Probability Numeracy: Measurement and Applications

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Beliefs about the probabilities of future events play a central role in many economic decisions over the life cycle.

How much to save

Depends on future health status

When to retire

Depends on survival

What to invest in

Depends on anticipated rates of return

Whether to purchase long-term care insurance

Depends on likelihood of needing it

Tradition of asking about intentions for forecasting purposes

“Do you plan to purchase a car within the next year?”

No

Yes

Juster (1966) on car purchases

Most purchases are made by non-intenders with small buying probabilities

Example: probability of purchase = 0.40;

answer “No”

Population frequency of “yes” = 0

Population frequency of purchase = 0.40

Similar problems likely occur in intentions data

Moving to nursing home

Losing a job

Alternative: subjective probability of purchase

"What are chances you will purchase a car within the next year?" 0.40

Aggregate to population 40% purchase

Juster (1966) on car purchases

Subjective purchase probabilities predict future purchases better than buying intentions

Subjective probability distribution

Individual's belief about probability distribution of some future event.

Examples:

- Probability a worker age 53 will work full-time at age 62
  - Point on "survival" in labor force
- Probability an individual age 55 lives to age 75
  - Point on subjective survival curve
- Probability of a stock market gain over coming 12 months
  - Point on cumulative distribution of stock gains

Main objective of collecting data on subjective probabilities

Understand inter-temporal decision-making

- Uncertainty about relevant future event
- What information does individual use in deciding?

Measure what individuals believe rather than

Make assumptions such as rational expectations.

Assume historical distribution of outcomes

For example, historical distribution of stock market gains.

This presentation

What are properties of subjective probabilities as elicited in household surveys?

- Measurement
- Predictive power for actual outcomes
- Response anomalies
    - Heterogeneity across domains and persons

We propose a **probability numeracy** measure to address heterogeneity

- Measurement
- Validation
- Use in stated preferences

Measurement of subjective probability

…give me a number from 0 to 100, where "0" means that you think there is absolutely no chance, and "100" means that you think the event is absolutely sure to happen.

Subjective survival probability

Among respondents aged less than 65

`What is the percent chance that you will live to be 75 or more?`

Also asked for target age of 85.

Thus ask about two points on individual's survival curve.

Additional subjective probabilities queried in HRS

Will income keep up with inflation?
Inheritance
Lose job
Live independently
Live free of cognitive impairment
Health decline
Health expenditures use up all of savings
U.S. will have major depression
Inflation
Among workers: work full-time after reaching age 62 (65)
Stock market gain over next 12 months.
Bequest (4 targets amounts)

Subjective probabilities are also collected in many other household surveys

SHARE (Europe)
ELSA (England)
KLOSA (Korea)
PSID (U.S.)
NLSY (U.S.)
SEE (U.S.)
JSTAR (Japan)
CHARLS (China)
SLP (Singapore)
LASI (India)
MHAS (Mexico)

