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**LIFE-CYCLE PORTFOLIO CHOICE: THE ROLE OF
HETEROGENEOUS UNDER-DIVERSIFICATION**

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Life-Cycle Portfolio Choice: The Role of Heterogeneous Under-Diversification*

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Abstract

The standard life-cycle portfolio choice model assumes that all agents invest in a diversified stock market index. In contrast recent empirical evidence, summarized in Campbell (2006) suggests that households' financial portfolios are under-diversified and that there is substantial heterogeneity in diversification. In the present paper I examine the effects of heterogeneous under-diversification in a life-cycle portfolio choice model with fixed per period participation costs and progressive social security. Progressive social security has a minor negative effect on participation rates and creates a negative relationship between stock share and permanent income/education. Realistically calibrated under-diversification generates moderate participation rates and conditional stock shares that are comparable with the empirical evidence. Moreover when it is negatively related to wealth or education it restores the positive relationship between income/education and equity shares found in the data.

Keywords: Portfolio choice, life-cycle, bequests, diversification, social security.

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1 Introduction

Traditional life-cycle portfolio choice models with intermediate consumption and uninsurable labor income have typically explored investors' decisions about how to allocate wealth between a risk-free and a risky asset. The assumption common to these models is that all agents face the same risky asset that can be interpreted as a stock index fund. This assumption is contradicted by abundant empirical evidence: studies by Curcuro et al. (2004) and Polkovnichenko (2005) for the US or Calvet et al. (2006 and 2007) for Sweden document that households invest in a limited number of individual stocks or mutual fund shares thus facing substantial idiosyncratic risk on their equity investment. The empirical evidence also suggests that the extent of individual portfolio diversification varies with observable characteristics broadly defined as financial sophistication of which wealth and education are important elements. In the present paper I make a first attempt at exploring the effects of portfolio under-diversification on household life-cycle asset allocation. I build on standard models like for example the ones presented in Campbell et al. (1999) and Gomes and Michaelides (2005) by constructing an economy populated by households that face a stream of uncertain and uninsurable earnings and borrowing and short sale constraints. These agents solve an optimal consumption-saving problem and a savings allocation decision between a risky and a risk-free asset. The key departure from the traditional framework is the assumption that agents are heterogeneous in their ability to construct a diversified portfolio. I achieve this in a reduced form by postulating an exogenous relationship between wealth or education/earning ability and the variance of the agent's stock portfolio and consistent with the empirical evidence I assume that the variance of equity investment is declining in the agent's wealth or education.¹ I also extend the basic model to include two more features that characterize the real world environment where agents make their financial decisions and that interact in interesting ways with heterogeneity in portfolio diversification. These two features are a fixed per period participation cost and the assumption that the replacement ratio of social security benefits is declining in the agent's past average earnings, a feature of the US — as well as of the other industrialized countries — social security systems.

The main results can be summarized as follows. First the per period cost of participating in the stock market is needed to obtain exits, a fact well documented by Vissing-Jørgensen (2002) and not achievable with the more standard assumption of an initial fixed entry cost. This assumption also generates a

¹As it will become clear when I describe the model I assume there are two types of agents defined by permanent differences in earning abilities. Since these can be interpreted as the result of differences in education from now on I will use the terms education, earning ability and permanent income interchangeably to identify the two groups.

hump shaped life-cycle profile of participation rates. Second, the introduction of a progressive formula for social security benefits has an important effect on conditional shares. In particular under such assumption it is optimal for lower permanent income households to invest a larger share of their wealth in the stock market than for high income households. To my knowledge this result is new and has not yet been incorporated in the investment strategies proposed by financial advisors. As it will become clear later a caveat applies here since the result depends on the assumption that all agents invest in an equally diversified stock fund. At a positive level the result runs against the empirical evidence suggesting the existence of a new asset allocation puzzle. The progressive social security formula also has a second effect: by inducing low income agents to hold little or no wealth at all it magnifies the impact of the fixed per period cost on participation rates. When both are realistically calibrated though their joint effect is too weak to bring average participation rates close to their empirical values. Third and most importantly the assumption of heterogeneous under-diversification of investors' stock portfolios has important implications for both participation rates and conditional shares. In particular the higher variance of stock returns faced by many agents reduces the participation rates substantially. The same mechanism also reduces conditional shares. The assumption thus provides an alternative explanation for the low participation rate and conditional shares to the one proposed in Gomes and Michaelides (2005) and based on Epstein-Zin preferences with heterogeneous risk aversion. Interestingly, because the ability to diversify is increasing with wealth or education, the reduction in conditional shares is stronger for households with lower economic status. Under the proposed calibration the effect is sufficiently strong to overturn the effect of progressive social security so that households with higher education/permanent income now want to invest larger shares in stock than the rest of the population, consistently with the empirical evidence. Finally, the life-cycle model fails along one dimension, that is, the life-cycle profiles of conditional stock shares are declining early in life. However the introduction of a bequest motive coupled with the assumption that bequests are indeed passed to the heirs allows the more complete model to also generate profiles of conditional shares that are virtually independent of age.

The rest of the paper is organized as follows. In the next subsection I briefly describe the literature on household portfolio allocation, both theoretical and empirical, that is most closely related to the present paper. In Section 2 I present the description of the life-cycle model; there I will also devote a subsection to the description of the empirical evidence about portfolio under-diversification that motivates the key assumption made in the paper. In section 3 I describe the choice of parameters, in section 4 I describe the results of the quantita-

tive analysis and in section 5 I present some brief conclusions. Finally two appendixes describe the details of the model with bequests and the numerical solution method used.

1.1 Related Literature

Starting from the nineties and possibly in response to the important changes in financial choices of American families a large literature has developed to study the issue of portfolio choice both empirically and theoretically.² In the empirical field, works by Poterba and Samwick (1997), Heaton and Lucas (2000), Bertaut and Starr-McCluer (2000) and Ameriks and Zeldes (2004), beside documenting the rise in stock market participation rates that occurred since the early nineties, have described a number of stylized facts about household portfolios in the US. These can be summarized as follows. First, despite the size of the equity premium and even after the recent surge, the participation rate is still only about 50 percent. Second, the participation rate is increasing in wealth and hump-shaped in age.³ Finally the share of stocks conditional on participation is roughly constant in both age and wealth. These findings for the US economy extend to a number of other industrialized countries like the U.K., Italy, Germany and the Netherlands as reported in the country studies presented in the volume edited by Guiso et al. (2001).

At the theoretical level the seminal work by Samuelson (1969) and Merton (1971) pointed to some key properties of portfolio decisions. Samuelson (1969) considered the problem of an agent with no labor income, power utility and facing i.i.d. returns in a complete and frictionless market and found out that the optimal share of the risky asset is independent of wealth and the horizon. Merton (1971) extended this result to the possibility of a constant labor income stream that can be fully capitalized and concluded that in this case the share of risky assets is constant in total — human plus financial — wealth implying that as the agent ages and his residual human wealth declines he should reduce his exposition to stocks as popular financial advisors suggest. In more recent times the advances in computational methods and computing power allowed scholars to solve models with realistic labor income risks and borrowing and short sale constraints thus merging the portfolio choice and precautionary saving framework. Among the many works produced in this framework are those of Heaton and Lucas (1997 and 2000) and Haliassos and Michaelides (2003) who consider infi-

²Two useful surveys of work done and open questions in this area are Guiso, Haliassos and Jappelli (2001) and Curcuru, Heaton, Lucas and Moore (2004).

³The relationship between participation and age is somewhat sensitive to the estimation procedure used. In particular when cohort effects are included it tends to be increasing. See Ameriks and Zeldes, (2004).

nite horizon problems and of Campbell, Cocco, Gomes and Michaelides (1999), Cocco, Gomes and Maenhout (2005), Gomes and Michaelides (2005) who look at finite horizon problems. These papers have delivered a number of interesting predictions. First, with an empirically plausible low correlation between labor earnings shocks and stock returns households would enter the stock market first and then diversify towards bonds only as their wealth grows. Second, as a consequence non participation can be justified only by adding some frictions in the form of fixed participation costs. Third, in a life-cycle setting, they predict that the share of stocks should be declining with age until retirement and then increasing again. The basic model has been modified in many directions in order to bring its predictions closer to the data or to consider a richer structure of assets or return processes. Lynch and Tan (2004) and Benzoni, Collin-Dufresne and Goldstein (2004) considered a cyclical and long-term correlation between stock returns and labor earnings respectively, Gomes, Michaelides and Polkovnichenko (2003) and Dammon, Spatt and Zhang (2003) studied the impact of differential tax treatment between assets; Davis, Kubler and Willen (2002) studied the impact on stock investment of the possibility of borrowing at a rate higher than the lending rate; Cocco (2004) and Yao and Zhang (2005) considered a more complex model where housing is added to bonds and stocks in the menu of assets available to the investor; finally a number of authors among whom Campbell and Viceira (1999) studied the impact of return predictability on stock demand.

A common feature of all of these models is the assumption that all investors face the same risky asset that can be thought of as a stock index fund. This assumption is at odd with reality since abundant empirical evidence documents that households invest in a limited number of individual stocks and stock mutual funds. The present paper is most closely related to the basic finite horizon models mentioned above. Its main goal is to explore the consequence of under-diversification of households' stock portfolios in that basic framework. Given the key importance of this assumption I will mention with some more detail the literature that documents this fact in a separate later sub-section.

2 The Model

In this section I will describe the model. The key results are obtained under the assumption of life-cycle agents, however an extension to the case when households have bequest motives and estates are actually passed to the descendant is also considered. Since the actual transmission of bequests coupled with the assumption of true altruism make the description of the model quite more complicated I will present the basic life-cycle model in the main text and leave the description of the full model to the appendix.

2.1 Demographic Structure

Time is discrete and the model period is assumed to be one year. I let a be the number of periods an agent has spent in the model and t be calendar year, although the distinction will become relevant only in the model with bequests. Agents enter the model as workers at age 21 so that real-life age is equal to model age plus 20. Every agent can live up to a maximum of $A = 69$ periods, corresponding to age 89. I allow for uncertain life-span by assuming that in every period there is a positive probability $1 - p_{a+1}$ that the agent dies. All agents retire after G periods of life in the model provided they have not died before; the value of G is chosen so that agents retire at real age 65.

2.2 Preferences

Agents value consumption but not leisure. Period utility is defined by a standard utility index $u(c_{i,t})$ and discounted at the rate β . Agents may also receive utility from the estate they leave to their descendant upon death. In that case I assume that they are truly altruistic so that they value the indirect utility the descendant will receive from enjoying the bequest. I denote with γ the further discount applied to the descendant's utility. A detailed description of the model when $\gamma > 0$ so that agents value leaving a bequest is deferred to the appendix.

