Working Paper 160/16

INFORMATION EFFECTS IN LONGEVITY-LINKED VS PURELY FINANCIAL PORTFOLIOS

Elisa Luciano
Antonella Tolomeo

April 2016

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Elisa Luciano[†] Antonella Tolomeo[‡]

First version January 28th - Last version March 12th

Executive Summary

Longevity linked products might transfer to financial markets part of the global exposure to longevity of pension funds and annuity providers, an exposure that exceeded $\$20tr$ already in the mid 2000. Their use has been advocated since the early 2000 and becomes more and more important as longevity increases worldwide.

It is not a priori evident, though, why an investor should stand ready to absorb longevity-linked products. On the one side, because their payoff depends on the longevity of a group of individuals or a population, their returns should be either little correlated or totally uncorrelated with individual income, insurance policies and other assets' returns. So, they should provide a powerful diversification tool. At the same time, since longevity-linked products are different from usual bonds and stocks, they are likely to be quite far from the understanding and information abilities of retail investors.

This paper models the optimal behavior of a rational investor facing the choice between a traditional (purely interest-rate based) and a longevity bond. When buying longevity bonds, he can decide to pay a fee and separate the information on different risks affecting its bond value, namely on interest rates and on the longevity performance of the population on which the bond is written. Or he can decide to remain uninformed and receive information only on the overall performance of the bond. In that case he saves on information fees (meant to include time and monetary fees to an intermediary themselves).

The paper provides conditions under which the optimal portfolio choice is the longevity bond and conditions under which it is not. In the latter case diversification into the longevity market is not beneficial to a small investor. An example in which the longevity component is calibrated to the survivorship of the Italian population and the interest rate is calibrated to the EURIBOR market is provided.

*We thank E. Barucci, B. Dumas, P. Guasoni, the participants to the XVII Workshop on Quantitative Finance, Pisa 2016, and to the III Workshop on Robust Decisions in Organisations and Markets, Venezia 2016, for helpful comments and suggestions. Financial supported by the Global Risk Institute, Canada, is gratefully acknowledged.

†University of Torino, Collegio Carlo Alberto and Netspar; ESOMAS Department, Corso Unione Sovietica 218/bis, 10134, Torino, Italy; e-mail: elisa.luciano@unito.it; phone number: + 390116705230 (5200).

‡University of Torino, Department of Statistics and Applied Mathematics, Corso Unione Sovietica 218/bis, 10134, Torino, Italy, e-mail: antonella.tolomeo@carloalberto.org, phone number: + 390116705170

Abstract

The development of a market for longevity bonds is considered beneficial to investors, because it offers diversification opportunities. However, understanding of both longevity and interest rate risks is required to rationally invest in longevity bonds. This paper models the optimal behavior of an investor facing the choice between a traditional and a longevity bond. When buying longevity bonds, he can decide to pay a fee and separate the information on different risks affecting its bond value, or to remain uninformed and receive a non-separating signal. The uninformed investor optimally filters his pooled signal. The paper provides conditions under which the optimal portfolio choice is the longevity bond and conditions under which diversification is not beneficial. A calibrated example is provided.

Keywords: Information costs, Optimal filtering, Longevity-linked bonds.

JEL Classification: G11, G14, G22.

Longevity linked products might transfer to financial markets part of the global exposure to longevity of pension funds and annuity providers, an exposure that exceeded $\$20tr$ already in the mid 2000. Their use has been advocated since the early 2000 (see Biffis and Blake (2009)) and becomes more and more important as longevity increases worldwide. Recently, market maker associations (such as the LLMA) as well as single arrangers have been very keen on developing a market for them. It is not a priori evident, though, why an investor should stand ready to absorb them. On the one side, because their payoff depends on the longevity of a group of individuals or a population, their returns should be either little correlated with individual income, insurance policies and other assets' returns, or uncorrelated with them. At the same time, since longevity-linked products are different from usual bonds and stocks, they are likely to be quite far from the retail investor's understanding. That happens because their returns depend both on interest rates and longevity indices, and the familiarity of an investor with the latter is even lower.