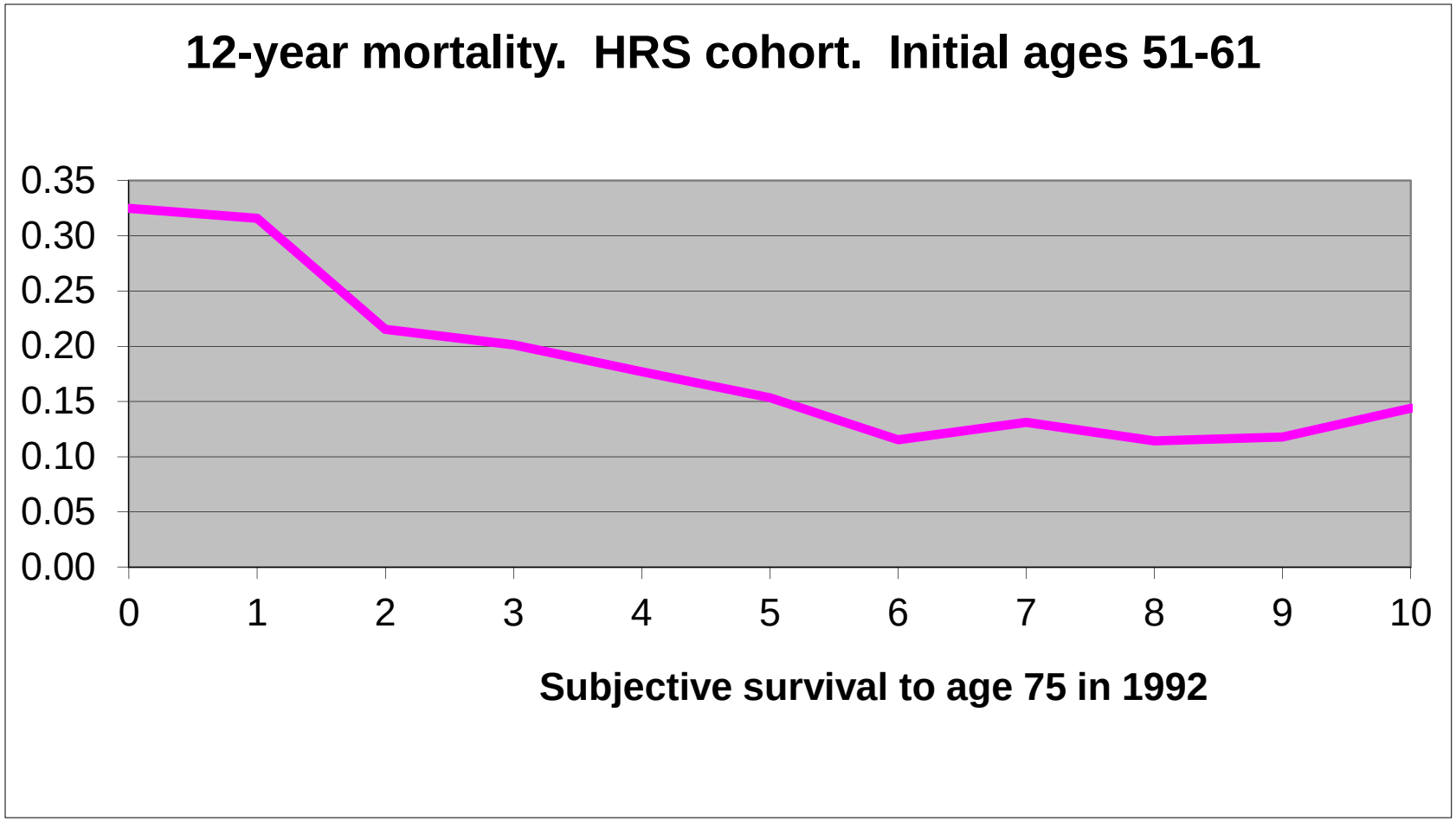
Properties

No predictive power for stock market gains, but predictive power for ownership

Owners more optimistic

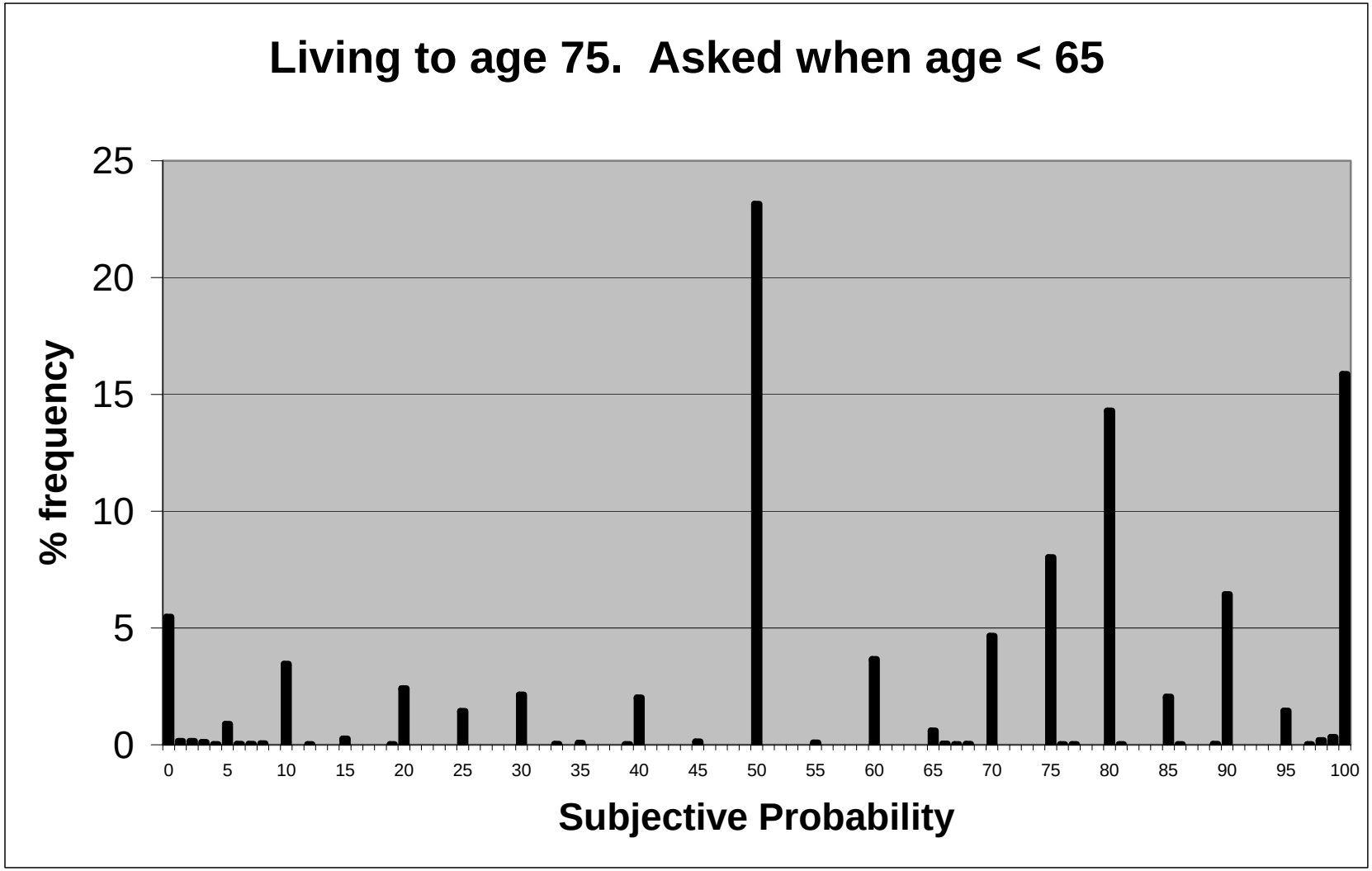
Good predictive power where respondent has personal information

- Working past age 62
- Enter nursing home
- Survival


12-year mortality. HRS cohort. Initial ages 51-61
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
1
2
3
4
5
6
7
8
9
10
Subjective survival to age 75 in 1992

But responses exhibit anomalies

- Focal point responses and rounding
  - 0%, 50% and 100%
    - 50% could be due to “epistemic” uncertainty: respondent doesn’t have well-formed probability distribution.
  - 25%, 75% etc.
  - Example from HRS


Living to age 75. Asked when age < 65
% frequency
25
20
15
10
5
0
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
Subjective Probability

Response anomalies (cont.)

- Violations of laws of probability
  - Probability of survival to age 85 greater than to age 75
- "Excessive" variation over time at individual level, even in same survey (white noise)
- Expectations about small-probability events tend to be upward biased
- Anchoring toward middle of scale.

Research has shown

- Heterogeneity in anomalies across domains
  - More rounding and uncertainty about stock market; less about working past age 62
- Heterogeneity in anomalies across people
  - Some individuals tend to say 50% across domains
    - May not understand probabilities

We develop a tool to classify individuals

- ability to think probabilistically
- to express subjective probabilities in household surveys

Eventual goal:

Use subjective probabilities more effectively to understand decision making under uncertainty.

Data

RAND American Life Panel

Internet based probability sample of US population

We use a subsample

Financial Crisis Surveys

Mostly monthly, some quarterly, November 2008 – January 2016

61 waves

Asked many subjective probability questions: 63

- Stock market gains, housing price gains, survival, gasoline prices, inflation, anticipated mortgage payment problems, etc.