2.3 Labor and Retirement Income

Investor's i labor income after a periods of life in the model is given by:

$$\log(y_{i,a,t}) = \theta_i + f(a) + z_{i,t} \quad (1)$$

for $a < G$. This formulation implies that there are three components that determine individual earnings. The first component denoted with θ_i is specific to the individual and fixed for the entire life time; it can be thought of as representing his earning ability as determined by education and other factors like genetics or the environment. The second component $f(a)$ is a deterministic function of age that is common to all individuals and is meant to capture the hump in life-cycle earnings that is observed in the data. Finally there is an idiosyncratic component $z_{i,t}$ which is assumed to follow an autoregressive process given by:

$$z_{i,t} = \rho z_{i,t-1} + v_{i,t} \quad (2)$$

and $v_{i,t} \sim N(0, \sigma_v^2)$ and independent over time.

After retirement the agent receives a pension benefit $b(\theta_i, z_{i,G})$ that depends on his permanent earning type and the earnings shock in his last period of

working life.⁴ This choice allows the model to capture some elements of the progressive US pension system without adding a further state variable.

The general notation for household income will be $Y_{i,a,t}$ where:

$$Y_{i,a,t} = \begin{cases} e^{\ln(y_{i,a,t})} & \text{if } a \leq G \\ b(\theta_i, z_{i,G}) & \text{if } a > G \end{cases} \quad (3)$$

2.4 Financial Assets

In the economy there are two assets in which the agent can invest. First a one period risk-free bond with price q and return $R_f = 1/q$. Second a risky asset called “stock portfolio” with return denoted $R_t(w)$ and defined by the equation:

$$R_{t+1}(w_{i,t}) - R_f = \mu + g(w_{i,t})\varepsilon_{t+1} \quad (4)$$

where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ is an i.i.d. innovation and μ is the expected excess return of the stock investment. Here $g(\cdot)$ is a function that satisfies the following two properties: first $g'(w_{i,t}) \leq 0$ and $\lim_{w_{i,t} \rightarrow \infty} g(w_{i,t}) = 1$ where $w_{i,t}$ is the wealth of agent i at time t . Its effect is that the variance of the return to the stock portfolio will be $g(w_{i,t})^2 \sigma_\varepsilon^2$ which as it can be seen is potentially wealth dependent. If we assume that $g'(\cdot) = 0$ over the whole range of possible wealth levels then we are back in the standard case in which all agents face the same return process on their stock portfolio, otherwise the model allows the variance of the stock investment to be declining in the agent’s wealth. In an alternative specification it will also be considered the case in which the variance of the return premium on the risky financial asset depends on the permanent income type, that is:

$$R_{t+1}(\theta_i) - R_f = \mu + g(\theta_i)\varepsilon_{t+1} \quad (5)$$

with the value of $g(\theta_i)$ being larger for low earning types. If we interpret earning types as education this would imply that more educated people hold better diversified portfolios. Since in both cases this is a key mechanism in driving the model results, an extensive discussion of the evidence in support of this assumption will be carried out in a separate subsection at the end of the model description.

The amount of bonds and stocks that household i holds at time t is denoted with $B_{i,t}$ and $S_{i,t}$ respectively and it is assumed that

$$B_{i,t} \geq 0 \quad (6)$$

$$S_{i,t} \geq 0 \quad (7)$$

⁴In the variable $z_{i,G}$ for simplicity but with some abuse of notation I use the second subindex to denote age rather than time.

meaning that the investor is prevented from borrowing against future labor income or retirement wealth and from selling short stocks.

Participation in the stock market requires payment of some costs. I allow for the possibility of two different costs. First as many authors have previously done I introduce an initial entry cost F_I that must be paid the first time one invests in a stock portfolio. This cost can be thought of as the cost needed to gather the initial information about the stock market in general; given its nature it creates the need for a new state variable in the problem.⁵ I denote this new state variable as $I_{F,i,t}$ where $I_{F,i,t} \in \{0, 1\}$. A value of the index equal to one means that the cost was paid before and a value of zero means that the cost was not paid before. Equation (8) below formalizes the fact that this initial information cost is paid only once. Second I allow for the possibility of a per-period participation cost, denoted F_p that must be paid in any period if the agent decides to invest in the stock market. This cost does not introduce state dependence and may be interpreted as extra time cost of filling tax forms or the monetary cost of brokerage fees.⁶ The index $I_{P,i,t}$ is used to denote payment of this cost if it takes the value of one or not payment and therefore not participation in the stock market if it takes the value of zero. With the notation for participation in the stock market given above we can write the following law of motion:

$$I_{F,i,t+1} = I_{F,i,t} + (1 - I_{F,i,t})I_{P,i,t} \quad (8)$$

that describes the evolution of the state variable used for payment of the initial entry cost.

2.5 The Investor's Optimization Problem

In this subsection I will describe the optimization problem for a life-cycle investor. The Bellman equation for an age a , type θ agent is given by:

$$V^{a,\theta}(w_t, z_t, I_{F,t}) = \max_{c_t, B_{t+1}, S_{t+1}, I_{P,t}} \{U(c_t) + \beta p_{a+1} V^{a+1,\theta}(w_{t+1}, z_{t+1}, I_{F,t+1})\}. \quad (9)$$

In the equation above w_t is the amount of financial resources available to the agent at the beginning of the period. As discussed in the previous subsections z_t is the persistent component of labor earnings and $I_{F,t}$ is the index representing payment of the entry cost. The maximization is performed with respect to consumption and the amount of bonds and stocks the agent buys to carry to the next period. Clearly if the agent buys a strictly positive amount of stocks,

⁵See for example Campbell et al. (1999) and Gomes and Michaelides (2005).

⁶See Vissing-Jørgensen (2002 and 2003).

that is, if $S_{t+1} > 0$, then the index $I_{P,t}$ that denotes payment of the per period participation cost takes the value of 1 otherwise $I_{P,t} = 0$. The maximization is performed under the constraints (6) and (7), the law of motion of the index of payment of first time entry cost (8) plus the budget constraint and the law of motion of financial resources described by equation (10) and (11) or (12) below. The budget constraint reads:

$$c_t + qB_{t+1} + S_{t+1} \leq w_t + Y_{a,t} - I_{P,t}F_P - (1 - I_{F,t})I_{P,t}F_I. \quad (10)$$

The resource constraint states that the expenditure in consumption, bonds and stocks cannot exceed the sum of financial wealth and income from labor or social security net of payment of the costs of participating in the stock market if the agent decides to do so. In turn these costs are equal to $F_I + F_P$ if participation occurs for the first time in the agent's life and it is F_P if the agent had participated before. Labor and pension income $Y_{a,t}$ are given by equations (1), (2) and (3). The law of motion of financial resources is given by the following equation:

$$w_{t+1} = B_{t+1} + R_{t+1}(w_t)S_{t+1} \quad (11)$$

that simply states that financial resources at time $t + 1$ are equal to the sum of the realized return on bonds and stock portfolios. This same equation will take the alternative form

$$w_{t+1} = B_{t+1} + R_{t+1}(\theta)S_{t+1} \quad (12)$$

when the case in which diversification is related to permanent income/education will be considered.

2.6 Discussion

In this subsection I discuss one of the key features of the model, that is, the assumption that agents invest in stock portfolios that are not fully diversified and that the level of diversification improves with economic status. I will briefly summarize the evidence supporting this hypothesis and then I will describe how this is used in the specific formulation of the model.

Documenting under-diversification in households' portfolios has proven difficult given the very strong requirements on the quality of the data needed to perform this type of analysis, yet in recent years substantial empirical evidence that points to under-diversification in stock portfolios as a widespread phenomenon has emerged.⁷ Curcuru et al. (2004) use the Survey of Consumer Finance; they label under-diversified those households with more than

⁷See Campbell (2006) for a description of the features that the ideal dataset needed to study household portfolio diversification should possess.

50 percent of their equity holdings in a brokerage account with fewer than 10 stocks and find that between 13 and 30 percent of households, depending on the issue of the dataset, meet this criterion. Also based on the SCF is the evidence reported in Polkovnichenko (2005). The author sorts households into wealth quintiles and then computes for the subset of equity holders the median share of directly held stocks and finds that it is declining over the wealth groups except at the top. He then computes the median number of directly held stocks for direct stock holders and documents that while in the bottom wealth groups this ranges between 1 and 2 in the top quintile it is 15 in most issues of the survey. The two findings jointly suggest that wealthier households hold better diversified stock portfolios. While the SCF has very detailed information on households' assets, still it lacks detailed information about allocations among asset classes within mutual funds and retirement accounts, moreover it does not report information about individual assets within each class so that it is not possible to measure the performance of individual portfolios. These shortcomings motivated the explorations of stock trade in specific retirement plans and brokerage houses to study the portfolio diversification of households. As an example Goetzmann and Kumar (2004) use the records of a large brokerage house to compute different measures of diversification and the Sharpe ratios for individual investors. Some of their findings most relevant for the present study are the following: while about half of the investors hold fewer than 3 stocks, about 5 percent hold 10 or more stocks. The differences in diversification persist when they look at portfolio variances which vary by a factor of more than two. Interestingly the average correlation among stocks in a given portfolio turns out to be independent of the number of stocks suggesting the better diversification occurs through larger number of stocks rather than better stock picking ability. Indeed stock picking ability seems constant across the 5 years considered in the study. Clearly the larger variance of portfolios with fewer stocks could be compensated by higher average returns, however the reverse happens in the dataset under study. This implies that there are large differences in risk-adjusted portfolio performance, however even the best diversified portfolios do not outperform the stock market index. Finally the two authors find that larger wealth and better education as proxied by occupation predict better diversification. The main limitation of this study is that the sample used in neither representative of the whole population nor does it cover the whole portfolio.⁸ Other interesting evidence about stock portfolio diversification is provided by the 1999 and 2002

⁸The two authors find that results are robust to a procedure to account for non representative sample. With respect to the issue of partial observation of investors' portfolio they find that substantial non-diversification would persist even if the money invested in the account represented only half of the portfolio with the rest invested in a diversified fund.

surveys “Equity Ownership in America” conducted by the Investment Company Institute. The survey asks a representative random sample of the US population of stockholders plus a high net worth sample a number of questions about their equity holdings and transactions plus other information like age, education, marital status, income and financial assets. The 2002 survey finds that the median number of individual stocks owned was 2 for stock holders with less than 25000\$ of financial wealth and 8 for those with more than 500000\$; the same pattern of increasing diversification was found when looking at the number of stock mutual funds and of both types of equities jointly. Finally the same pattern emerged when participating households were sorted by income and in the other survey year. Overall this last piece of evidence confirms that also when considering all financial wealth of a representative sample, under-diversification remains a widespread phenomenon and that it becomes less strong with higher income and wealth.