This paper provides a model to assess the optimal behavior of a rational investor facing the choice between a traditional and a longevity bond. We assume that an investor maximizes the long run rate of growth of wealth, and that the longevity risk factor is uncorrelated with the financial risk one. A priori, information is imperfect, and the investor adopts optimal filtering to process it. However information can be made perfect (each signal entering into it can be distinguished) by paying a fee. So, the investor can decide to stick to traditional bonds or, as an alternative, to invest in longevity bonds, after having decided whether to get perfect information on the latter return dynamics, or to get only partial information about them. Information disclosure will be described as in Guasoni (2006): an uninformed investor observes only the "combined" return from the financial and longevity component of the new financial product, while an informed investor receives separately the information on the financial and longevity factors.

Our study extends previous theoretical models on the demand for new financial products with a longevity component. We add a new effect to the literature which stresses the role of longevity products as hedges against shocks to one's own longevity (such as Cocco and Gomes (2012)). In the context of a calibrated life-cycle model, Cocco and Gomes (2012) shows that the benefits of longevity linked products are substantial. Instead of looking at the hedging properties of longevity products, we look at their general diversification properties (assuming zero correlation with other financial risks) and at the drawbacks of investing in a product on which information is scarce.

To anticipate on our results, we provide the conditions under which an investor buys longevity bonds and the conditions under which they prefer traditional bonds. In the former case we show that, when fees are low, the investor acquires information and he does not when fees are high. In the latter case he does not acquire information. A calibrated example follows in which diversification does not pay.

The outline of the paper is as follows: in Section 2 we describe the portfolio choice of an investor with a risky traditional bond only and we determine the optimal logarithmic utility derived from it and a riskless asset. In Section 3 we extend the model to assets with a longevity component, and characterize the optimal filtering of information in that case. We determine the maximum cost or fee that the investor is willing to pay to acquire information, using Scolozzi and Tolomeo (2015). At that point, each investor can choose between four scenarios: he can buy financial bonds or longevity bonds, being informed or not. In Section 4 we study under which conditions on information fees the investor selects one specific scenario. In Section 5 we calibrate the model. Last, we summarize and outline further research.

1. Longevity modelling

This section gives some basic notions of stochastic mortality/longevity modelling, the set up that will be adopted below do describe longevity risk.

Mortality has been recently described by means of Cox or doubly stochastic counting processes. Mortality modelling via Cox processes has been introduced by Milevsky and Promislow (2001) and Dahl (2004). In this approach, the time of death is the first jump time of a Poisson process with stochastic intensity. The existence of a stochastic mortality intensity generates systematic mortality/longevity risk. If the intensity process is an affine diffusion process, then the survival function can be derived in closed form. To see this, let us introduce a filtered probability space $(\Omega, \mathbf{I}, \mathbb{P})$, equipped with a filtration which satisfies the usual properties of right-continuity and completeness. On this space, let us consider a non negative, predictable process $\kappa_x(t)$, which represents the mortality intensity of an individual or head belonging to generation x at (calendar) time t. We introduce the following

Assumption 1 *The mortality intensity κ_x follows a process of the type:*

$$d\kappa_x(t) = a(t, \kappa_x(t))dt + \sigma(t, \kappa_x(t))dW_x(t) \tag{1}$$

where W_x is a standard one-dimensional Brownian motion[1] *and the regularity properties for ensuring the existence of a strong solution of equation (1) are satisfied for any given initial condition $\kappa_x(0) = \kappa_0 > 0$.*

Given this assumption on the dynamics of the death intensity, let τ be the time to death of an individual of generation x. We define the survival probability from t to $T \geqslant t$, $S_x(t,T)$, as the survival function of τ under the probability measure $\mathbb{P}$, conditional on the survival up to time t:

$$S_x(t,T) := \mathbb{P}\left(\tau \geqslant T \mid \tau > t\right)$$

It is known since Brémaud (1981) that - under the previous assumption - the survival probability $S_x(t,T)$ can be represented as

$$S_x(t,T) = \mathbb{E}\left[\exp\left(-\int_t^T \kappa_x(s)ds\right) \mid \mathcal{F}_t\right] \tag{2}$$

where the expectation is computed under $\mathbb{P}$. When the evaluation date is zero ($t = 0$), we simply write $S_x(T)$ instead of $S_x(0,T)$.