Multiple times...as many as 61 occasions

In waves 58, 60 and 61 administered probability numeracy questions

Have 2,878 observations with

- data on subjective probabilities from waves 1-61
- and probability numeracy

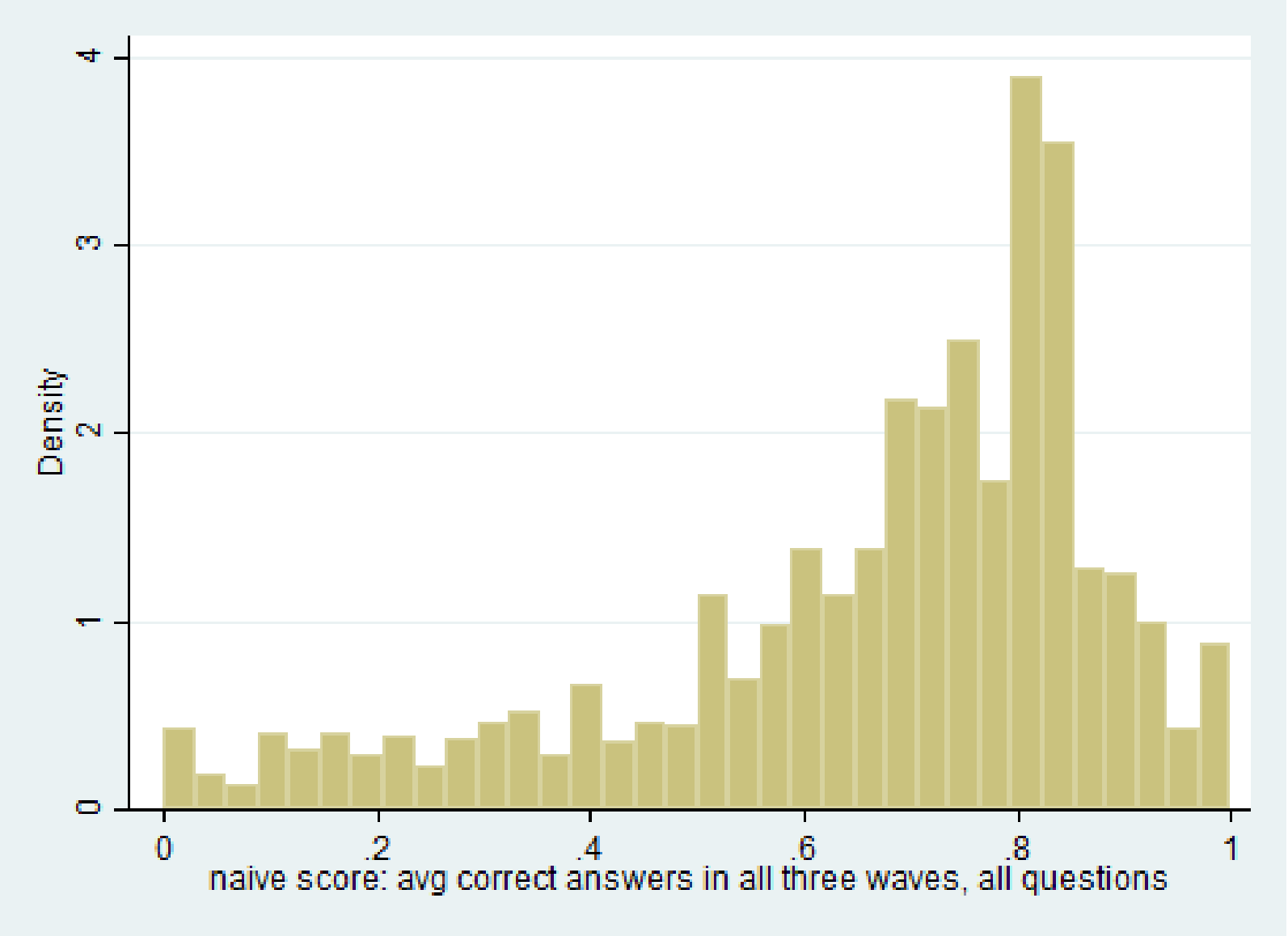
13 probability numeracy questions

Show subset

		Difficulty	Frequency correct
Q1	10 white balls, no red. Probability draw is white?	Medium	0.768
Q3	7 white, 3 red. Which is more likely?	Easy	0.879
Q4	7 white, 3 red. Probability of red?	Medium	0.702
Q6	Chance of rain is 70%. Probability of not rain?	Easy	0.871
Q7	Chance of rain is 70%. Can chance of rain both today and tomorrow be 80%?	Hard	0.243

Q8	Positive autocorrelation in rain and 50% marginal. Probability of rain two days in a row can be what? {ranges given}	Hard	0.151
Q9	Chance it rains in your town and Paris are both 50% and independent. Probability of raining in both cities?	Hard	0.136
Q10	Fair coin comes up head 3 times. Probability of next one being tail?	Medium	0.677
Q12	Chance it rains in your town and Paris are both 10% and independent. If rains in your town, what is probability of raining in Paris?	Medium	0.644
Q13	Fair coin comes up head. Probability next is tail?	Easy	0.865

Distribution of average number of correct answers



But want to account for

Some questions more difficult than others

Not everyone responded to all three waves

Correct for that

Some faced more difficult questions on average

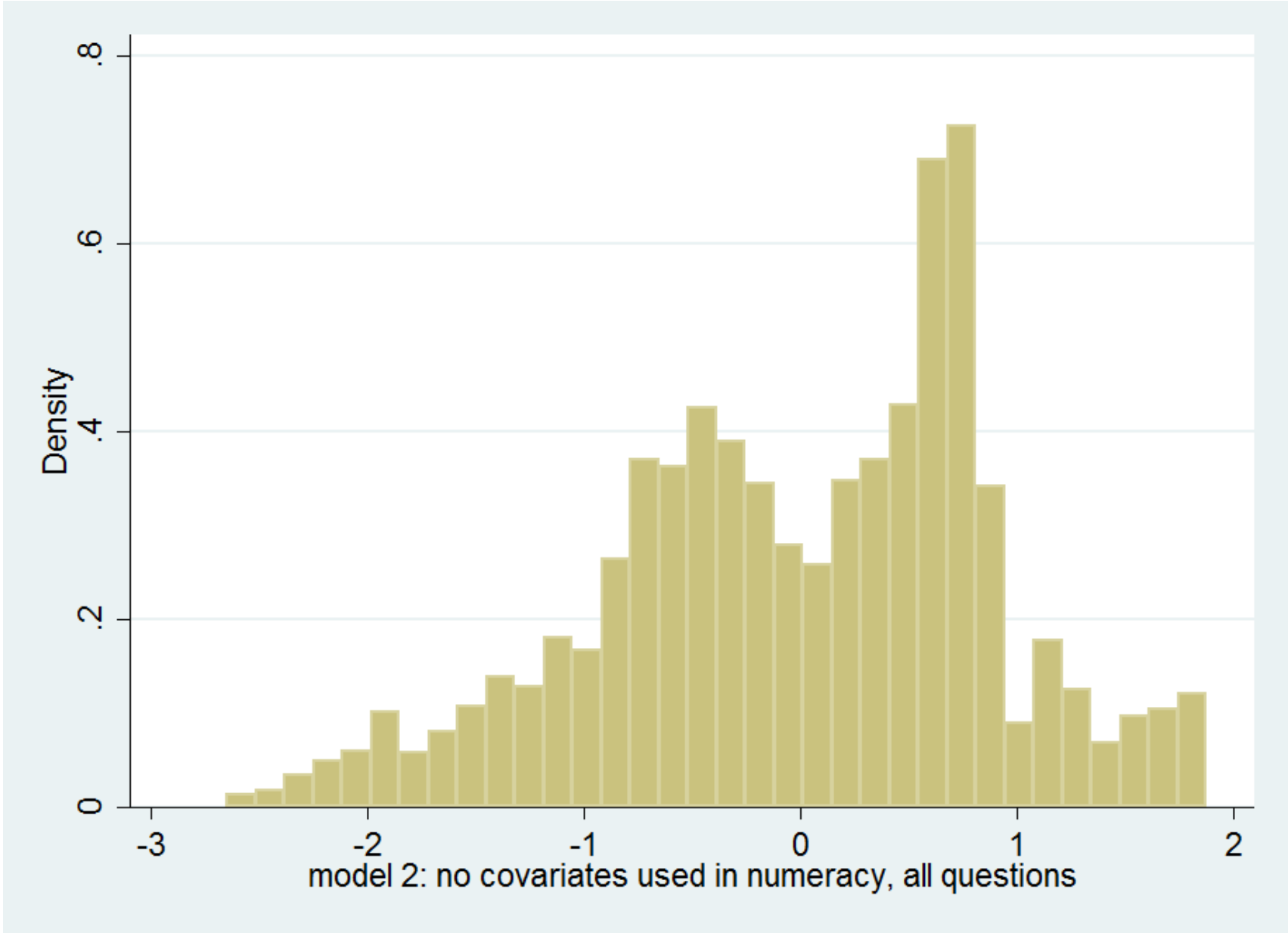
Randomized question format

Allowed “don’t know” for some, not for others

Early placement in survey vs. late (fatigue)

Developed and estimated a model of latent probability numeracy

Normalized to mean 0 and standard deviation of 1.0



Questions most discriminating

- 10 white balls, no red. What is the probability draw is white (red)?
- 7 white, 3 red. What is the probability of white (red)?
- Fair coin comes up head 3 times. What is the probability of next one being tail?
- Chance it rains in your town and Paris are both 10% and independent. If rains in your town, what is the probability of raining in Paris?