Finally, the most extensive study of household portfolio diversification is carried out by Calvet et al. (2006, 2007). Calvet and his co-authors exploit a panel of data collected by the Swedish government’s statistical agency, Statistics Sweden, covering the whole population and collecting, beside information on various demographic variables, a detailed description of households’ assets that include individual assets within each class. They find that idiosyncratic volatility is an important part of households’ portfolio volatility and that there is a wide dispersion in both portfolio volatilities and return losses caused by under-diversified investment. They also find that variables like wealth and education predict better diversification of risky portfolios although the total dollar cost of under-diversification is higher for better diversified households due to more aggressive investment strategies. Finally they find that once predicted under-diversification is taken into account the cost of non participation in the stock market is substantially reduced for those that actually stay out of it.

Summarizing, the studies presented above suggest a number of facts. First households do not invest in fully diversified index funds but choose portfolios made of a limited number of individual stocks and mutual fund shares. Second, there is a wide dispersion in the volatility and return performance of households’ portfolios. Finally, the extent of portfolio diversification depends on a number of household characteristics, with financially more sophisticated households as defined by among else larger wealth and better education, being better diversified. Taken together the evidence reported above motivates the assumption made in the present paper that stock investment does not attain for most households the historical performance of the stock market index. While in reality individual household portfolios differ in both average and standard deviation of returns, for the sake of simplicity in the model I represent hetero-

geneous under-diversification by assuming differences in the variance of returns only. Also for the same reason, in relating the variance of the return on the stock portfolio to observable characteristics I consider only one possible determinant at a time: wealth in the baseline model, education/permanent income as a sensitivity analysis.

3 Parameter Calibration

In this section I describe the choice of the model parameters used in the simulations. Most of the parameters are taken from other studies while a few are chosen so as to match some key targets taken from US data.

3.1 Preference Parameters

Preferences in the model are defined by three parameters. First the period utility index is of the standard iso-elastic form $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and the coefficient of relative risk aversion σ is set to 7 a value in the range normally chosen in this literature. The other two parameters are β and γ , the discount factor on own and descendant's utility. I consider two possible values of γ : first I set it to 0 to consider the life-cycle model, then I set it to 0.1 when I extend the model to include bequests. This value is chosen because it generates a reasonably small ratio between the flow of bequeathed wealth to total wealth in any period: more precisely the model generated ratio is 1.8, close to the estimate of 1.4 presented in Gale and Scholz (1994).⁹ The value of β is instead determined endogenously so that once all the other parameters are given, the average wealth-earnings ratio in the population is 5, a value taken from the estimates in Budría-Rodríguez et al. (2001) and Díaz-Giménez et al. (1997). Targeting the wealth-to-earnings and bequest-to-wealth ratios is important because the profile of financial wealth and its ratio to human wealth over the life-cycle are key determinants of the decision to participate and on optimal stock shares respectively.

⁹The ratio targeted here corresponds to the sum of bequests and inter-vivos transfers reported by Gale and Scholz. This seems more appropriate than targeting the bequest to wealth ratio because in the current model all intergenerational transfers occur through bequests. Unfortunately even if one sets $\gamma = 0$ the model ratio exceeds the target. The reason is that in real life a large part of bequests are left by the last surviving spouse to the descendants. Often this is the female in a couple who usually dies later than the male. Here the structure of the household is not modeled and survival probabilities are taken from the male mortality tables, so that bequests are left somewhat earlier in the life cycle, thus tend to be larger than in reality.

3.2 Labor Income Process and Pensions

In order to fully characterize the labor income process we need to specify two different sets of parameters. First I fix the function $f(a)$ that describes the deterministic life-cycle profile of earnings. This is taken from the profile estimated by Cocco, Gomes and Michaelides (2005) for high-school graduates; when aggregated over five year periods the profile is also consistent with the one estimated by Hansen (1993) for the general population. Second we need to specify permanent earnings differences and the stochastic process that determines the yearly evolution of household earnings. To do that, first I fix the standard deviation of the innovation $v_{i,t}$ to the value of 0.025 which is consistent with the different estimates available for AR(1) processes of earnings.¹⁰ Then I fix the permanent component of individual earnings: I assume that θ_i can take two values and choose them so as to match the Gini index of earnings for first year workers. Finally I set ρ , the autocorrelation coefficient of the AR(1) process of earnings to 0.97 so that I can match the Gini index of earnings in the general population. Interestingly and reassuringly the resulting autocorrelation coefficient is very close to the ones directly estimated on PSID data by Hubbard et al. (1994) and Storesletten et al. (2004).

An important issue is the calibration of the social security system. This is because in the US economy replacement ratios used to compute retirement benefits are progressive. For this reason agents with high earnings will need to accumulate more wealth relative to their earnings to finance retirement consumption when compared to low earners. In order to perform the calibration I proceed in two steps. First I compute the average life-time earnings conditional on an agent's type and last year of work earnings. This forms the base used to compute the pension benefit the agent receives during retirement. Second the formula used in the US economy is applied to this average lifetime earnings.¹¹ This formula fixes two bend points at 0.20 and 1.24 times average earnings and attributes a benefit that is 90 percent of earnings up to the first bend point, 32 percent from the first to the second and 15 above that.¹² Retired households also receive social security payments in the form of medical and hospitalization benefits that are independent of their earnings history, so that I also add a fixed component to the benefit and set it approximately equal to 19 percent of average earnings, a value consistent with the one reported in Huggett (2000). To

¹⁰See Hubbard, Skinner and Zeldes (1994) and Huggett and Ventura (2000).

¹¹This calibration method is not perfect since in general two different agents of the same type that receive the same earnings shock in the last period of working life will have different past earnings histories and therefore also different average lifetime earnings. This method though is the best that can be done without adding a further state variable to keep track of average past earnings.

¹²See Huggett and Ventura (2000) or Social Security Online (2004).

understand the implications of the progressive formula of social security benefit I also consider the case of constant replacement ratio fixed at the average level of the US economy.

3.3 Asset Returns and Participation Costs

I assume that the constant return to bonds R_f is 2% and that the average equity premium is 4% a value that is somewhat below the historical one but is the one commonly used in this literature (e.g. Campbell et al. (1999), Cocco (2001) or Gomes and Michaelides (2005)). As far as the standard deviation of the risky return is concerned I fix its base value at 16% a value consistent with the historical evidence about the volatility of the stock market index. While this value will be used in some initial simulations in others I will explicitly recognize the fact that households typically differ in the degree of diversification of their stock portfolios and that this improves with, among else, wealth and education. In order to calibrate the variation in the variance faced by different households I will use the figures reported in Goetzmann and Kumar (2004) and especially Calvet et al., (2006, 2007). Goetzmann and Kumar report a ratio between the standard deviation of portfolios with 15 or more stocks and that of portfolios with 2 stocks of 1.74. While this may be indicative of the order of magnitude of differences in portfolio variances it is affected by the fact that their data consider only individual stocks in a single brokerage account and not the whole portfolio of the sampled agents. On the other hand Calvet et al. (2006) report the distribution of the standard deviation of risky portfolio – that includes by definition individual stocks as well as stock mutual funds — of a representative sample of Swedish households. They find a median standard deviation of 20.7 %, a ratio between the 75th to 25th percentile of standard deviations of 1.43 and a ratio between the 90th and the 10th percentile of 2.53.¹³

In the baseline model, where diversification is assumed to depend on wealth, I postulate the following function:

$$g(w) = 1 + \frac{1}{1 + e^{\delta(w-\bar{w})}} \quad (13)$$

and set $\bar{w} = 30$ and $\delta = 0.2$ implying that agents at the borrowing constraint face almost twice the volatility of the stock market index and that this volatility is approximately reached around four times average wealth. This implies that the ratio of the largest to smallest observed standard deviation of the stock portfolio return in the model is somewhat below 2. In light of the numbers presented above this seems to be a reasonable estimate of the difference in

¹³These numbers are based on Table 4 in Calvet et al. (2006).

stock portfolio volatility. For robustness a smaller range of only 1.5 will also be considered. Similarly in the sensitivity analysis where stock portfolio volatility is made dependent on education/permanent income I consider two possible cases, that is, $g(\theta_{low}) = 2$ and $g(\theta_{low}) = 1.5$ while $g(\theta_{high})$ is set equal to 1 in both cases so that high education individuals indeed face the stock market index return process while low education investors face a higher variance.

Next we have to calibrate the two different costs that agents face to participate in the stock market. There is no empirical estimate of the initial entry cost F_I so I set it to 0.075 which is equivalent to about 3 percent of the yearly average wage, near the value used for example by Gomes and Michaelides (2005). There have been instead efforts to estimate the per period participation cost. Work by Vissing-Jørgensen (2002 and 2003) and by Paiella (2001) have found values between 50 and 200\$ so that I fix F_P to 0.01. Given model average earnings this value is consistent with a dollar amount that roughly falls in the middle of the interval reported above.

4 Results

For convenience of exposition I organize results into two subsections. In the first one I start with the standard case where all households investing in risky assets face the volatility of the stock market index. Under this general assumption I start with a benchmark case that is very similar to the base cases considered in Cocco, Gomes and Maenahout (2005) or Gomes and Michaelides (2005) and then examine in turn the effects of introducing a fixed per-period participation cost and a realistically calibrated, progressive social security system. In the second subsection I move to the central theme of the paper, that is, the effects of under-diversification on life-cycle portfolio allocation. Each economy is obtained from the previous one by adding the relevant marginal feature and re-calibrating the subjective discount factor so as to keep the aggregate wealth-to-earnings ratio constant across experiments.

4.1 The Models with Diversified Stock Investment

4.1.1 The Baseline Model

The first model I present is a benchmark case where each agent receives a pension benefit that is a constant fraction of average past earnings conditional on his earning type and last year of work earning shock. The replacement ratio is fixed at the average replacement ratio implied by the calibrated social security system of the later experiments. This replacement ratio turns out to be 0.502. No fixed per-period cost is assumed. The value of β is 0.89; with this value the

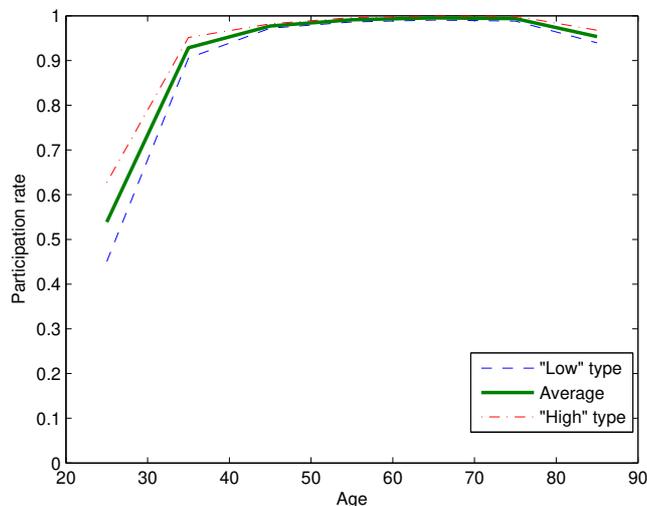


Figure 1: Life-cycle participation rates

wealth-earnings ratio in the economy is 5.02. Results are reported in Figures 1 and 2; relevant variables are reported by 10 year age groups. The first of the two figures reports participation rates by age. The thick continuous line represents the average participation rate by age groups in the economy. As it may be seen this rate is low at young ages since agents have not yet accumulated enough wealth to make it convenient to pay the initial entry cost. As households move into mid age and wealth accumulation to finance retirement consumption picks up the participation rate jumps up to reach a plateau of almost 100 percent; later in life it stabilizes basically reflecting the fact that in the absence of any further cost of staying in the stock market all agents with positive wealth will hold at least some stock in any period of life. The graph reports two more lines: the dashed line represents the participation rate for low earning ability households and the dash-dot line does the same for the high ability households. As expected the participation rate for high ability types is higher than the average and that of low earning types is lower. This reflects the fact that the latter have on average lower earnings thus less wealth than the former so that a lower fraction will accumulate enough to pay the initial entry cost. It is worth noticing though that this difference is not large.

