 + (n-2)$\cdot$ 9,200) $\cdot$ 0.77	0.44
15-19	(9,200 + 9,340 + (n-2)$\cdot$ 9,200) $\cdot$ 0.77	0.45

Sources: Working time: *Report on the American Workforce.* 1999. U.S. Department of Labor. Table 3-6. Expenditures: Table 9 in *Expenditures on Children by Families. 1998 Annual Report*, United States Department of Agriculture. Miscellaneous Publication Number 1528.

Table 2: Cost of Children

Benchmark PAYG and FF

	τ_{ss}	K	L	Y	C	K/Y	$r(1-\tau_k)$	$w(1-\tau_{ss}-\tau_l)$
PAYG	0.115	0.757	1.785	1.333	0.549	2.839	0.047	0.357
FF	0.00	0.694	1.889	1.344	0.536	2.582	0.053	0.394
(%)		-8.3	+5.8	+0.8	-2.4			+10.1

	Both f and s Alive				Only s Alive			
	LL	HL	LH	HH	L	H	Average	Growth
Fertility								
PAYG	1.83	1.56	1.65	1.11	1.85	1.61	1.67	1.24%
FF	1.88	1.89	1.78	1.73	1.80	1.76	1.84	1.46%
(%)	+2.7	+21.2	+7.9	+57.3	-2.7	+9.3	+10.2	
Assets (Changes in %)								
(%)	+0.5	-22.2	+3.5	-23.7	-4.1	-19.4	-8.3	
Welfare Gains from FF Reform (%)								
All	+0.09	-0.85	+0.75	-0.66	+2.35	+1.88	+0.41	
Newborns	+1.56	+0.63	+2.19	+0.88	—	—	+1.42	
Demographics (%)							H-Skill	Retired
PAYG	46.0	9.2	9.5	12.3	15.4	5.5	26.6	18.7
FF	44.8	10.1	9.2	13.5	14.8	5.6	28.9	17.5

Political Support

Generation	1	2	3	4	5	6	7	All
(%)	100	71	10	28	42	81	89	60

Table 3: Steady State Results: Benchmark Steady States

Return on Benchmark PAYG				
Conditional on Father's Survival				
f's Age	Both f and s Alive			
at Death	LL	HL	LH	HH
65	<0	<0	<0	<0
70	0.5	3.3	<0	0.5
75	3.8	6.9	<0	3.3
80	5.2	8.4	<0	5.3
85	6.0	9.0	<0	6.1
In Expectation				
	LL	HL	LH	HH
$E[r_{SS}]$	2.7	5.7	<0	3.4
	All			
$E[r_{SS}]$	2.4			

The after-tax return on capital is 4.7%.

Table 4: Return on Social Security. Benchmark Steady State PAYG with $\theta = 0.44$.

Case 1: Certain Lifetimes $\psi = 1$

	τ_{ss}	K	N	Y	C	K/Y	$r(1-\tau_k)$	$w(1-\tau_{ss}-\tau_l)$
PAYG	0.169	0.692	1.624	1.215	0.531	2.846	0.046	0.331
FF	0.00	0.644	1.764	1.252	0.532	2.570	0.054	0.393
(%)		-6.9	+8.6	+3.0	+0.2			+18.7

	Both f and s Alive				Only s Alive			
	LL	HL	LH	HH	L	H	Average	Growth
Fertility								
PAYG	1.52	1.37	1.35	1.10	—	—	1.42	1.06%
FF	1.71	1.66	1.52	1.33	—	—	1.62	1.44%
(%)	+12.5	+21.2	+12.6	+20.9			+14.1	
Assets (Changes in %)								
(%)	-0.0	-14.1	+7.5	-12.0	—	—	-6.9	
Welfare Gains from FF Reform (%)								
All	+0.11	-1.37	+1.85	-0.31	—	—	+0.09	
Newborns	+3.11	+1.74	+4.80	+2.90	—	—	+3.14	
Demographics (%)							H-Skill	Retired
PAYG	61.6	11.1	12.6	14.7	—	—	25.8	27.6
FF	61.3	11.3	12.5	14.9	—	—	26.2	25.0