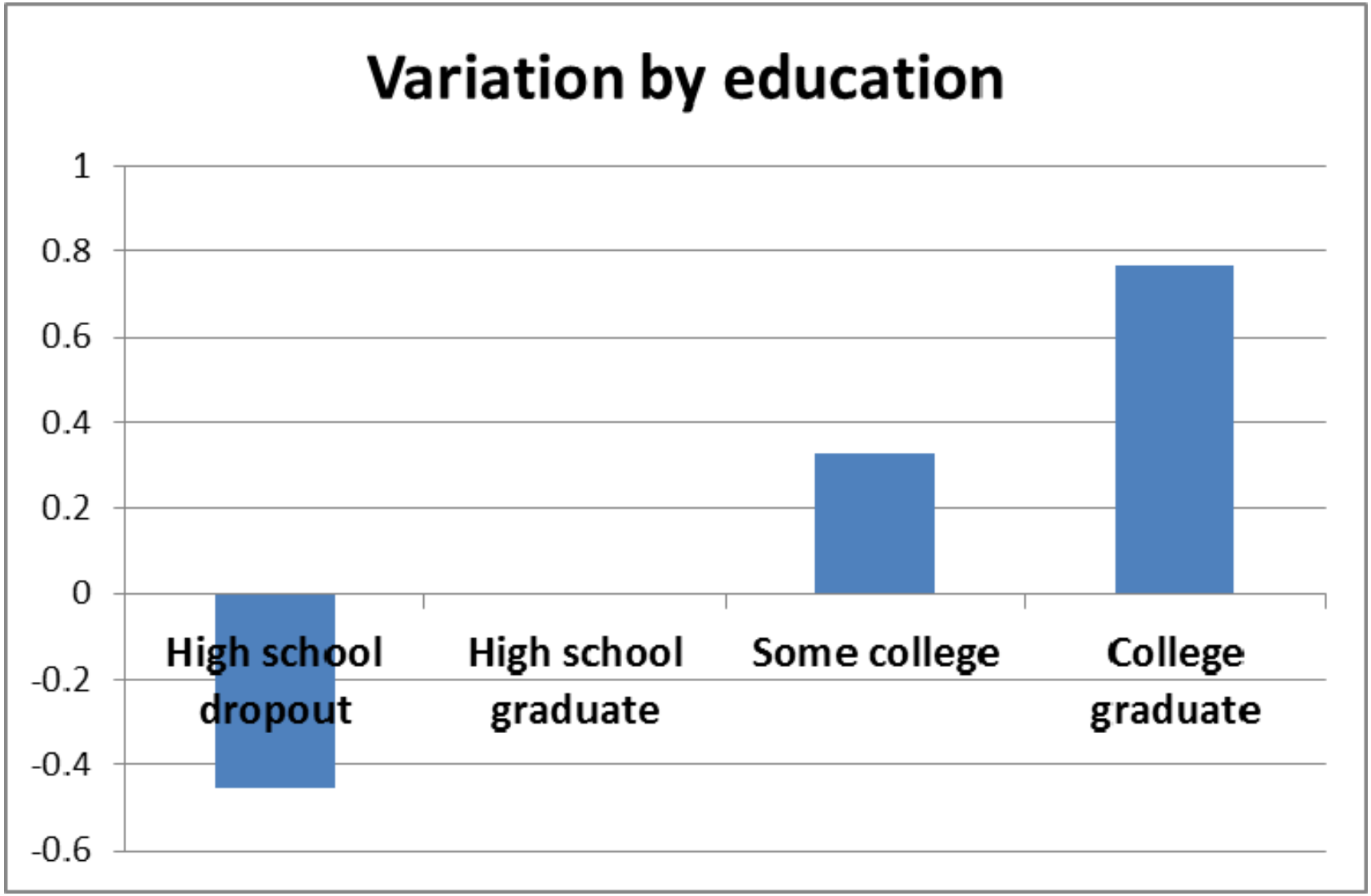
All medium hard questions

Side note: asking earlier in survey increased probability of correct answer by about 0.04.

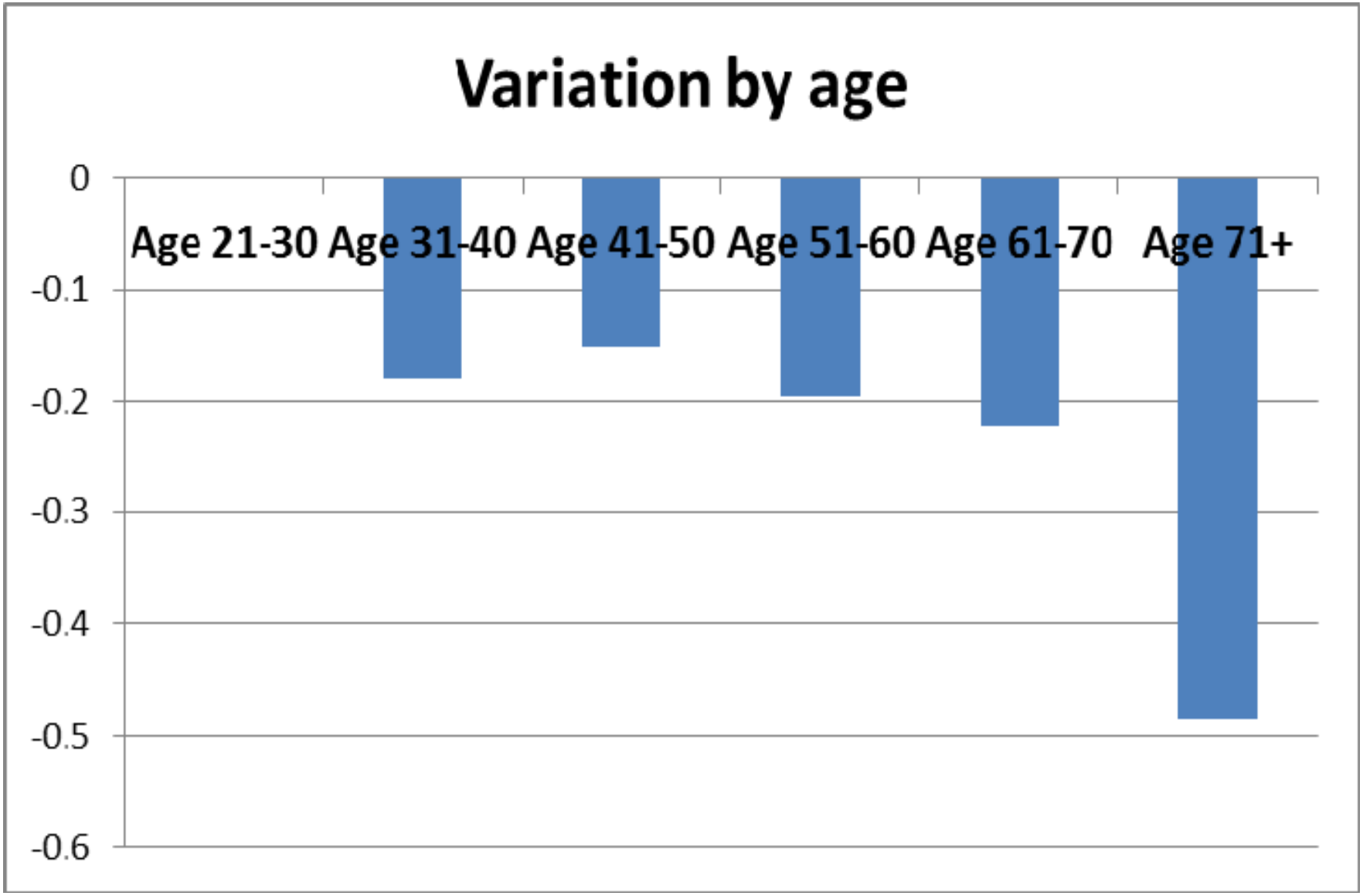
Characteristics of those more probability numerate