Figure 2 reports the life-cycle profiles of the conditional stock share. These profiles reproduce the well known result that upon paying the entry cost agents would invest 100 percent of their wealth in stocks; after that the portfolio share

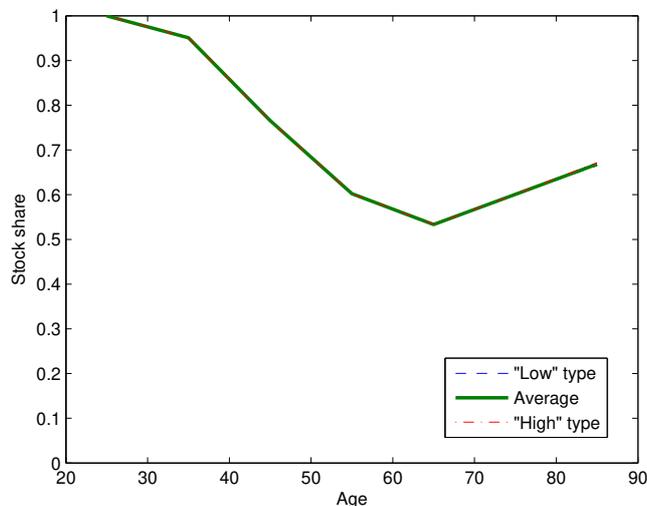


Figure 2: Life-cycle conditional stock share

of stocks declines monotonically and substantially until retirement and then increases somewhat towards the end of life. This profile is in contrast to the empirical evidence that suggests that the conditional share is always well below 100 percent and is roughly flat or slightly increasing in age. A second feature of the model generated profiles is that the average conditional share is the same for both earning ability types over all of the life-cycle. The intuition for these results is well known and will be explained by way of Figure 3. The figure reports for each age, the ratio between average financial wealth and the average present discounted value of the remaining stream of earnings and pensions until death.¹⁴ For ease of reference, in what follows, I will call the latter quantity human wealth. The intuition for the portfolio result is that earnings, even though uncertain, are a better substitute for the risk-free bond. Agents with CRRA utility facing i.i.d. stock returns want to keep a constant share of their total wealth in the risky asset so that when financial wealth is low relative to human wealth they would like to invest all of their financial portfolio in stocks while as financial wealth becomes larger they would diversify more and more towards bonds. A comparison between Figure 2 and 3 clearly shows this. The ratio of

¹⁴Here average wealth is simply the simulated average wealth for each earning type, age group. The same is true for the present discounted value of earnings; the results are obtained when future earnings are discounted at a 3 percent rate, but they do not change significantly when the discount rate is fixed at 2 or 4 percent.

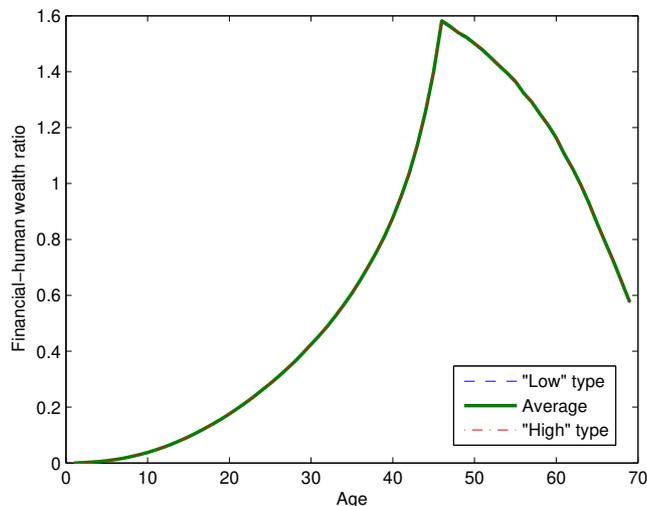


Figure 3: Life-cycle financial to human wealth ratio

financial to residual human wealth starts from 0 at age 21, since with no bequests all agents enter working age with no wealth at all. It then picks up quickly as agents start to accumulate for precautionary reasons first and to finance retirement consumption then, while at the same time the shortening of the remaining horizon reduces human wealth. After retirement then consumption of the accumulated wealth reduces the ratio once more. The inverted V-shape that results mirrors the V-shaped pattern of the conditional stock share. While this is no new result the point that is worth stressing is that under proportional replacement ratios the pattern of wealth accumulation of high earning types is simply a scaled up version of that of low types with the scaling factor being the same as the one of earnings, thus the ratio of the financial to residual human wealth is the same for both types of agents. This is reflected in a life cycle profile of the conditional stock share that is the same for both types of agents.

4.1.2 Fixed Per-Period Participation Cost and Progressive Social Security

In the baseline model I now introduce a fixed per period cost of participating in the stock market. Since the pattern of wealth accumulation is only marginally affected by this change there is no need to re-calibrate the value of β which is then left at 0.89 as before, generating an average wealth-earnings ratio of 5.03 almost identical to the one in the previous experiment. Results are reported

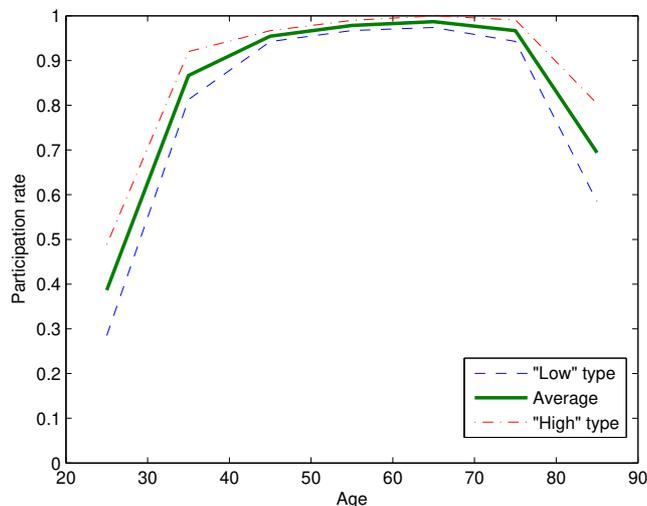


Figure 4: Life-cycle participation rate

in Figures 4 and 5. The first of the two figures reports the life cycle participation rates. As in the previous case the participation rate is relatively low in the age group 20 to 30 and then increases rapidly as households accumulate wealth to reach a peak of almost 100 percent in mid-life. The novel element here is that there is substantial exit of stock market participants which shows up in the decline in participation rates down to 70 percent late in life. This decline gives the life-cycle profile of participation rates a characteristic inverted U shape. This result brings the model closer to the data since exit is a well documented phenomenon.¹⁵ As far as the hump in participation rates over age is concerned, this is commonly observed in the data from a number of different industrialized countries. However whether this indeed results from the pattern of individual decisions is not yet a completely resolved issue since results vary with the identification assumption used in the estimation.¹⁶ The results for the conditional stock share, reported in Figure 5, do not show significant changes compared to the benchmark case: the profile still starts from 100 percent in the youngest age group, declines until around retirement age and then picks up slightly as agents approach the maximum allowed age. This is not surprising

¹⁵See for example Vissing-Jorgensen, (2002).

¹⁶The most complete work on this issue is Ameriks and Zeldes (2004). They find a hump shaped profile when age and time effects are included and an increasing one with cohort and time effect only. They also notice that in the TIAA-CREF some old people completely get out of the stock market upon withdrawal suggesting that the hump does exist.

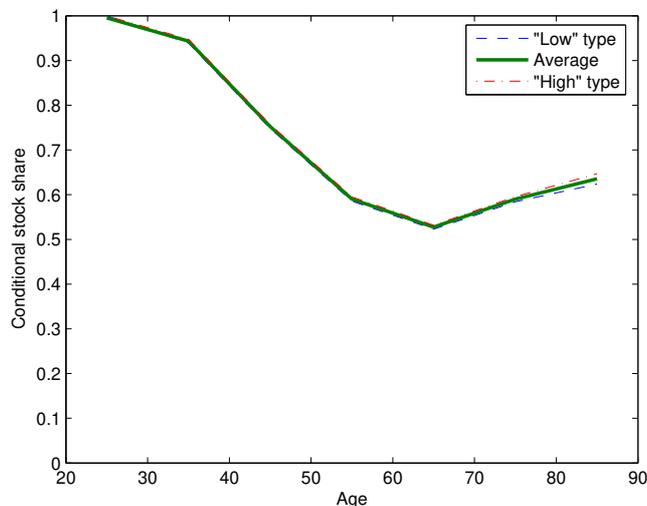


Figure 5: Life-cycle conditional stock share

since they are driven by the evolution of the financial to human wealth ratio and this is not affected by the introduction of the fixed per period participation cost.

The next step is to add a progressive social security system with the benefit formula defined in the calibration section and modeled according to the rules of the US social security. In this case the discount factor β needs to be increased to 0.92 in order to keep the wealth to earnings ratio at the target ratio of 5. While this implies that the average life-cycle profile of wealth and the financial to human wealth ratio is not substantially altered from the previous cases, when we look at the two earning ability groups separately the picture is different. With progressive social security, households that have higher earnings face a lower replacement ratio, so that they need to accumulate more assets to smooth their consumption past retirement age, compared to lower earnings households. Even though a high type household may have a higher expected replacement ratio than a low type household if it experiences a sufficiently worse history of earnings shocks, on average high type households will have higher earnings, thus lower replacement ratios.¹⁷ This can be seen in Figure 6 where the curve representing the financial to human wealth ratio for the high earning types lies above the one for low types. The difference is minor in the first decades of working life — when

¹⁷Recall here that the benefit formula is applied to average earnings conditional on both the household earning type and its last year of life earnings shock.

