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RATE OF GROWTH OF POPULATION, SAVING AND WEALTH IN THE BASIC LIFE-CYCLE MODEL WHEN THE HOUSEHOLD IS THE DECISION UNIT

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Rate of Growth of Population, Saving and Wealth in the Basic Life-Cycle Model when the Household is the Decision Unit

by Carlo Casarosa* and Luca Spataro†

Abstract

In this paper we explore the impact of the life-cycle dynamics of family composition on the aggregate wealth-income ratio and on the aggregate propensity to save in the hypothesis of life-cycle behaviour. We depart from Modigliani-Brumberg's basic model by assuming that the household, rather than the individual, is the relevant economic unit. In this framework we first explore the single household's life-cycle paths of consumption, saving and wealth and point out the impact on such paths of the timing of births and of the rearing period of the children. We then show that both in a stationary economy and in economy with a steadily growing population the life-cycle dynamics of family composition affects strongly the aggregate wealth-income ratio and the distribution of wealth among the age-cohorts. Further and more importantly, we show that in an economy with a steadily growing population the aggregate propensity to save and the rate of growth of population move in opposite directions for a wide range of values of the timing of births and of the number of children per-household.

Classification JEL: D31, D91, J13.

Keywords: Life-Cycle, Saving, Timing of Births, Population Growth.

1 Introduction

In their 1954 seminal paper Modigliani and Brumberg (M.-B. henceforth) explored the macroeconomic features of an economy in which agents behave in a life-cycle manner. In the simplest specification of their model M.-B. assumed that all agents are equal, earn the same income throughout their active lives and no income during retirement and have preferences which, given the assumption of zero rate of interest, imply a constant rate of consumption throughout their lives. Under these assumptions in the active period the individual propensity to save is positive and constant, while in retirement is negative and constant. Building on this microeconomic behaviour M.-B. showed that in a stationary economy the aggregate propensity to save is zero and that, if the economy grows steadily because of population's growth, the aggregate propensity to save is positive and an increasing function of the rate of growth of population. The explanation of the latter result (hereafter M.-B. Proposition) is very simple: as the rate of growth of population increases, the relative weight of the active workers, who have a positive propensity to save, increases, while the relative weight of the pensioners, who dissave, decreases. Consequently the aggregate propensity to save increases.

Few months after the publication of M.-B.'s paper, M.R. Fisher (1956) criticized the utility function adopted by M.-B. as a bachelor's utility function and modified it to take into account variations in the composition of the household over the life-cycle. In their comments to this paper, Modigliani and Ando (1957) (hereafter M.-A.) accepted Fisher's contribution as a natural extension of the original theory and suggested to substitute the original assumption about the consumption rate of the individual with the assumption that the household finds it convenient to maintain a constant rate of consumption per "equivalent member". On this basis, they came to the conclusion that, since in the real world the life-

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cycle of family size has a very humped shape, the life-cycle of family consumption must have a similar shape too. This implies that, if the household earns a constant income over its active life, its propensity to save over its active life is U shaped.

Some years later Tobin (1967) presented a generalization of M.-B.'s simple life-cycle behaviour. In particular, Tobin adopted a household model very similar to the one adopted by Fisher and M.-A. and, using USA data on family size and income, derived the life-cycle path of household's consumption, saving and wealth under different "adult equivalent" coefficients, different values of the difference between the real rate of interest and the rate of growth of per-capita income and different subjective discount rates.

Tobin's results were obtained in the context of an economy in which income per-head is growing at a constant rate, while Fisher and M.-A. had assumed stationary income. However, the message of the three papers was the same: if the variable size of the family is taken into account, the life-cycle profile of the individual agent's propensity to save is much more complex than the one suggested by the original M.-B. model. This conclusion had important implications for the relationship between the aggregate propensity to save and the rate of growth of population. However, Fisher, M.-A. and Tobin did not draw such implications plausibly because they were interested in different aims.