Regression of score on

- Sex
- Race/ethnicity
- Education
- Number series score
- Age
- Marital status
- CESD depression score
- Health



Score has mean zero and standard deviattion 1.0



Score has mean zero and standard deviattion 1.0

Probability numeracy score and quality of answers on 63 subjective probabilities assessed up to 61 times
Do the less numerate give lower quality responses?

Indicators of low quality

- Do not conform to laws of probability
  - Not monotonic
  - Sum to more than 1.0
- DK (don't know)
- 50% responses
- Variation over time at the indiviudal level
- Overstate small probabilities

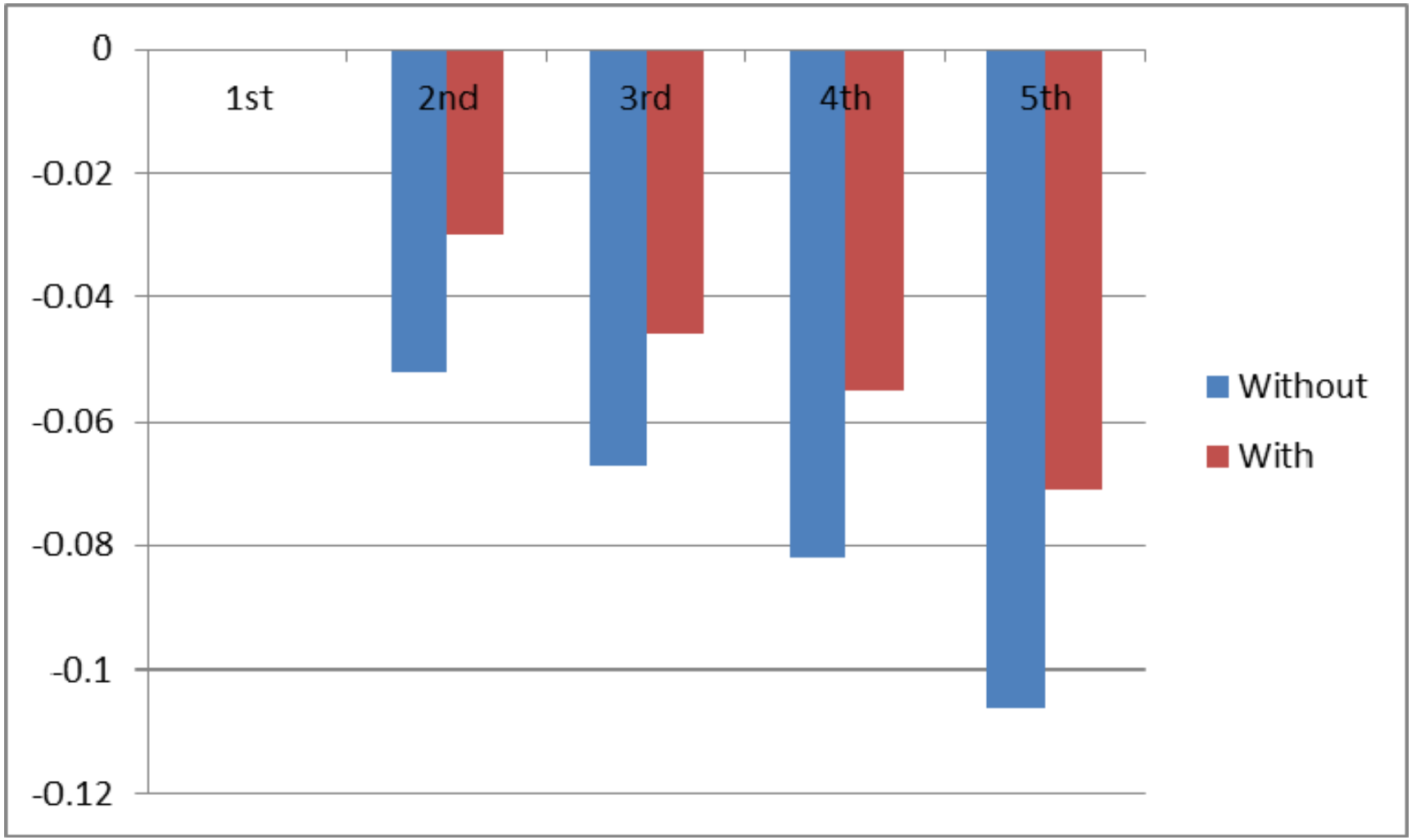
Regressions of these indicators of quality on