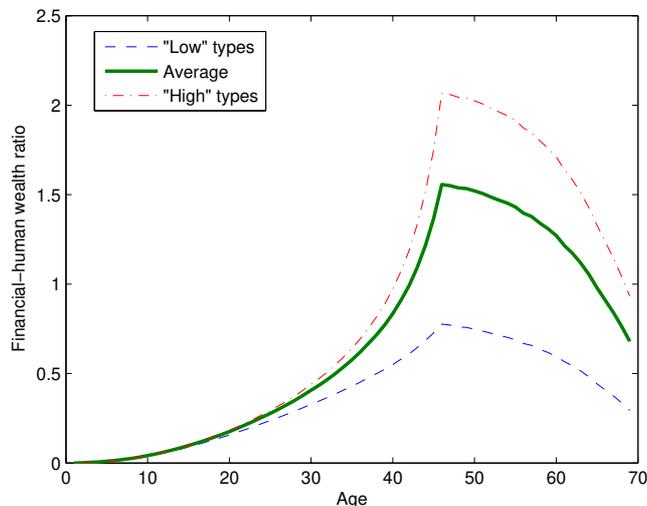


Figure 6: Life-cycle financial-to-human wealth ratio

saving occurs mainly for precautionary reasons — but becomes more pronounced in the decade before retirement and even more after that. The effects that this has on households' stock market decisions are reported in Figures 7 and 8. The first of the two figures shows that there is a reduction in participation rates for both types of agents especially very late in life. Under progressive social security, those agents facing bad shocks late in their career have very high expected replacement ratios, hence they will choose to enter retirement with very little or no wealth at all and rely entirely on pension benefits to finance their consumption. In both cases the effect is not to participate in the stock market, either because they don't have any asset or because they don't have enough to make it worth paying the per period cost.¹⁸ If we look at the conditional portfolio share of stocks in Figure 8 we see that once again the profiles start from a 100 percent share in the first decade of life and then substantially decrease until retirement age, after which they stabilize. The other important feature that emerges from the graph is that early in life the conditional share is about the same for the two types of agents; as retirement approaches high earning types start to choose on average a reduced exposition to

¹⁸Indeed an important chunk of the extra non-participation induced by the US formula of pension benefits is the consequence of agents with 0 wealth late in life. This is shown in plots of the fraction of agents with positive wealth against age that are not reported to economize on space.

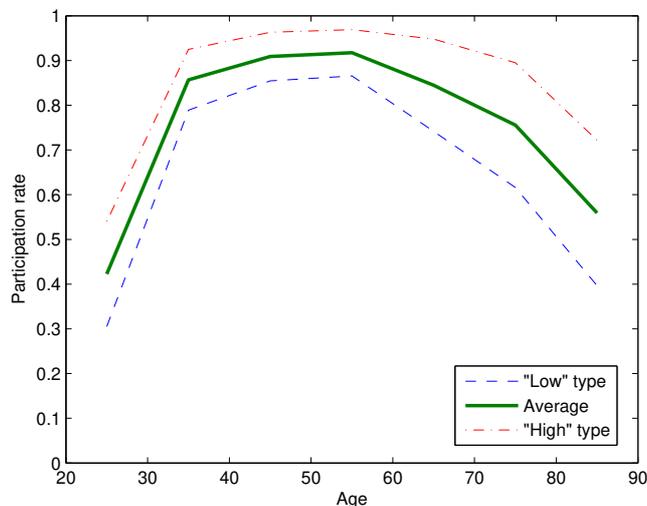


Figure 7: Life-cycle participation rates

stock risk compared to low earning types and the difference becomes substantial after retirement. This is not surprising in light of the well known intuition behind the behavior of conditional stock shares given in the previous subsection and the result in Figure 6 that shows substantially higher financial-to-human wealth ratios for high types during retirement. A few comments are needed about this result. It is common among financial advisors to suggest investment strategies that relate the share of portfolio to be invested in the stock market to age. A popular advice is that the share of risky assets should decline with age, with other factors like job security and risk tolerance to fine tune the allocation.¹⁹ The result obtained here is that the suggested strategies omit a key factor, that is, the level of life-time income, especially as retirement approaches or past retirement age. Because of the progressive nature of the pension benefit formula, low permanent income households implicitly hold a larger position in the risk-free asset represented by their future earnings and pension income relative to their tradeable wealth and would benefit the most from exploiting the equity premium, while high permanent income households — who hold a relatively smaller position in risk-free non financial wealth — should try to diversify more

¹⁹Jagannathan and Kocherlakota (1996), Ameriks and Zeldes (2004) and Cocco, Gomes and Meanhous (2005) report this kind of advice and examine how sound it is from the view point of economic theory. One can also consult the web sites of investment companies like for example the Vanguard Group at <http://www.vanguard.com>.

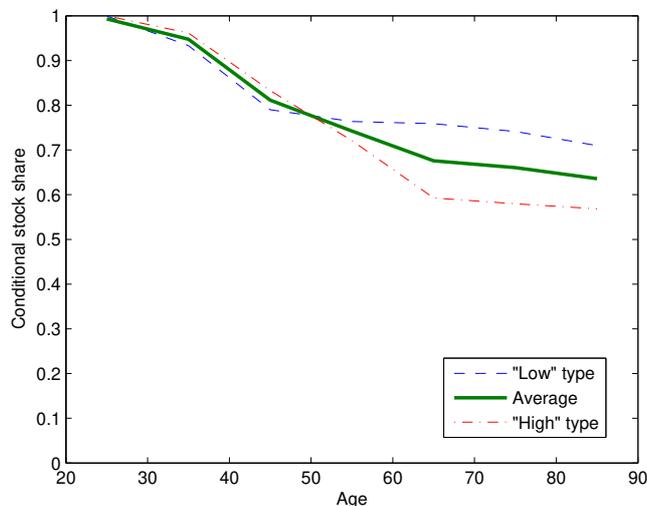


Figure 8: Life-cycle conditional stock share

to bonds to avoid excessive exposure to stock market risk. At a positive level the available evidence — see for example Haliassos and Bertaut (1995), Kennickell et al, (2000) and Vissing-Jørgensen (2002) — points to a positive relation between non-financial income and the share of stocks adding a new puzzling fact to portfolio choice theory.²⁰ A similar conclusion can be reached if we interpret types in the model as education groups. Under this interpretation the model predicts larger conditional shares for less educated people which is contrary to the empirical findings.²¹ The caveat here is that this result is obtained under the assumption that all households invest in the stock market index if they decide to hold equities.

²⁰Some caution should be exerted before drawing definitive conclusions from those studies since neither is a perfect empirical counterpart to the plot showed above. The positive link between nonfinancial income and the stock share found by Vissing-Jørgensen (2002) is conditional on a complete set of variables that may affect this choice; the one found in Haliassos and Bertaut (1995) and Kennickell et al. (2000) is unconditional. The one in this paper is something in between since it reports the share of stock by permanent income, conditioning on age.

²¹Regression analysis that find a positive relationship between education and conditional stock shares based on SCF data are Bertaut and Starr-McCluer (2000) and Curcuro et al. (2004).

Table 1: Average participation rates and conditional shares

	Participation rate	Conditional share
Benchmark	0.89	0.74
Fpc, prop. s.s.	0.83	0.72
Fpc, prog. s.s.	0.77	0.80

4.1.3 Average Participation Rates and Conditional Stock Shares

Before turning to the role of under-diversification I will present, with the help of Table 1 summary results about the average stock market participation rate and conditional shares and how they are affected by the fixed per period cost and progressive benefit formula. The participation rate in the benchmark case is 0.89, a value that is very high compared to what is observed in reality. We saw that both a small fixed per period participation cost and progressive social security improve the performance of the model enabling it to obtain a hump-shaped life-cycle profile of participation rates. When we try to quantify this effect though, it seems rather small. Adding a fixed participation cost of the size suggested by the studies of Paiella (2001) and Vissing-Jørgensen (2002) only reduces this participation rate to 0.831, just 6 percentage points below the benchmark case. When this is combined with the more spread out wealth accumulation profiles that result from a progressive benefit formula a further decrease of 6 percentage points in participation rates results. Overall then the reduction compared to the benchmark case is of only 12 percentage points.

As far as the average conditional stock share is concerned we can see that this is counter-factually very high in all the three models considered: it is 73.8 % in the benchmark case, it goes down to 71.8 % in the model with a fixed per period participation cost and it goes up to 80.0 % when progressive social security is added as well.

4.2 The Model with Under-Diversified Stock Investment

In the last section we saw that the benchmark life-cycle portfolio choice model produces a very high participation rate that quickly increases in the first part of life and then stabilizes at 100 percent. It also generates conditional stock shares that start at 100 percent in the first decade and then follow a V-shaped pattern. Both facts are at odds with the empirical findings. Motivated by these failures I introduced a fixed per-period participation cost and a progressive pension benefit formula. These new features allow the model to produce stock market exits and an inverted U shaped pattern of participation rates. However, quantitatively the reduction in the average participation rate falls short of the one

needed to match the empirical evidence. Moreover the conditional stock share is still at odds with the evidence since it is very high, monotonically declining and higher for low earning ability types. For these reasons in this section I introduce the possibility that agents don't actually purchase the stock market index, they buy instead some portfolio of individual stocks and stock mutual funds and that higher wealth households have better opportunities and/or abilities to diversify so that the variance of this risky portfolio declines with the total amount of assets held. It will be shown that this can dramatically improve the ability of the model to reduce both the participation rate and the conditional stock share. Also, the reduction in the conditional stock share is larger for low permanent earning types so that high types would now invest a larger share in stocks as the data suggest. Finally I perform two robustness checks. First I complete the model by adding a bequest motive and actual transmission of wealth through the generations. I show that while this does not change the main qualitative results, it still helps bringing the conditional stock share close to the flat age profile found empirically. Second on this final version of the model I check the sensitivity of the results to a smaller difference in the variance of the stock portfolio among households and to the possibility that these differences are driven by the permanent earning type rather than by wealth.

4.2.1 A Baseline Model with Under-Diversified Stock Investment

In this section I consider the model where agents face a variance of their stock portfolio return that is declining with their wealth holdings. As it will be shown below the fact that now the risk of holding stocks is greater implies that households will participate less frequently and hold a smaller share conditional on participation. In turn, since portfolios become more heavily tilted towards the low return and safe bond wealth accumulation will proceed at a lower pace. This forces an increase of the discount factor to 0.94 in order to keep the wealth to earnings ratio close to its target. Results for this case are shown in Figure 9 and 10 for the participation and conditional share life-cycle profiles respectively. A look at Figure 9 reveals that participation rates are substantially reduced: this reduction occurs among all agents except mid-life high earning types and it is particularly strong for low earning types whose participation rate now barely reaches a peak of 50 percent among the 60 to 70 year old group. The reason for this result is the following. The total benefit from participating in the stock market can be decomposed into the product of three elements: the total amount of wealth, the optimal stock share and the risk adjusted return premium.²²

²²Vissing-Jørgensen (2002) and (2003) present this decomposition in a formal way in the context of her estimation of stock market participation costs.