Political Support

Generation	1	2	3	4	5	6	7	All
(%)	100	84	3	10	23	81	99	57

Table 5: Steady State Results: Certain Lifetime Steady States

Case 2: Equal Survival Probability $\psi_H = \psi_L$

	τ_{ss}	K	N	Y	C	K/Y	$r(1-\tau_k)$	$w(1-\tau_{ss}-\tau_l)$
PAYG	0.116	0.733	1.730	1.292	0.547	2.836	0.047	0.357
FF	0.00	0.711	1.836	1.330	0.542	2.673	0.051	0.402
(%)		-3.0	+6.1	+2.9	-0.9			+12.6

	Both f and s Alive				Only s Alive			
	LL	HL	LH	HH	L	H	Average	Growth
Fertility								
PAYG	1.72	1.48	1.53	1.10	1.67	1.44	1.58	1.09%
FF	1.87	1.80	1.67	1.45	1.79	1.62	1.77	1.38%
(%)	+8.7	+21.6	+9.2	+31.8	+8.9	+12.5	+12.0	
Assets (Changes in %)								
(%)	+0.0	-15.4	+1.9	-16.5	-4.3	-13.2	-3.0	
Welfare Gains from FF Reform (%)								
All	+0.37	-1.26	+1.13	-1.04	+2.98	+2.27	+0.63	
Newborns	+2.08	+0.75	+2.85	+1.03	—	—	+1.87	
Demographics (%)							H-Skill	Retired
PAYG	47.5	8.4	9.7	11.1	15.4	5.7	25.4	19.5
FF	47.3	8.7	9.7	11.6	14.9	5.7	26.3	18.0

Political Support

Generation	1	2	3	4	5	6	7	All
(%)	95	68	11	27	74	82	91	63

Table 6: Steady State Results: Equal Survival Probability Steady States

Case 3: Equal Survival Probability $\psi_H = \psi_L$ and Productivity $\varepsilon_H = \varepsilon_L$								
	τ_{ss}	K	N	Y	C	K/Y	$r(1-\tau_k)$	$w(1-\tau_{ss}-\tau_l)$
PAYG	0.134	0.589	1.775	1.220	0.495	2.415	0.058	0.320
FF	0.00	0.573	1.884	1.257	0.504	2.281	0.062	0.369
(%)		-2.7	+6.1	+3.0	+1.8			+15.5
	Both f and s Alive					Only s Alive	Average	Growth
Fertility								
PAYG	1.66					1.79	1.67	1.21%
FF	1.80					1.81	1.80	1.39%
(%)	+8.4					+1.1	+7.8	
Assets (Changes in %)								
(%)	-2.2					-5.1	-2.7	
Welfare Gains from FF Reform (%)								
All	+3.33					+5.74	+3.79	
Newborns	+5.27					—	+5.27	
Demographics (%)							H-Skill	Retired
PAYG	77.1					20.9	—	18.8
FF	77.4					20.6	—	18.0
Political Support								
Generation	1	2	3	4	5	6	7	All
(%)	100	99	86	86	90	97	100	94

Table 7: Steady State Results: Limited Heterogeneity Steady States

Case 4: Exogenous Fertility								
	τ_{ss}	K	N	Y	C	K/Y	$r(1-\tau_k)$	$w(1-\tau_{ss}-\tau_l)$
PAYG	0.115	0.601	1.785	1.233	0.501	2.438	0.057	0.331
FF	0.00	0.641	1.785	1.261	0.525	2.546	0.054	0.391
(%)		+6.7	0.0	+2.3	+4.8			+18.1