1. Probability numeracy only
2. Probability numeracy and personal characteristics

Monotonicity

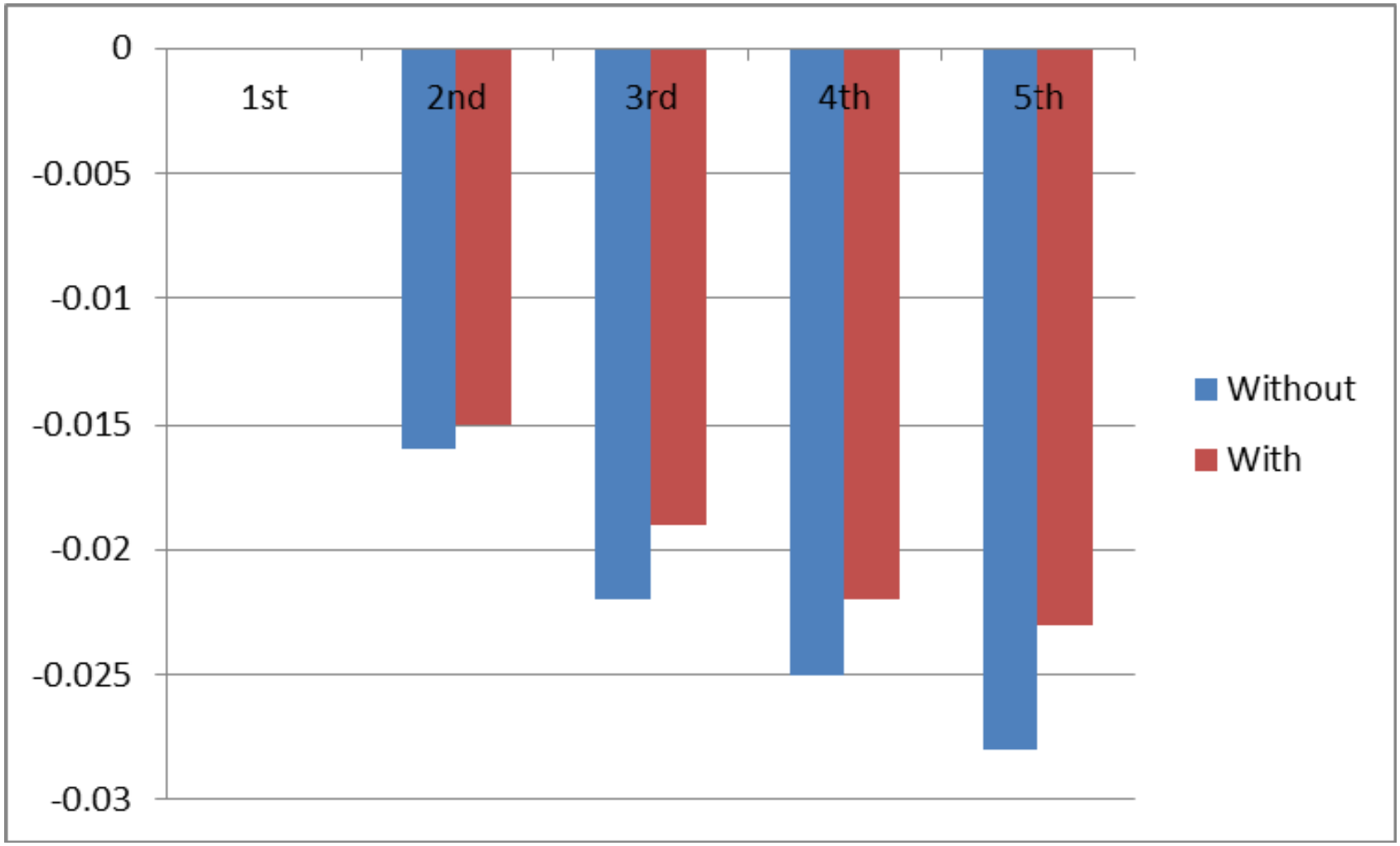
- 13 subjective probability pairs such as
  - Live to age 75 and live to age 85
  - Stock market goes up, and stock market goes up by more than 20%
- Measured up to 61 times over ALP waves
- Fraction of answers with non-monotonic answers
  - Probability survive to 85 > probability survive to 75
  - Probability stock market goes up by more than 20% > Probability stock market goes up

Variation in violation of monotonicity by probability numeracy quintiles without and with covariates



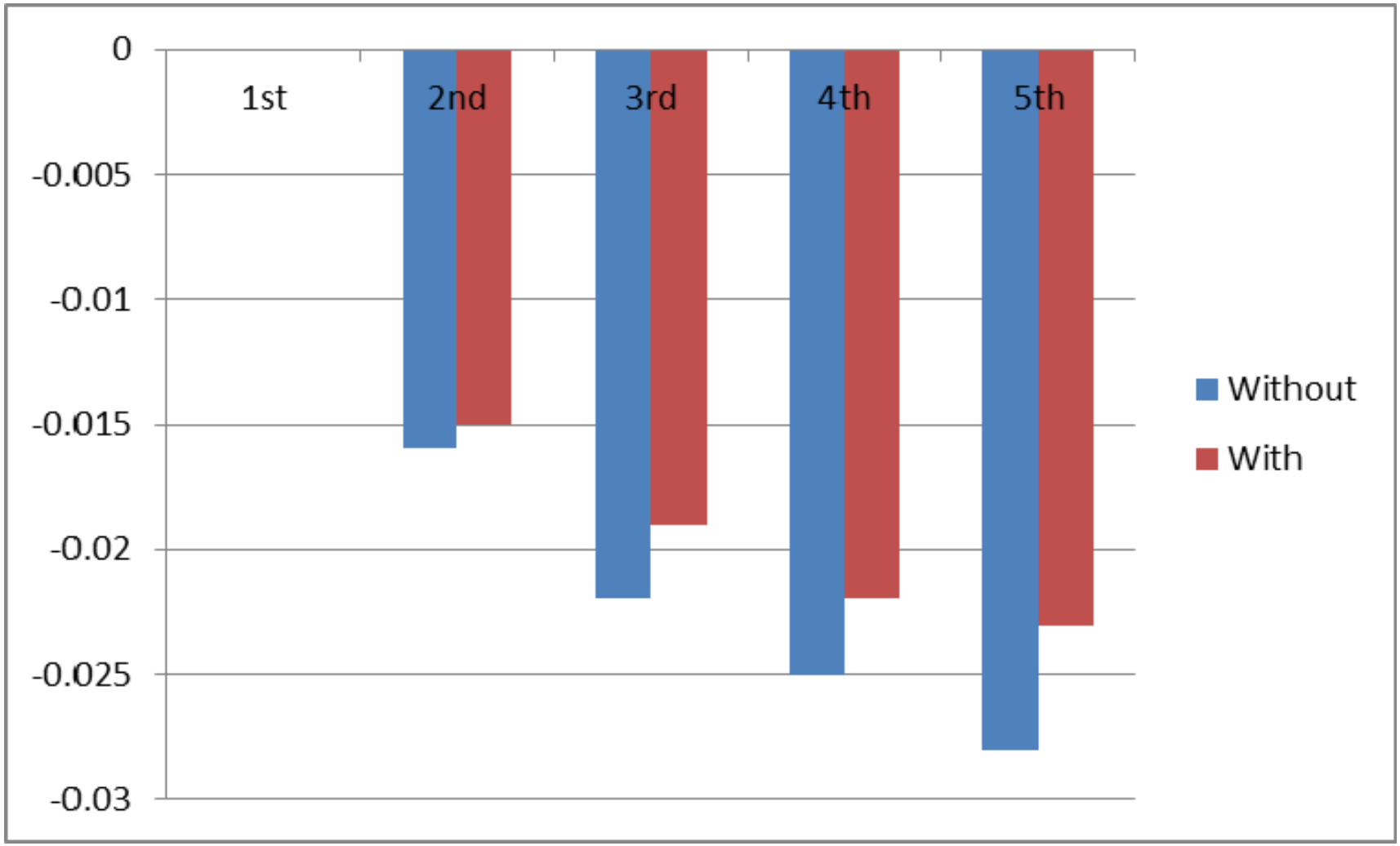
Rate in first quartile: 0.137

Variation in average fraction of DK by quintiles of probability numeracy, without and with covariates



Rate in first quartile: 0.030

Variation in average fraction of 50% responses by quintiles of probability numeracy, without and with covariates