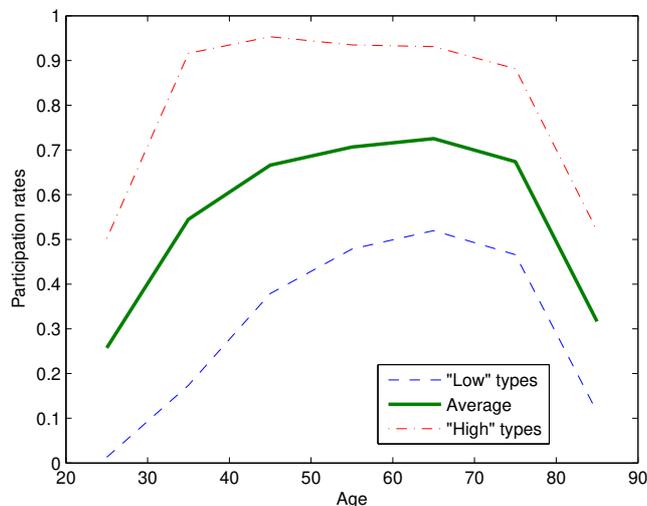


Figure 9: Life-cycle participation rates

The increased variance of returns associated with small holdings of wealth reduces both the equity premium corrected for risk and, as it will be shown in the next paragraph, the optimal stock share. As a consequence the benefit from participating in the stock market are greatly lowered especially for low wealth agents. A large number of them then decides either not to pay the initial cost and never enter the stock market or to stop paying the cost earlier and quit it sooner after entering for the first time. This result is reminiscent of the empirical finding of Calvet et al. (2007) according to which once one takes into account predicted portfolio diversification of households that do not participate in the stock market their benefit from participation would be rather small and for this reason their choice easy to justify by minor participation costs.

The impact of the increased variance of stock returns for small amounts of holdings is also strong when we look at the conditional stock shares. A look at Figure 10 shows that the average conditional share is greatly reduced except in the first decade of working life. For most of the life-cycle it is only about 30 percent while it was about 70 percent in the previous models. The share held by high earning types now lies well above the ones of low earning types. The intuition behind this result is the following: now agents face a variance of returns that is higher than the one on the stock market index so they all want to reduce their exposure to stocks. However because this variance declines as the agent becomes wealthier the reduction in stock shares is smaller for high

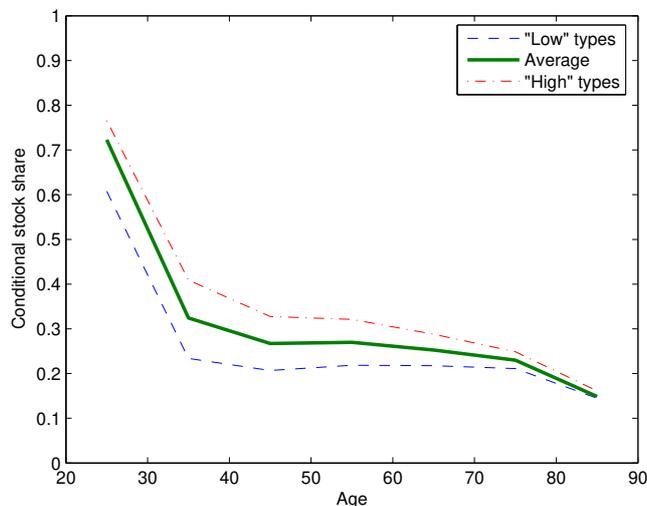


Figure 10: Life-cycle conditional stock share

permanent earnings and wealthier agents than for the rest of the population. It turns out that this effect is opposite in sign and stronger than the one induced by the progressive formula for social security benefits reverting the pattern of conditional stock shares by permanent income/education. Despite the success of this formulation in reducing the average conditional stock share, it is still true that its life-cycle profile is declining with age especially in the first decades of life.

4.2.2 Introducing Bequests

I next turn to the results that are obtained when agents have bequest motives and estates are actually passed to the descendent household. The introduction of bequests increases wealth accumulation late in life, so that more impatience is needed to keep the average wealth-earnings ratio constant. A value of 4.95 for this ratio is obtained by lowering β to 0.90. The life cycle profile of participation rates is depicted in Figure 11; the figure shows no important changes compared to the model with under-diversification but no bequests. The introduction of intergenerational transmission of wealth seems to reduce somewhat the participation rate, especially at young ages. This is because while a few agents inherit early in life so that they can pay the entry cost and start to invest in the stock market, all of them are more impatient which reduces wealth accumulation in

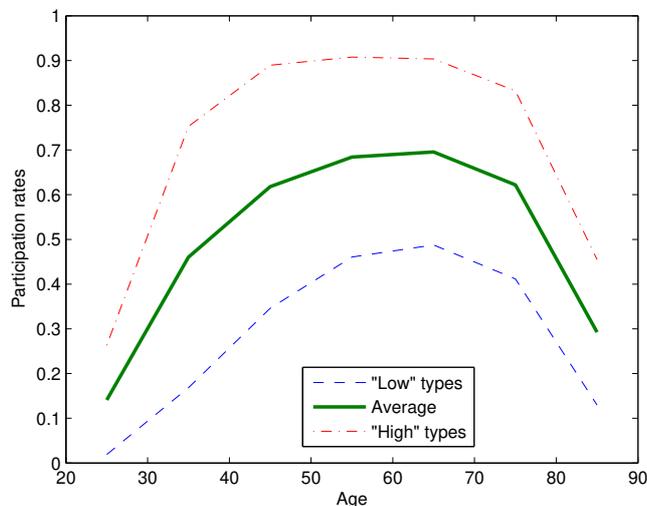


Figure 11: Life-cycle participation rates

the first part of life reducing participation in the stock market as well. The results for the share invested in stocks conditional on participation are reported in Figure 12 : the most notable difference is that now the profile for the average conditional stock share in the population is virtually constant from the 30 to 40 year old group until the oldest group and the decline in the share observed between the first two decades of life is reduced. When moving from the 20 to 30 year old group to the next one the conditional stock share declines from slightly below 50 percent to about 30 percent. In the model without bequests the decline was from 70 to 30 percent. This difference is the consequence of the fact that when intergenerational transmission of wealth is allowed some young agents may receive substantial bequests; this increases their financial to human wealth ratio reducing their optimal stock share and consequently the average of their age group.

4.2.3 Average Participation Rates and Conditional Stock Shares

To close this section I report once again figures for the average participation rate and conditional stock share. This is done in Table 2. The first line of the table corresponds to the last line of Table 1, that is, a model where social security is progressive, there is a fixed per period participation cost, but there are neither wealth related diversification opportunities nor intergenerational transmission

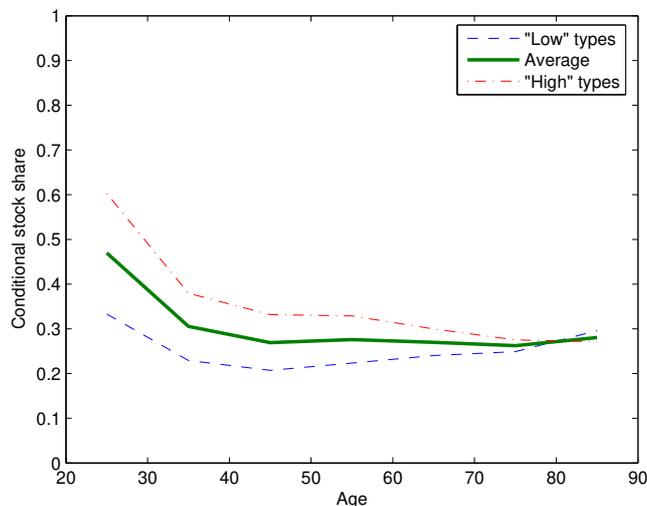


Figure 12: Life-cycle participation rates

Table 2: Average participation rates and conditional shares

	Participation rate	Conditional share
No beq, cvr	0.77	0.80
No beq, vvr	0.57	0.33
Beq, vvr	0.51	0.31

of wealth. As said in the previous section the average participation rate is 0.771 in this case. When heterogeneous under-diversification is introduced the participation rate is reduced in a quite substantial way, from 77.1 percent down to 56.8 percent. In the model with bequests the participation rate further goes down to 50.9 percent, a value that is indeed only slightly above the one observed in the most recent issues of the Survey of Consumer Finances.

When we look at conditional stock shares the effects of under-diversification are even more dramatic. In the model where all agents invest in the stock market index the conditional share is 80.0 percent while accounting for under-diversification reduces it to 33.2 percent in the life-cycle case. In the version of the model with bequests the conditional share is further reduced although by a minor amount, that is, from 33.2 to 31.3 percent. These numbers are closer to their empirical counterpart but are somewhat below it. I will check in the next subsection how they change in response to different specifications and calibrations of heterogeneity in diversification.

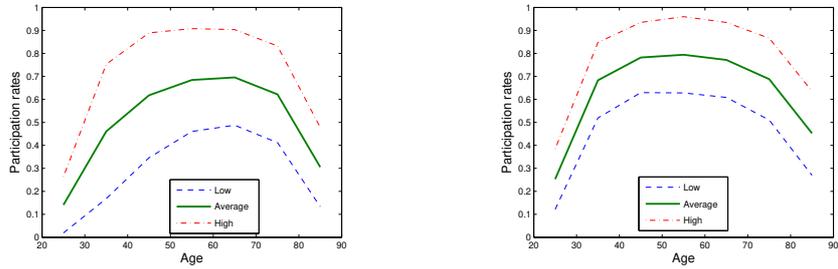


Figure 13: Participation rates: $1 < g(w) < 2$ left, $1 < g(w) < 1.5$ right

4.3 Sensitivity Analysis

In this section I perform some sensitivity analysis on the formulation of heterogeneous under-diversification. This is done using as a benchmark the last model presented in this section, that is, the one that includes bequests.

First I explore the case where the standard deviation of the return on the stock portfolio is wealth dependent but the ratio between the maximum and minimum standard deviation of stock portfolios for different investors is bounded above by 1.5 instead of 2. The results are presented in Figure 13 for participation rates and 14 for conditional shares. In both figures the left panel depicts the baseline case, while the right panel represents the case with reduced maximum variance of returns. A look at Figure 14 reveals that average conditional shares for the whole population are higher. This is because now low wealth investors face a standard deviation of their stock portfolio that is at most 1.5 times the one on the public equity index. The increase in the optimal portfolio share coupled with the larger risk-adjusted premium brought about by the reduction in volatility increases the benefits of stock market participation. As it can be seen from Figure 13 this moves upwards the average life-cycle profile of participation rates. This is especially true for low earning type agents who now enter the stock market earlier and reach a peak participation rate of about 60 percent compared to 50 percent in the baseline case. Two more features of the calibration with reduced under-diversification are, first, that now high earning agents invest larger shares in risky assets than low earning agents only up to retirement age, while afterwards the effect of progressive social security becomes dominant and reverts the result. Second, the profile of conditional stock shares displays a steeper downward slope and becomes flat only starting with the 40 to 50 year of age group.