Suppose that

Assumption 2 *The drift $a(t,\kappa_x(t))$ and the instantaneous variance $\sigma^2(t,\kappa_x(t))$ have a linear affine dependence on $\kappa_x(t)$.*

So, drift and variance coefficients are of the form:

$$a(t,\kappa_x(t)) = b + c\kappa_x(t)$$

$$\sigma^2(t,\kappa_x(t)) = d \cdot \kappa_x(t)$$

where $b, c, d \in \mathbb{R}$. Under Assumption 2 standard results on functionals of affine processes allow us to provide a closed form for the survival probability

$$S_x(t,T) = e^{\alpha(T-t)+\beta(T-t)\kappa_x(t)}$$

where $\alpha(\cdot)$ and $\beta(\cdot)$ solve the following system of Riccati differential equations (see for instance Duffie et al. (2000)):

$$\begin{cases} \beta'(t) = \beta(t)c + \frac{1}{2}\beta(t)^2 d^2 \\ \alpha'(t) = \beta(t)b \end{cases} \tag{3}$$

under the boundary conditions are $\alpha(0) = 0$ and $\beta(0) = 0$.

[1]The extension of the mortality intensity definition to a multidimensional Brownian motion is straightforward.

Close to the notion of instantaneous death intensity $\kappa_x(t)$, one can define also the survival mortality, that will be named J_2 below and used in modelling longevity bonds. J_2 is evidently the opposite of a. The drift of our specification for J_2 will satisfy Assumption 2. In this paper will we not use the survival function, if not for the calibration.

2. Investor's preferences and Financial bonds

The representative investor maximizes the long run rate of growth of wealth. He can invest in a riskless bond (or cash) and participate either in a market in which a risky bond subject to one risk source or factor (the risky interest rate), or in a market where a longevity-linked bond subject to two risk sources is traded (the risky interest rate and an intensity of longevity).

For the sake of simplicity we normalize the riskless rate to zero, so that the riskless asset is worth $B_0 \in \mathcal{R}$ at all times.

Let $J_1(t)$ represent the instantaneous risky interest rate. In this paper the risk factor $J_1(t)$ follows an Ornstein-Uhlenbeck process

$$dJ_1(t) = -\lambda_1 J_1(t)dt + dW_1(t) \tag{4}$$

where $\lambda_1 > 0$ and $W_1(t)$ is a Brownian motion. Call $\mathcal{F}(t)$ the augmented filtration generated by $W_1(t)$. On $(\Omega, \mathcal{F}, P, \mathcal{F}(t))$ the risky asset has the price dynamics

$$\frac{dM(t)}{M(t)} = \left(\mu - \sigma\lambda_1 J_1(t)\right)dt + \sigma dW_1(t). \tag{5}$$

with $\mu > 0$, $\sigma > 0$. The idea behind the above assumption on the asset dynamics consists of having a fixed return $\mu dt > 0$ and a temporary return or shock dJ_1 with a mean-reverting component. The higher is the mean-reverting parameter λ_1, the less persistent will be the shock.

We solve the logarithmic utility maximization problem for the investor. The investor seeks for

$$\sup_z \left(\lim_{T\to\infty} \frac{ln(W)}{T}\right) \tag{6}$$

where, in the current market, $W = X + B_0$, $X(0) = x$ and $z = \pi$ is the fraction of wealth invested in M. The budget constraint or self-financing condition is:

$$\frac{dX(t)}{X(t)} = \pi(t)\frac{dM(t)}{M(t)} \tag{7}$$

with $\frac{dM(t)}{M(t)}$ given by (5). The total amount invested in M at time t is

$$X(t) = x \exp\left[\int_0^t \left[\mu - \sigma\lambda_1 J_1(s) - \frac{1}{2}\pi(s)^2\sigma^2\right] ds + \int_0^t \pi(s)\sigma dW_1(s)\right]. \tag{8}$$