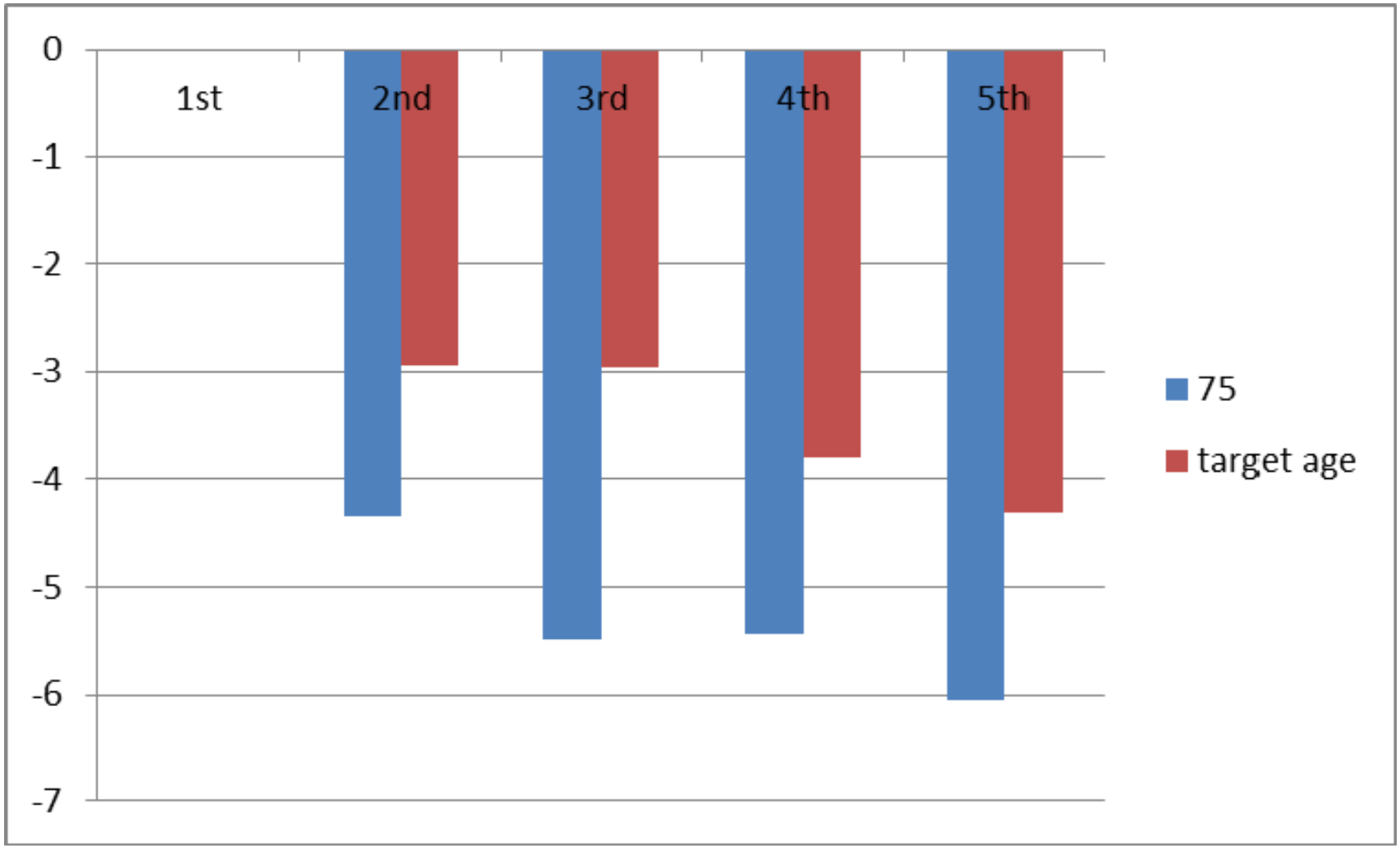
Rate in first quartile: 0.207

To measure excess variation in subjective probability (white noise) calculate standard deviation of subjective probability at individual level over many waves

For example variation in subjective survival to age 75.

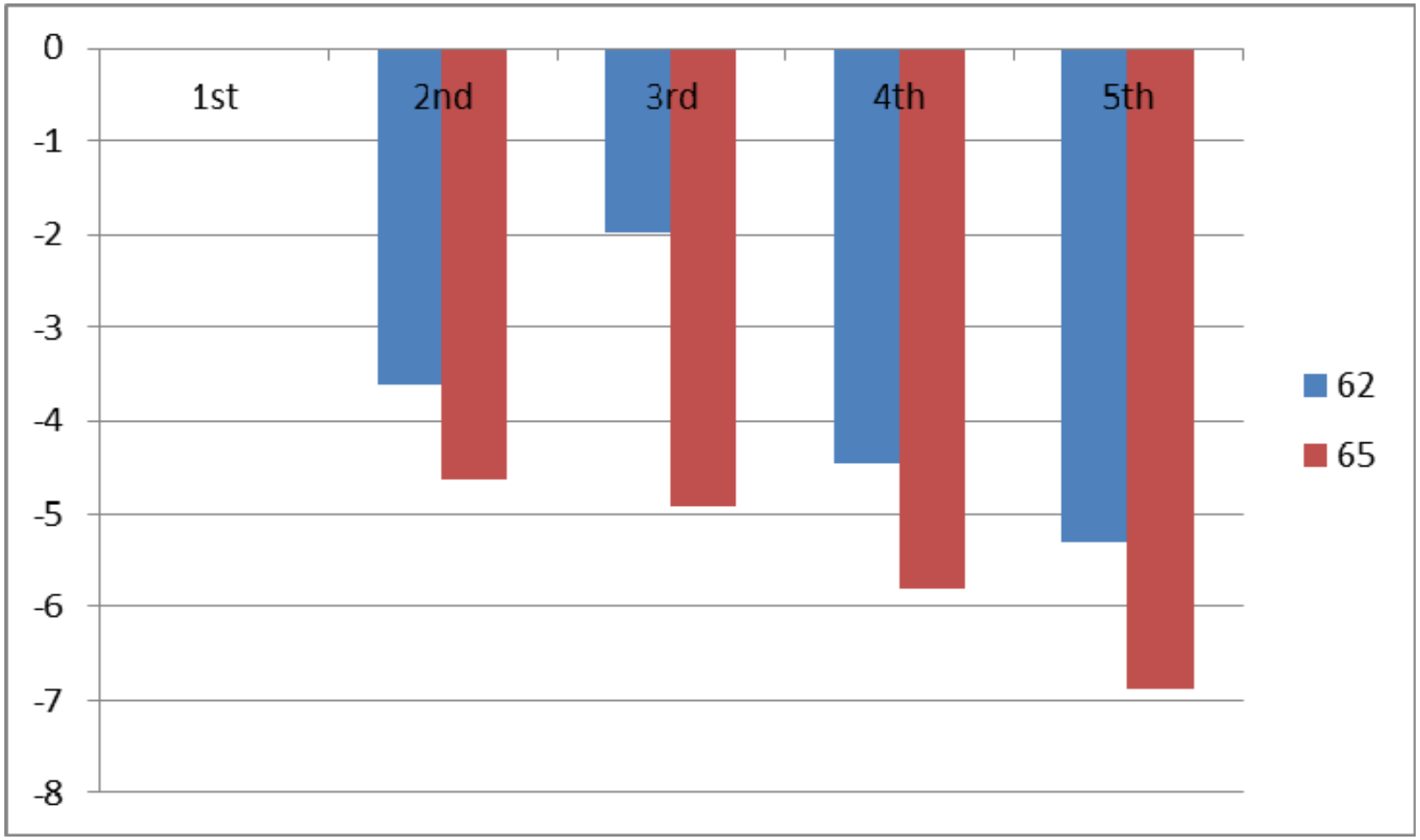
Regression of standard deviation on probability numeracy score and covariates

Variation in standard deviation of subjective survival responses by quintiles of probability numeracy.