In the second sensitivity exercise I assume that the ability to diversify the stock portfolio depends positively on permanent income/education and again

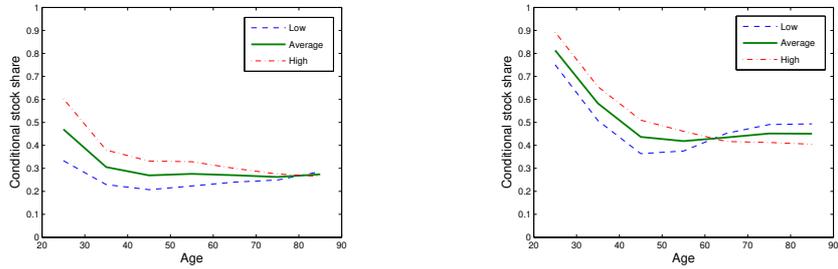


Figure 14: Conditional shares: $1 < g(w) < 2$ left, $1 < g(w) < 1.5$ right

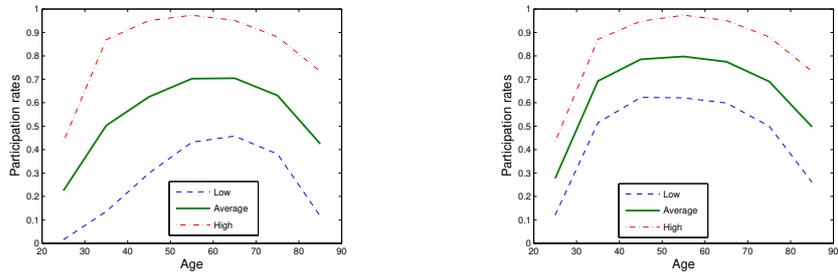


Figure 15: Participation rates, $g(\theta_{low}) = 2$ left, $g(\theta_{low}) = 1.5$ right

consider two possible ratios between the highest and lowest standard deviation of returns, namely 2 and 1.5. Figure 15 presents results for the participation rates. The left panel refers to the case in which high income types face the stock market index return process, while low income types face double its volatility. Here we see that there is a large reduction in average participation rates that peak at only 70 percent in mid-life. This reduction is entirely due to the low earning ability group whose average participation rate never reaches 50 percent while in the high earning ability group virtually all agents participate in the stock market in mid-life. As we can see from the right panel of the figure, when the standard deviation of returns faced by the low earning types is reduced to 1.5 times the one on the index we see that the average life-cycle profile of participation moves up; once again this shift is entirely driven by the low income group. Comparing this profile to the one in Figure 7 though we see that the downward movement in the participation curve is still sizeable.

Finally Figure 16 plots the shares of risky assets conditional on participation for these two experiments. Comparing it with Figure 8 we see that in both cases the reduction in the average stock allocation is quite substantial as can be seen from the downward movement in the thick line. This reduction is especially

pronounced when the variance faced by the low earning types is twice the one on the stock index fund; in this case the average life-cycle profile exhibits a somewhat declining profile centered around a value of 50 percent, quite in line with the empirical evidence. The right panel of the figure shows that when heterogeneity in under-diversification is less pronounced the average profile of conditional stock shares is a little higher with a more downward sloping stretch early in life; from about 85 percent in the 20 to 30 year old group to about 55 percent in mid-life. The other notable feature shown by Figure 16 is that now the profile of conditional shares for the high permanent income/education group lies well above the one for the other group. The intuition behind this result is straightforward: when the standard deviation of stock returns declines smoothly with wealth some low earning types that are very lucky in their income realizations may indeed earn more than unlucky high earning types and end up accumulating more wealth, so that the difference in the volatility faced on average by investors in the two groups are less pronounced. This translates into a smaller effect on conditional shares. When diversification is made dependent on the earning type, all agents in the high group face the stock market index volatility and all agents in the low group face the larger volatility thus making the difference in their choices much stronger.

The sensitivity analysis is completed by Table 3 where results about the average participation rates and conditional stock shares are reported. The top line refers to the baseline model with under-diversification described in the previous section when the participation rate was 51 percent and the conditional stock share 31 percent. In the second line I report the figures for the case when under-diversification still depends on wealth but it is less heterogeneous: now the average participation rate climbs up to 64 percent and the conditional share to 50 percent. In the last two lines instead I report the figures for the models in which under-diversification depends on permanent income/education. In the third line this is done for the case with the larger difference in stock return volatility. We can see that in this case the average participation rate is 54 percent and the average conditional stock share is 63.7 percent. In the last line we can see that when the ratio of the return volatility faced by the two types of agents is reduced to 1.5 the average participation rate climbs up to 65 percent and the average conditional share in risky assets to 65 percent as well.

To conclude the section I will briefly summarize and discuss the results obtained. The empirical evidence suggests that both wealth, income and education predict better ability to construct well diversified equity portfolios. The results in the sensitivity analysis section were obtained by considering wealth and education/permanent income separately in the function defining under-diversification. Considering both at the same time would add more flexibility

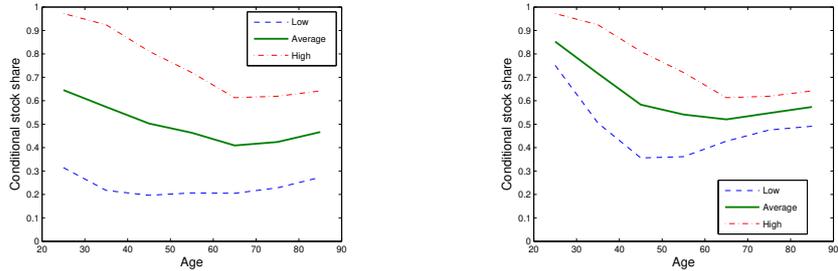


Figure 16: Participation rates, $g(\theta_{low}) = 2$ left, $g(\theta_{low}) = 1.5$ right

Table 3: Average participation rates and conditional shares

	Participation rate	Conditional share
$1 < g(w) < 2$	0.51	0.31
$1 < g(w) < 1.5$	0.64	0.50
$1 < g(\theta) < 2$	0.54	0.64
$1 < g(\theta) < 1.5$	0.65	0.65

leading to an improved ability of the model to fit the data. However I won't pursue this possibility further here. The analysis conducted so far is sufficient to highlight the great importance that heterogeneous under-diversification can have in rationalizing a few fact of life-cycle portfolio allocation. First, under a realistic calibration it explains the low conditional stock shares. Second, the lower optimal shares and the reduced risk adjusted equity premium reconcile large non-participation with moderate fixed costs. Moreover the increased non participation occurs among low wealth, less educated agents that would otherwise hold a more under-diversified stock portfolio which is consistent with the empirical result in Calvet et a. (2007). Finally it implies that higher education/permanent income agents hold larger conditional shares of risky-assets, a fact that is more consistent with the empirical evidence but that would not emerge under a realistic representation of the US formula that determines social security benefits.

Since participation rates and conditional shares are strongly affected by wealth levels and the ratio between financial and nonfinancial resources, I tried to constrain the model to generate appropriate values of the average wealth to earnings ratio. This was done by calibrating the value of the discount factor. One caveat applies here: the wealth-earnings ratio targeted in the calibration includes all wealth but it is well known that a substantial part of this wealth is

held in the form of housing which is omitted in the present model.²³ Introducing housing though would reinforce the conclusions reached here. With regard to the participation rate, housing crowds out financial wealth and with smaller financial wealth the benefits of participation in the stock market would be reduced so that an even lower degree of under-diversification could justify more non participation for any given level of fixed costs. The effects on conditional share are less evident, however if one takes the approach of Gomes and Michaelides (2005) and models housing as an expenditure that represents a fixed share of income without giving utility this would reduce the present value of relatively safe non-financial income. This effect cancels out, at least qualitatively with the reduction in the amount of financial wealth leaving the ratio between the two, hence the conditional shares, likely unchanged. On the other hand Cocco (2005) finds, in a setting where investment in housing is explicitly modeled, that house price risk crowds out stock; once again this conclusion would reinforce the results presented here as an even smaller amount of under-diversification could rationalize the moderate fraction of stock in financial portfolios observed in the data.

5 Conclusions

In the present paper I have considered an extension of the standard life-cycle asset allocation model that allows for heterogeneous under-diversification, progressive social security and fixed per period participation costs. The results that emerged are summarized here for convenience. First the introduction of a fixed per period participation cost is needed to obtain exits along the life-cycle and generates a hump shaped profile of participation rates over age. Second a progressive formula to determine social security benefits makes it optimal for low education/permanent income agents to invest larger shares of wealth in the stock market than high education/permanent income agents. Moreover by generating more agents with low or no wealth at all it magnifies the effect of participation costs in reducing participation rates, although the two mechanisms jointly are not sufficient to bring the model close to the data. Third, recognizing the lack of diversification in stock portfolios has a strong impact in reducing both the participation rate and the conditional stock share. Moreover when under-diversification is assumed to be a stronger problem for low wealth or education/permanent income households it restores a positive relationship between

²³Cocco(2005) and Yao and Zhang (2005) have considered housing in their model. In their model though, housing introduces one more state and choice variable, which given the complexity of the current model is not very desirable. A simpler way to consider housing is the one in Gomes and Michaelides (2005).

financial income and conditional stock shares. Finally under some calibrations intergenerational transmission of wealth helps make profiles of the conditional stock share independent of age as the empirical evidence suggests.

Among the different results summarized above, the key contribution is to show how the lack of diversification in stock portfolios, a fact that has recently received much attention in the empirical literature, can be a fundamental determinant in household life-cycle decisions concerning the stock market both at the extensive and intensive margin. In particular, I showed that under a reasonable choice of parameters it can jointly explain both low stock market participation and conditional share providing an alternative mechanism to the one proposed by Gomes and Michaelides (2005) and based on Epstein-Zin preferences with heterogeneous risk aversion.

Appendix

A The Model with Bequests

In this appendix I describe the dynamic programming formulation of the household's problem in the case when an active bequest motive is present and estates are actually passed through the generations. I complete the description of the demographic structure by assuming that all agents have a kid at age 35. With this assumption we can divide a household life into two qualitatively different periods. Bequests are altruistic, hence they depend on the descendant's indirect utility from receiving them. Since this is defined only after the kid enters the labor force at age 21 a household won't have an active bequest motive until reaching 55 years of age. At the same time before that age the parent household can still be alive so the agent is the potential recipient of an inheritance. We can then split an agent's life into a first period up to age 54 when he can receive a bequest but does not value leaving one: I call "early life" this first part. Afterwards and until death the agent cannot receive any more a bequest but values leaving one: I call "late life" this second part. In the next two subsections I describe in turn the dynamic programming problem solved by an agent in the two different stages of life. In order to simplify the notation I will omit the index i that denotes the household. For the same reason I will omit the index that denotes the agent's permanent earning type.