	Both f and s Alive				Only s Alive			
	LL	HL	LH	HH	L	H	Average	Growth
Fertility								
	—	—	—	—	—	—	1.71	1.24%
Assets (Changes in %)								
(%)	+11.6	+0.7	+9.9	+0.3	+7.3	+1.9	+6.7	
Welfare Gains (%)								
All	+4.18	+2.87	+5.20	+3.92	+6.73	+6.24	+4.57	
Newborns	+6.13	+4.93	+7.11	+5.94	—	—	+6.06	
Demographics (%)							H-Skill	Retired
	44.1	10.2	9.1	13.7	15.1	5.7	29.7	18.9

Political Support								
Generation	1	2	3	4	5	6	7	All
(%)	100	99	98	97	97	97	100	98

Table 8: Steady State Results: Exogenous Fertility Steady States

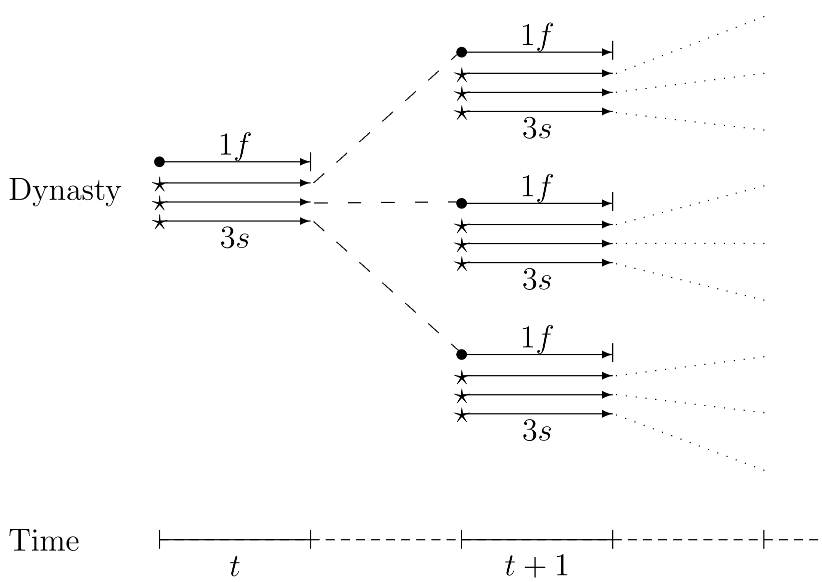

Figure 1: Timeline for Dynasties

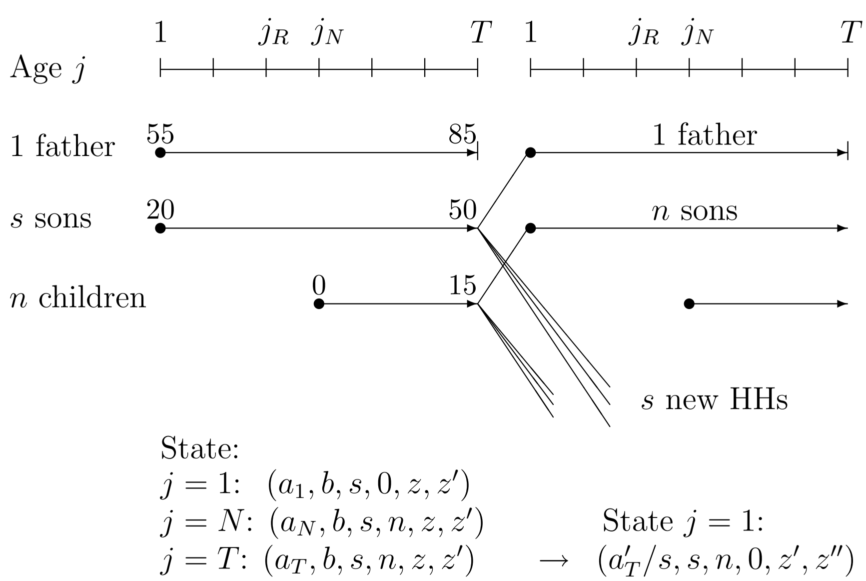

Figure 2: Timeline for Households

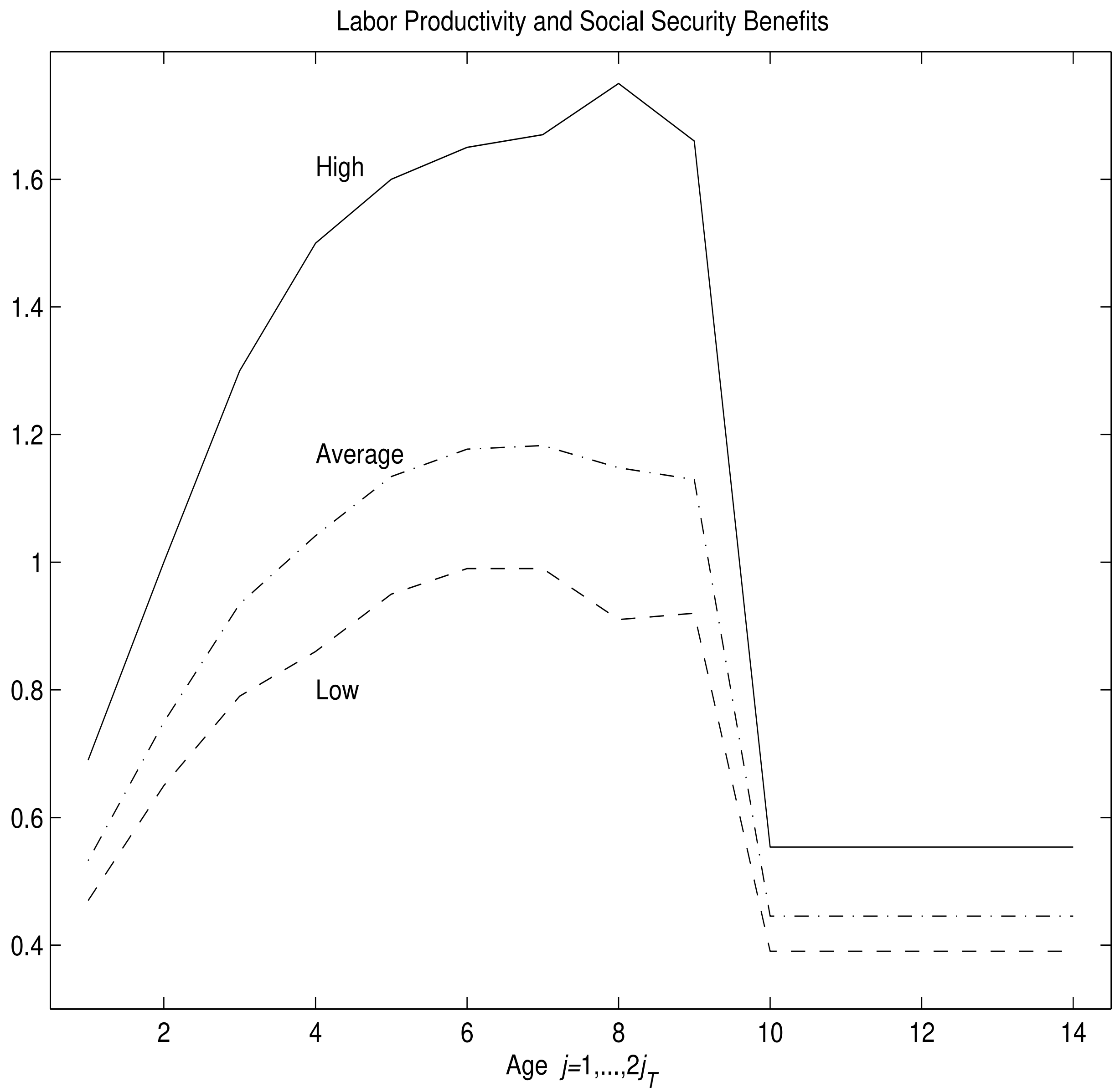

Figure 3: Age Profiles of Labor Productivity and Social Security Benefits.