It is proven in Luciano and Tolomeo (2016) the following Theorem

Theorem 2.1. *The optimal strategy $\pi(t)$, and the asymptotic log utility $u(x)$, which solve problem (6) subject to (7) are*

$$\pi(t) = \frac{\mu - \sigma\lambda_1 J_1(t)}{\sigma^2}, \tag{9}$$

$$u(x) = \frac{\mu^2}{2\sigma^2} + \frac{\lambda_1}{4}. \tag{10}$$

3. Longevity bonds

Suppose now that the investor has access to a riskless bond and a longevity bond, instead of the risky bond. The longevity bond depends on the same fixed component μdt and on the same temporary, short term interest rate we had above, as well as on a longevity rate. It actually depends, as we specify in few lines, on a weighted average of them. By definition, the longevity rate is the complement to one of the mortality rate of a given population and generation, as described above.[2] Let the longevity intensity of a given generation x follow an Ornstein-Uhlenbeck process

$$dJ_2(t) = -\lambda_2 J_2(t)dt + dW_2(t) \tag{11}$$

Let as define $Y(t)$ as the weighted sum of the two processes J_1 and J_2

$$Y(t) = p_1 J_1(t) + p_2 J_2(t) \tag{12}$$

with $p_1, p_2 > 0$ and $\sum_{j=1}^2 p_j^2 = 1$. The longevity bond is assumed to evolve according to

$$\frac{dN(t)}{N(t)} = \mu dt + \sigma dY(t). \tag{13}$$

The investor has incomplete information because, while he knows μ and σ and the weights p_j, he does not observe separately the realizations of the processes J_j. If he decides not to acquire specific information about the dynamics of the single factors, he just observes the "pooled signal" or risk factor Y, that is the total return he gets from the bond. As an alternative, the investor can decide to get full information on both risk factors (on the componenets of his return) that

[2] More complex longevity bonds are described in Biffis and Blake (2009). A longevity bond is not an annuity or term insurance on the investor's life time. In that case he would probably be able to separate at least the longevity signal (or factor) from the financial ones.

allows him to observe separately J_1 and J_2. It follows from (13) that the longevity source of risk J_2 has a stronger effect on the *uninformed* investor. The higher is the mean reversion parameter λ_2 associated to it. The greater is the difference between λ_2 and λ_1, with $\lambda_2 > \lambda_1$, the greater is the "gap" in terms of information between an *informed* and an *uninformed* investor. If the investor decides to stay uninformed, his filtration is $(\mathcal{F}_U(t))_{t\in[0,+\infty)}$. The filtration for the informed investor is $(\mathcal{F}_I(t))_{t\in[0,+\infty)}$. $\mathcal{F}_I(t)$ denotes the augmented filtration generated by $W_1(t)$ and $W_2(t)$, while $\mathcal{F}_U(t)$ is the augmented filtration generated by $Y(t)$ alone. Further, $\mathcal{F}_U(t)\subset\mathcal{F}_I(t)$.

It is proved in Guasoni and Tolomeo (2016) that the longevity bond dynamics for the *uninformed* investor are

$$\frac{dN_U(t)}{N_U(t)} = \left[\mu + \sigma\left(\tilde{J}_1(t) + \tilde{J}_2(t)\right)\right]dt + \sigma d\tilde{W}(t) \tag{14}$$

where $\tilde{J}_1(t)$, $\tilde{J}_2(t)$ are

$$d\tilde{J}_1(t) = -\lambda_1\tilde{J}_1(t)dt + \hat{j}_1 d\tilde{W}(s) \tag{15}$$

$$d\tilde{J}_2(t) = -\lambda_2\tilde{J}_2(t)dt + \hat{j}_2 d\tilde{W}(s) \tag{16}$$

and $\tilde{W}$ represents the innovation process obtained from the filtering procedure.[3]

For the *informed* investor the longevity bond dynamics can be written substituting (12) in (13) as follows

$$\frac{dN_I(t)}{N_I(t)} = \left(\mu - p_1\sigma\lambda_1 J_1(t) - p_2\sigma\lambda_2 J_2(t)\right)dt + \sigma dW_I(t) \tag{17}$$

where $W_I(t) = p_1W_1(t) + p_2W_2(t)$ is an $\mathcal{F}_I(t)$ Brownian motion.