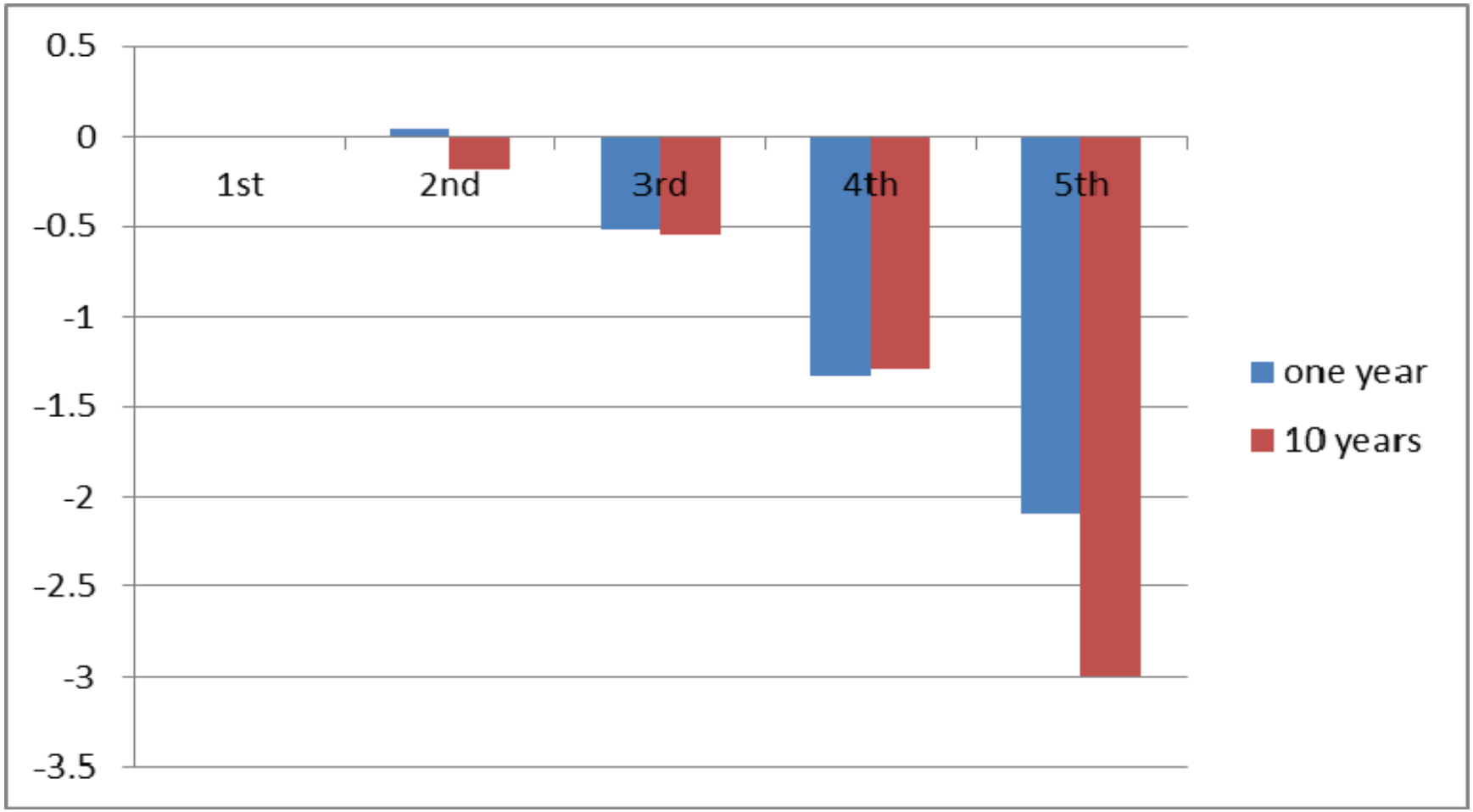
Constant: 16.3

Variation in standard deviation of subjective probability of working past 62 or 65 by quintiles of probability numeracy.



Constant: 14.8

Variation in standard deviation of subjective probability of stock market gains by quintiles of probability numeracy.



Constant: 17.4

Predictive power of subjective probability and its relation to probability numeracy

Whether job loss over 12 months (Y/N) regressed on subjective probability of a job loss, probability numeracy and interaction, without and with covariates

	without	With
Job-loss expectations	0.465	0.433
Probability numeracy	-0.057	-0.038
Probability numeracy X expectations	**0.089**	**0.087**

All coefficients significant

Increase of one standard deviation in probability numeracy increases coefficient on expectations by 0.089

Do more probability numerate people use subjective probabilities better in decision making than the less numerate?

Method

Stated preference for an insurance product

- Those with a higher subjective probability of the (bad) outcome should find the product more attractive.

Insurance in the event of

- Job loss
- Disability (inability to work)
- House value declines
- Stock market declines
- Nursing home
- Longevity: pay off if survive to age 75

Respondents asked to rank insurance policies as

- very good deals
- somewhat good deals
- neither good nor bad deals
- somewhat bad deals
- very bad deals

Two randomizations

- Price
- Introduction that explained that payments were inflation adjusted. Otherwise nothing stated about inflation.

Linear regression of insurance policy assessment (somewhat good or very good), with covariates

	job-loss	disability	Nursing	survival
Numeracy	-0.030	-0.021	-0.065	-0.052
	[0.029]	[0.031]	[0.029]**	[0.025]**
Expectations	0.204	0.143	0.223	0.064
	[0.061]***	[0.058]**	[0.046]***	[0.035]*
Expectations X Numeracy	**0.171**	**0.139**	**0.135**	**0.095**
	[0.057]***	[0.054]***	[0.054]**	[0.033]***
Constant	0.279	0.460	0.120	0.083

Example: one standard deviation increase in numeracy would increase the impact of an increase in expectations from 0.204 to 0.375 (job loss)

Neither expectations nor numeracy have explanatory power for insurance against housing price decline or stock market decline.

	housing	stocks
Numeracy	-0.058	0.003
	[0.033]*	[0.034]
Expectations	0.051	-0.067
	[0.092]	[0.073]
Expectations X Numeracy	0.144	0.081
	[0.102]	[0.074]

Summary and conclusions

Subjective probabilities have been successful in prediction

E.g. predicted increasing labor force in U.S. 60 or older

Less successful in explaining behavior

Perhaps due to heterogeneity in use and expression

- Some fraction of population uses probabilities effectively
- Some fraction does not or cannot express them
- Failing to distinguish amounts to a mis-specificaiton

We presented a measure of probability numeracy

Objective: address heterogeneity in response to subjective probability queries

- Response anomalies are unevenly distributed across people
- Some people don't use or can't express subjective probabilities
  - We need to find out what they use in intertemporal decisions
  - Unlikely to be subjective probabilities as elicited
- But other people express subjective probabilities consistently and use them (at least in stated preferences)

We developed a 13 question battery

Subset of four questions does quite well

Put on other household surveys

Separate population

Use subjective probabilities where meaningful

Use something else (?) where not meaningful