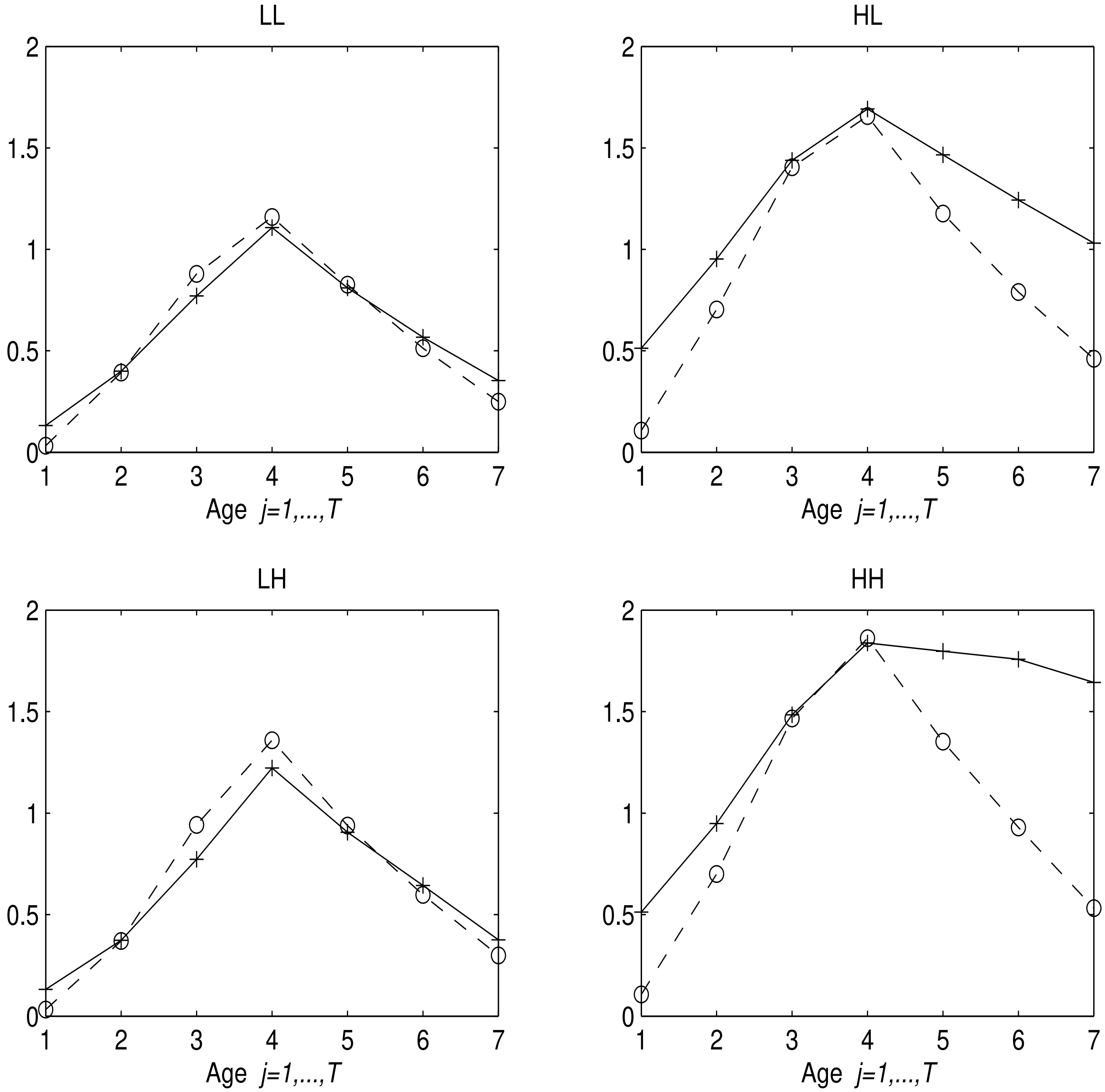

Figure 4: Average Wealth in Complete Households. PAYG: Full line (+). FF: Dashed line (○).