In the subsequent literature the variable size of the households along the life-cycle has been taken into account in empirical work. In fact, there are many studies[1], which confirm that the time path of family consumption is hump-shaped and others, partly inspired by Modigliani himself (1965, 1970)[2], which introduce dependency rates and, in particular, children dependency rates, or some proxy for the age structure of population, in the explanation of aggregate saving. Some of the authors of these studies and other economists[3] have perceived that, if one takes into account the variable size of the family along the life-cycle, under some circumstances the aggregate propensity to save might be inversely correlated to the rate of growth of population. However, in our view, the nature of these circumstances has not been sufficiently explored. This is unfortunate, since it has left the potential of the life-cycle theory of saving partially unexploited. In the present paper we take some steps in the direction of filling the gap, essentially by stressing the relevance of the timing of births both for the relationship between the aggregate propensity to save and the rate of growth of population and for the distribution of wealth.

The paper is organized as follows: in section 2 we modify the M.-B.'s "stripped down" (or basic) version of the life-cycle model by introducing the constant per-adult-equivalent consumption hypothesis and in section 3 we show the micro implications of our hypothesis and in sections 4-6 the macro implications. In sections 4 and 6 we examine the impact of the timing of births on the aggregate wealth-income ratio and on the distribution of wealth in a stationary economy (section 4) and in an economy with a steadily growing population (section 6). Finally, in section 5, the core of the paper, we examine the implications of our hypothesis for the relationship between the aggregate propensity to save and the rate of growth of population in an economy in which population grows at a constant rate.

2 The basic life-cycle model with constant per-adult-equivalent consumption

The M.-B. basic life-cycle model is characterized by the following assumptions:

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[1] See the reviews by Browning and Lusardi (1996) and Attanasio (1999).

[2] See, Leff (1969), Fry and Mason (1982) and Mason (1987), Higgins and Williamson (1997), Cook (2005).

[3] See, for instance, Castellino and Fornero (1990), p. 85.

1) each working agent maximises his utility function with respect to his own lifetime consumption under the constraint of his lifetime resources;
2) agents work, with certainty, N years and from the moment they enter the labour force they live, again with certainty, L years;
3) labour productivity is constant;
4) during their working life workers get a constant yearly income, Y, and this income is the same for all workers;
5) during their retirement period, lasting L-N years, workers do not receive any income and, therefore, they are able to consume only if, during their working life, they have accumulated a sufficient amount of wealth;
6) the real rate of interest is zero;
7) agents have identical preferences and the latter are such that each agent finds it convenient to maintain a constant level of consumption throughout his life.

In what follows we maintain assumptions 1-6 but, following Fisher and M.-A., we substitute the household to the individual as the decision unit and, consequently, we substitute assumption 7) with the following:

7') households have identical preferences and the latter are such that each household finds it convenient to maintain a constant level of per-adult-equivalent consumption through time[4].

Finally, for the sake of simplicity, we introduce the following additional assumptions:

8) each household is formed by two adults of the same age and by their children, if any;
9) only one of the two adults works outside the household and earns income Y;
10) the two adults generate and rear a certain number of children who stay with their parents for M years and then leave and form immediately a new household;
11) parents know with certainty the number of children they are going to generate, the length of the rearing period and the timing of births, T;
12) children leave the family before the working adult retires;
13) the number of males and females in each age-cohort is the same.

3 Microeconomic behaviour

Given assumptions (1)-(6), (7'), and (8)-(13), the constant per-adult-equivalent consumption of any household is given by:

$$(1) \qquad C = \frac{YN}{2L + fqM}$$

[4] Such an hypothesis can be rationalized by assuming that households solve the following optimization problem:

max $\sum_{s=1}^{L} \beta^{s-1} \frac{N_s (C_s / N_s)^{1-\delta}}{1-\delta}$, subject to $\sum_{s=1}^{L} \frac{(Y - C_s)}{(1+r)^{s-1}} = 0$, where N_s is the number of equivalent adults in the household and assuming that r=0 and β=1 (see Ando and Kennickell (1987) or Blinder et al. (1983)).