As in intermediate step towards our final goal, we spell out the conditions under which the investor participates in the longevity-market as an *uninformed* (U) or as an *informed* (I) investor. For this reason we solve the logarithmic utility maximization problem in the two cases, namely (6) with $W = X_i + B_0$, $i = U, I$, and $z = \pi_i$, $i = U, I$. The self-financing condition for the *uninformed* investor is

$$\frac{dX_U(t)}{X_U(t)} = \pi_U(t)\frac{dN_U(t)}{N_U(t)} \tag{18}$$

The condition in (18) is different for the *informed* investor because we introduce for him an *information fee* ϕ. It stands for a cost per unit of time t that the *informed* investor pays to access

[3] Notice that

$$\hat{j}_1 = \frac{\lambda_1 - \sqrt{p_1^2\lambda_2^2 + p_2^2\lambda_1^2}}{p_1(\lambda_1 - \lambda_2)}, \qquad \hat{j}_2 = \frac{\sqrt{p_1^2\lambda_2^2 + p_2^2\lambda_1^2} - \lambda_2}{p_2(\lambda_1 - \lambda_2)}.$$

to the full information on risk factors. The self-financing condition for the *informed* investor is

$$\frac{dX_I(t)}{X_I(t)} = \pi_I(t)\frac{dN_I(t)}{N_I(t)} - \phi dt. \tag{19}$$

Theorem 3.1. *The optimal strategies π_i, $i = U, I$ and the asymptotic log utilities u_i, $i = U, I$, which solve problem (6) subject to (18) for the uninformed investor, (6) subject to (19) for the informed one, exist. They are*

$$\pi_U(t) = \frac{\mu + \sigma\left(\tilde{J}_1(t) + \tilde{J}_2(t)\right)}{\sigma^2} \tag{20}$$

$$u_U(x) = \frac{\mu^2}{2\sigma^2} + \frac{\lambda_1(1+p_2^2) + \lambda_2(1+p_1^2) - 2\sqrt{\lambda_2^2 p_1^2 + \lambda_1^2 p_2^2}}{4} \tag{21}$$

$$\pi_I(t) = \frac{\mu - \lambda_1\sigma p_1 J_1(t) - \lambda_2\sigma p_2 J_2(t)}{\sigma^2} \tag{22}$$

$$u_I(x,\phi) = \frac{\mu^2}{2\sigma^2} + \frac{p_1^2\lambda_1 + p_2^2\lambda_2}{4} - \phi. \tag{23}$$

The previous results allow us to consider under which conditions an investor participates in the longevity-market as informed or uninformed. The level of indifference fees which match $u_U(x)$ in (21) and $u_I(x,\phi)$ in (23) is

$$\phi_N^* = \frac{1}{2}\left(\sqrt{\lambda_2^2 p_1^2 + \lambda_1^2 p_2^2} - \lambda_2 p_1^2 - \lambda_1 p_2^2\right). \tag{24}$$

The level of indifference fees ϕ_N^* is also equal to $u_I(x,0) - u_U(x)$ and in Luciano and Tolomeo (2016) it is shown that it is positive. Note that ϕ_N^* does not depend on μ and σ, which are common knowledge, but only on the weights p_j and the mean reversion parameters λ_1, λ_2, which enter the risk factors J_1, J_2 and their weighted sum Y, and therefore determine the value of information. If the level of ϕ remains below the value ϕ_N^* it is better to be informed, because

$$\phi < \phi_N^* \Rightarrow u_I(x,\phi) > u_U(x). \tag{25}$$

If the level of ϕ is greater than the value ϕ_N^* it is better to be uninformed

$$\phi > \phi_N^* \Rightarrow u_I(x,\phi) < u_U(x). \tag{26}$$

4. Investor's choice

We now investigate the scenarios mentioned above, for a numerical case. In the following cases we consider equal weights p_j since we are interested in examining the impact of the mean reversion parameters λ_j of the forecast errors on utilities.

4.1. Case a) $\lambda_2 > \lambda_1$

Assume that the mean reversion parameters are increasing, $\lambda_2 > \lambda_1$, so that the second shock is less persistent. We show below that the standard diversification effect holds, and it is better to invest in the longevity bond than to stick to a traditional bond.