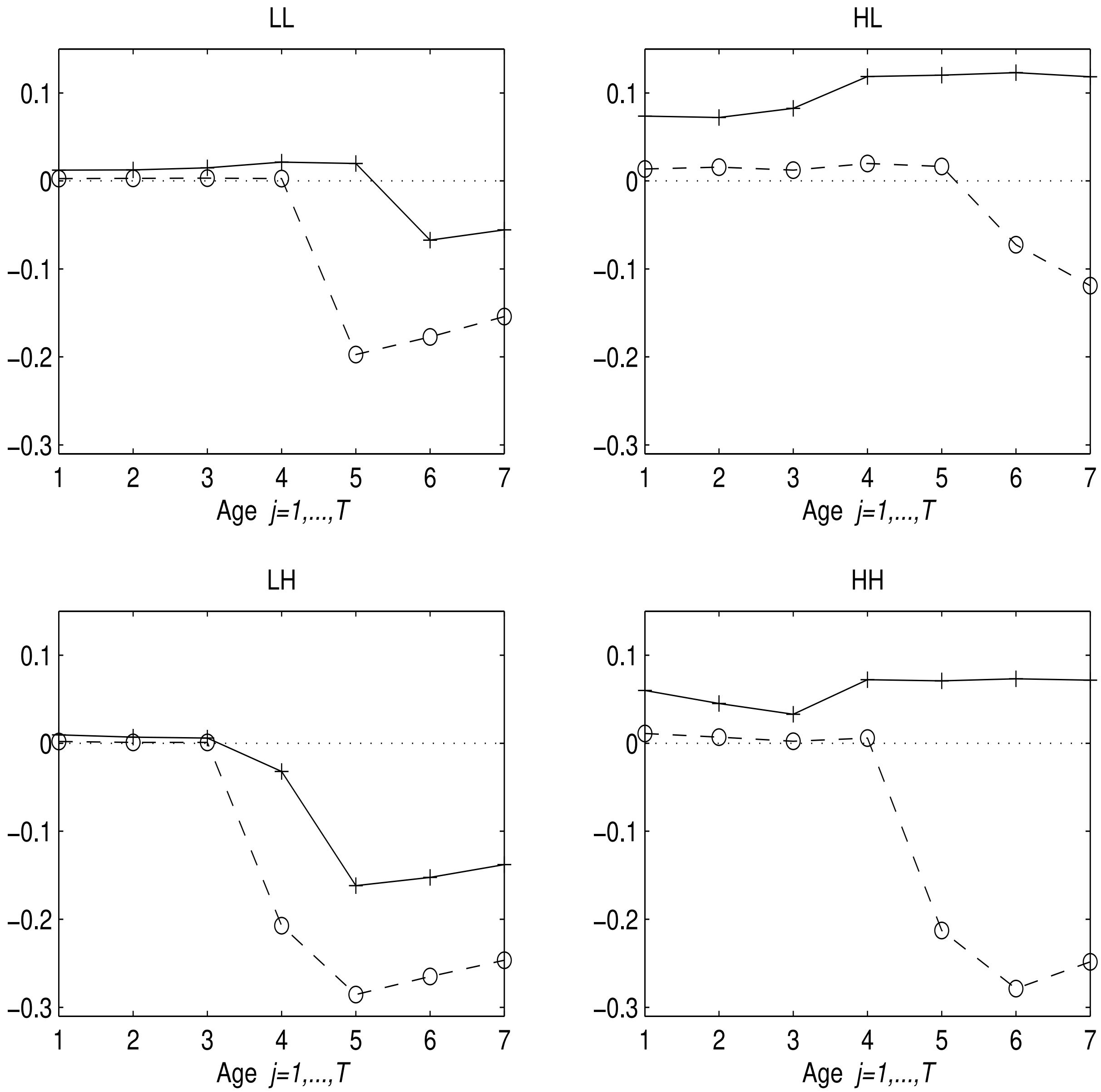

Figure 5: Average Intervivo Transfer from Father to One Son in Complete Households. PAYG: Full line (+). FF: Dashed line (∘).