A.1 Early Life Problem

Given the description of the model in the main text the value function problem in the early stage of life is:

$$V^a(w_t, z_t, I_{F,t}, I_{s,t}) = \max_{c_t, B_{t+1}, S_{t+1}, I_{P,t}} \left\{ u(c_t) + \beta p_{a+1} \left\{ I_{s,t} [p_{a+36} E_t V^{a+1}(w_{t+1}, z_{t+1}, I_{F,t+1}, 1) + (1 - p_{a+36}) E_t V^{a+1}(w_{t+1} + \bar{W}^{a+36}, z_{t+1}, I_{F,t+1}, 0)] + (1 - I_{s,t}) E_t V^{a+1}(w_{t+1}, z_{t+1}, I_{F,t+1}, 0) \right\} \right\}. \quad (14)$$

In the above equation the index I_s takes the value of one if the agent's parent is alive at time t and zero otherwise. The interpretation of the equation is the following: on the left-hand side we have the value function of an agent who is a model periods old and whose states are given by his financial resources w_t , his labor earnings shock z_t and the two indexes that say if the agent had previously paid the initial entry cost and if the agent's parent is alive. This value is the maximized value of the sum of the utility flow from current consumption $u(c_t)$ and future discounted utility where the maximization is performed with respect

to consumption, the amount of bonds and stocks to carry to the next period and payment of the participation cost. In turn the continuation value can be either of the two following possibilities. If the agent's parent had died before then the index $I_{s,t}$ takes the value of zero so that continuation utility is the last term of the Bellmann equation $E_t V^{a+1}(w_{t+1}, z_{t+1}, I_{F,t+1}, 0)$, that is, the indirect utility of an agent that has grown one year older and whose parent is dead, given his resources and labor income shock. The other alternative is that the agent's parent is alive at time t so that $I_{s,t} = 1$ in which case continuation utility is given by the term in square brackets. With probability p_{a+36} the agent's parent survives so that the household's continuation utility will be the utility of a one year older household whose parent is still alive given resources w_{t+1} and the labor income shock: this is the term $E_t V^{a+1}(w_{t+1}, z_{t+1}, I_{F,t+1}, 1)$ in the Bellmann equation above. With probability $1 - p_{a+36}$ the parent will die next period in which case the continuation utility will be $E_t V^{a+1}(w_{t+1} + \bar{W}^{a+36}, z_{t+1}, I_{F,t+1}, 0)$, that is, the utility of a household that has grown one year older, with no alive parent and with resources that on top of its own personal funds include some "expectation" about the size of the bequest. The notation \bar{W}^{a+36} points to the fact that the assumption made here is that the agent does not know his parent's wealth and uses the average value of wealth in the population of households of the same age and earning type.²⁴ In general we may assume that heirs have some information about parental wealth but that this is not perfect. The assumption made here that they assume parental wealth is average among their cohort and earning type corresponds to a case of limited information and it is made to reduce the already high computational burden imposed by the program structure. Some discussion is needed to justify this assumption. The way we model how an agent forms his expectation about how much he will inherit affects his decisions since if he expects to receive a larger bequest he will save less. Consequently the modeling choice made here implies that some agents will over-save and some will under-save compared to the case where they had more precise information about parental wealth. The goal of the paper though is to study average life-cycle profiles so that it is reasonable to think that these deviations from a more detailed informational assumption will compensate each other and therefore will be minor.²⁵ Finally notice that in principle a descendent household can die before its parent does but I rule

²⁴To avoid further complications in the computation types are assumed to be perfectly correlated across generations.

²⁵An alternative choice would have been to assume that the agent knows his parent's current wealth and labor earnings shock and uses his decision rules to forecasts the bequest he will receive. This assumption though would imply the addition of two more state variables; moreover such perfect forecast assumption is not necessarily more tenable than the one made here.

out altruism between one member of a dynasty and the previous one. The description of the early life problem is completed by the resource constraint and the law of motion of the household's financial resources. These are the same as those of the economy without bequests so the reader is referred to equations (10), (11) and (12) given in the main text.

A.2 Late Life Problem

In the second stage of life, when an agent has an active bequest motive but cannot inherit any more the value function problem takes the following form.

$$\begin{aligned}
V^a(w_t, z_t, I_{F,t}, I_{s,t}) = & \max_{c_t, B_{t+1}, S_{t+1}, I_{P,t}} \left\{ u(c_t) + \beta I_{s,t} \{ p_{a+1} \times \right. \\
& [p_{a-34} E_t V^{a+1}(w_{t+1}, z_{t+1}, I_{F,t+1}, 1)] + (1 - p_{a-34}) E_t V^{a+1}(w_{t+1}, z_{t+1}, I_{F,t+1}, 0)] \\
& \left. + (1 - p_{a+1}) p_{a-34} \gamma E_t V^{a-34}(w_{t+1} + \bar{W}^{a-34}, z_m, 0, 0) \right\} + \\
& \beta (1 - I_{s,t}) p_{a+1} E_t V^{a+1}(w_{t+1}, z_{t+1}, I_{F,t+1}, 0). \quad (15)
\end{aligned}$$

The state variables of this problem are formally the same as in early life but the interpretation of the index $I_{s,t}$ is now different because it refers to the son's living status, with a value of one meaning that he is alive. As usual the value function of an age a agent is the maximized value of the sum of the utility from the flow of current consumption plus continuation utility with the maximization performed with respect to consumption, the amount of financial assets carried to the next period and the stock market participation decision. In turn the continuation utility can be either of the following two possibilities. First if the index $I_{s,t}$ takes the value of zero the agent has no living descendant, so with probability p_{a+1} he survives and enjoys utility $E_t V^{a+1}(w_{t+1}, z_{t+1}, I_{F,t+1}, 0)$ — the last term in equation (15) —, that is, the value of an agent who has grown one year older and does not have a living descendant, given his financial resources and labor efficiency units. Alternatively the agent may have a living descendant. In this case with probability $p_{a+1} \times p_{a-34}$ both parent and descendant survive to the next period so continuation utility will be given by $E_t V^{a+1}(w_{t+1}, z_{t+1}, I_{F,t+1}, 1)$, that is, the value to an $a + 1$ year old agent whose son is still alive, with probability $p_{a+1} \times (1 - p_{a-34})$ the parent survives but the son dies so the continuation utility will be the one of an $a + 1$ year old agent whose descendant is dead described by the term $E_t V^{a+1}(w_{t+1}, z_{t+1}, I_{F,t+1}, 0)$. Finally and more interestingly with probability $(1 - p_{a+1}) \times p_{a-34}$ the agent himself dies but his son survives so that the transmission of a bequest occurs. Given the altruistic assumption in this case the value to the parent household will be given by $\gamma E_t V^{a-34}(w_{t+1} + \bar{W}^{a-34}, z_m, 0, 0)$, that is, the parent uses the value function of an agent who is 34 years younger than himself next period

which corresponds to the age of his son. The parent household needs to form some estimate of the state his son is in and again I make the assumption that the parent does not have such information and simply takes his son to be “average”. This means that if he leaves resources w_{t+1} then he expects his son will have resources $w_{t+1} + \bar{W}^{a-34}$ where \bar{W}^{a-34} is the average wealth of agents in the cohorts and earning type cell the son belongs to. The parent also assumes that the descendant received the median labor earnings shock z_m and that he has not paid the fixed entry cost. The justification of this choice is similar to the one given when describing the early life problem: on the one hand it is computationally convenient, on the other hand a more sophisticated choice would not have a mayor impact on average life-cycle profiles that are the object of this study. Also a further discount factor $\gamma < 1$ is applied to the descendant’s utility to capture imperfect intergenerational altruism. The description of the late life problem is completed by the resource constraint and the law of motion of financial resources that are the same as the ones reported above when describing the early life problem.

B Numerical Solution Method

In this appendix I will give a brief and informal description of the numerical methods used to solve the life-cycle and dynastic models presented in the paper. In the life-cycle model we know that the value function at age $A+1$ is uniformly equal to zero. Then we can substitute the zero function in the right hand side of equation (9) and perform the maximization to get the decision rules and find V^A , the value function at age A . In the same way we can work backward up to age $a = 1$. Notice that because of the per-period participation cost, at each age we need to find two functions: V^{a,I_p} for $I_p = 0, 1$, that is, the continuation indirect utility in case of non participation and participation in the stock market and then pick the upper envelope.²⁶ At each state space point the optimization must then be performed twice, first with respect to bonds only and then with respect to the two assets jointly and subtracting the participation costs in the budget constraint. The one dimensional optimization is performed by using Brent’s algorithm; the two dimensional problem is solved by exploiting the fact that for any function $f(x, y)$ we can write $\max_{x,y} f(x, y) = \max_x \{ \max_y f(x, y) \}$ and then applying Brent method along both dimensions. The advantage of Brent method is that it exhibits super-linear convergence, so it is faster than bisection — thus also of direct search — but does not require concavity of the

²⁶To simplify notation here I omit the arguments of the V functions.

objective, a property that is violated here because of the fixed cost.²⁷ Once the decision rules are obtained they are simulated for 200000 agents over 6500 periods. Life-cycle profiles are then obtained by averaging over 100 cross-sections to smooth out the effects of a particular sampling history of stock return realizations.

The numerical solution to the dynastic model is more complicated for two reasons. The first one is that under the assumption of altruistic bequest motives we need to know the descendant's utility to compute the progenitor's continuation utility. However the descendant's value function coincides by definition with the progenitor's one. This creates a circularity that is overcome by way of a standard fixed point algorithm. So let $\{V_0^a\}$ for $a = 1, 2, \dots, A$ be the initial guess for the value function, which I take to be the zero function for simplicity. This is used as the descendant's utility to solve by backward induction — as illustrated in the previous paragraph — the parent households' dynamic program. This gives rise to a new guess for the value function $\{V_1^a\}$ for $a = 1, 2, \dots, A$ that can be in turn used as a new descendant's utility. In this way we get a sequence of approximations to the true value function $\{V_0^a\}, \{V_1^a\}, \dots, \{V_N^a\}, \dots$ converging to the fixed point solving the problem. As usual the process is interrupted when two consecutive guesses for the value function are sufficiently close to each other, that is, when $\max\|\{V_N^a\} - \{V_{N-1}^a\}\| < \epsilon$ for some specified small ϵ which I set to 10^{-5} . The policy functions thus obtained are then used to simulate the economy in the same way as the economy with life-cycle agents. The second problem arises because, as it was described in Appendix A, both parent and descendent household need to forecast each other's wealth and to do so they use average wealth in the relevant age group. For this reason we need to iterate on the average life-cycle profiles of wealth as well. More precisely, I start with an initial guess of the wealth profile for each type, solve the dynamic programming problem and simulate decision rules to get a new guess for the life-cycle profile of wealth. The procedure is repeated until two consecutive wealth profiles are sufficiently close to each other.

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