Under the assumptions of case a), indeed, from the optimal log utilities in (10), (21), (23) and the fact that the difference $u_I(x, 0) - u_U(x)$ is positive at $\phi = 0$, decreasing in ϕ and null at ϕ_N^*, while the difference $u_U(x) - u(x)$ is positive for $\lambda_2 > \lambda_1$, as proven in Luciano and Tolomeo (2016), it follows that

$$u(x) < u_U(x) < u_I(x, \phi) \qquad\qquad \textit{when } \phi < \phi_N^* \tag{27}$$

Therefore, the scenarios open to the investor depend on the actual cost-of-information fees:

1. If the actual level of fees is lower than the level of indifference fees, there is an advantage for the investor to be informed and participate in the market that includes the longevity factor. Indeed, it is evident from (27) that

$$\phi < \phi_N^* \Rightarrow u_I(x, \phi) \tag{28}$$

2. If the actual level of fees exceeds the threshold ϕ_N^*, the optimal choice is to buy the longevity bond and being uninformed. In this scenario the cost-of-information is too high. Consequently, the optimal utility is obtained by selecting the market with greater diversification (without paying any cost of information). The investor utility is greater than the utility that he could get otherwise

$$\phi > \phi_N^* \Rightarrow u_U(x) \tag{29}$$

since $u_U(x) > u_I(x, \phi)$ because of (26) and $u_U(x) > u(x)$ according to Luciano and Tolomeo (2016).

So, under optimal filtering, log utility and the assumptions on weights and mean reversion parameters introduced so far, the investor always exploits diversification (buys the longevity bond) and at most decides not to be informed.

If for instance the mean reversion parameters are $\lambda_1 = 1$ and $\lambda_2 = 2$ we obtain the values of derived utilities and switching fees as shown in Table 1[4]

The following Figure 1 synthesizes the analysis provided points 1 and 2 for the case a).

[4] Hereafter we will not consider in the computation the term $\dfrac{\mu^2}{2\sigma^2}$, since it is common to all utilities and does not affect the level of indifference fees.

Table 1: Case a) - Comparing Log Utility

	Asymptotic Log Utility			
$\mathbf{p_j}$ **equal**	$\mathbf{u(x)}$	$\mathbf{u_U(x)}$	$\mathbf{u_I(x,0)}$	$\phi^*_{\mathbf{N}}$
	.25	.334	.375	.041

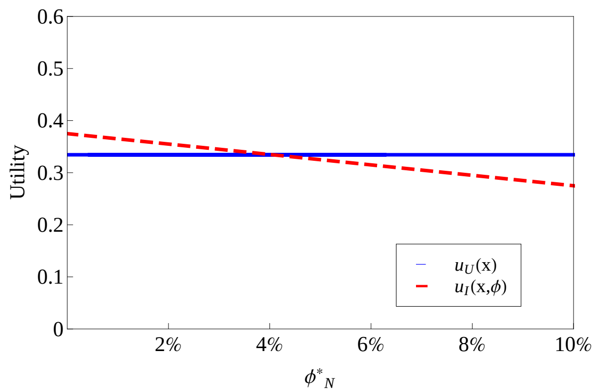

Figure 1. Level of indifference fees ϕ^*_N, given $\lambda_1 = 1$, $\lambda_2 = 2$ and equal weights p_j.

4.2. Case b) $\lambda_2 < \lambda_1$

Consider now the case in which the mean reversion parameters are decreasing, from the first to the second factor, so that the second shock is more persistent than the first. The optimal choice of the investor is to stay undiversified in the market with the traditional bond, independently of the cost of information. As shown in the numerical example below in Table 2, buying information on a longevity bond which in this case is more noisy than the traditional bond is of no value. For instance, given the mean reversion parameters equal to $\lambda_1 = 2$, $\lambda_2 = 1$ and equal weights p_j, the maximized utility values for this case are as in Table 2

Table 2: Case b) - Comparing Log Utility

	Asymptotic Log Utility			
$\mathbf{p_j}$ **equal**	$\mathbf{u(x)}$	$\mathbf{u_U(x)}$	$\mathbf{u_I(x,0)}$	$\phi^*_{\mathbf{N}}$
	.5	.334	.375	.041

It follows from Luciano and Tolomeo (2016) that this result holds for all admissible weights, $\sum_{j=1}^{2} p_j^2 = 1$, since

$$u(x) > u_I(x,\phi) > u_U(x) \qquad\qquad for\ \phi < \phi_N^*, \tag{30}$$

$$u(x) > u_U(x) > u_I(x,\phi) \qquad\qquad for\ \phi > \phi_N^*. \tag{31}$$

5. An example

In this section we provide an example of choice. We consider a traditional bond that pays a fixed return $\mu = 3\%$ plus a short interest EURO-denominated rate, and a longevity bond that adds to that payoff a longevity intensity. For simplicity, we assume that the two factors enter with the same weight in the product design: $p_1 = p_2 = 1/\sqrt{2}$ As a short interest rate we take the six months EURIBOR (source: Bloomberg). We calibrate its parameters, including the initial level of the EURIBOR itself, to the 5-years term structure as of March 4th 2016. The longevity intensity is calibrated to the 65 year-old Italian males population (source: Istat), setting the volatility to the same level as for the short rate. By so doing, we get the following model parameters: [5] $J_1(0) = -.2002\%$, $J_2(0) = .52\%$, $\mu = 3\%$, $\sigma = .00207\%$, $\lambda_1 = 28,96\%$, $\lambda_2 = 7.02\%$. In the Figure 2 we plot the observed and fitted survival probabilities for the Reference population

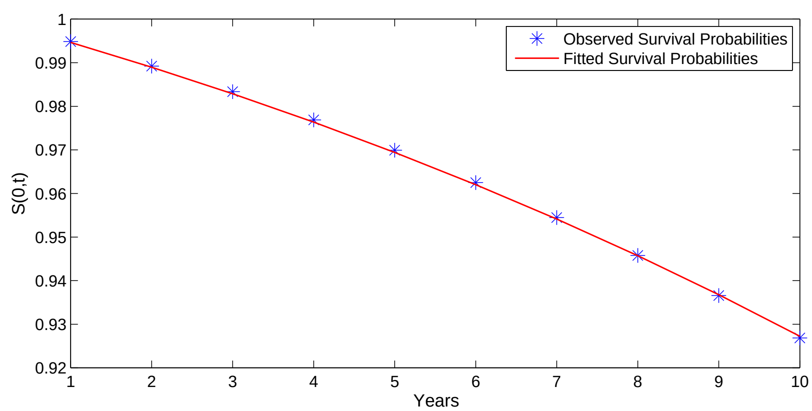

Figure 2. Observed and fitted survival probabilities for the Reference Population.

With these two factors available, in principle an investor can either buy the financial bond, or the longevity one. In the latter case, he should get information as long as the fees are below

$$\begin{aligned}\phi_N^* &= \frac{1}{2}\left(\sqrt{\lambda_2^2 p_1^2 + \lambda_1^2 p_2^2} - \lambda_2 p_1^2 - \lambda_1 p_2^2\right)\\ &= 1.4\%\end{aligned}$$

and no information above. In practice, since the calibrated values make us enter into case b), he

[5]Please note that on march 4th the OIS interest rate was $-.238\%$ (source: Bloomberg).

should invest in the traditional bond and should not diversify as shown in Theorem 2.1. This is therefore an example in which, even with moderate fees, there is no incentive to diversify because the longevity component of the returns is very noisy. The fraction invested in the traditional bond is

$$\pi(t) = 2,100,399.07 - 7.147 \cdot 10^{-5} J_1(t)$$

and the utility follows

$$u(x) = 1,050,199.61.$$

Let us remind the reader that the result does not depend on the weights p_1 and p_2.

6. Summary and Further Research

In this paper we investigate diversification benefits in a longevity bond market with information costs, and the optimal behavior of a rational investor facing them. Assuming that the interest rate and longevity factors are uncorrelated, the investor maximizes the long run rate of growth of wealth. He can invest in traditional financial products or, as an alternative, he can invest in a longevity bond, after having decided whether to get perfect or only partial information about its return dynamics. When information is imperfect the investor adopts optimal filtering to process it. To get perfect information (each signal entering into it can be distinguished) the investor has to pay a fee. We provide conditions under which an investor buys longevity bonds, both when he decides to remain uninformed and when he does not, and the conditions under which he prefers traditional bonds.

We completed the analysis with a calibrated example, both for the interest rate and for the longevity factor and we showed the impact on the specific decision to buy a logevity versus a traditional bond. We hope this helps to visualize that the benefits provided by the diversification of longevity bonds in comparison with traditional bonds can be offset by lack of knowledge or noise in the longevity component.

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