where C is per-adult-equivalent consumption, f the number of children generated by the household and q the adult-equivalent of each child. Therefore the amount of consumption of the household in any year is given by:

$$(2) \qquad C_h = \frac{(2+qd)YN}{2L+fqM}$$

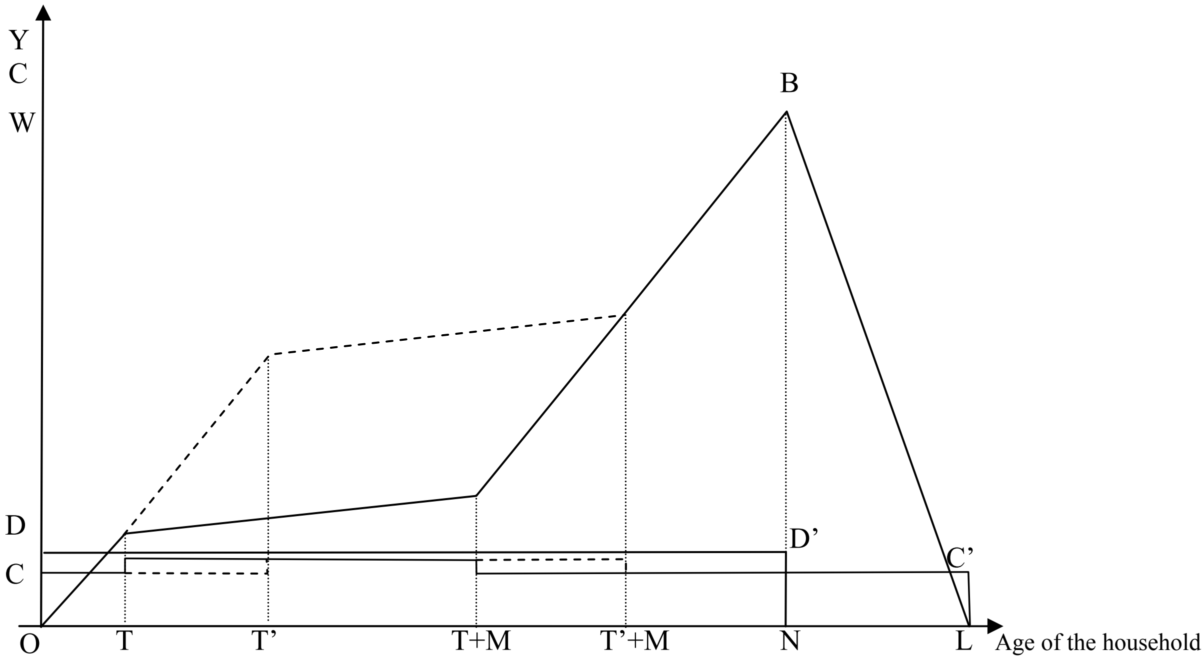
where d is the number of dependent children which are present in the household in that year.

From (4) it is evident that the time paths of consumption, saving and wealth of the household depend on the parameters which determine the constant per-adult-equivalent consumption of the household. In the following we examine some such paths and the influence, on them, of T, q and M, in the simplifying hypothesis that the household generates two twins.

As a starting point we take the values given by M.-B. and M.-A. to L, N and M, *50*, *40*, and *19*, respectively, and assume q=*0.4.* In this and in the following paragraphs this set of numerical assumptions is taken as our *benchmark*.

Let us now consider, first, the case of a household which generates its children with timing T. The broken line CC' of Fig.1, which describes the time path of consumption, shows that the household consumption rises after the children are born, remains constant as long as the children remain within the family and goes back to the initial level after both children have left. Symmetrically, household's saving, which is given by the vertical distance between the DD' line and the CC' line, decreases when the children are born, remains constant till the children stay with the family and goes back to the initial level when the children leave. Finally, during retirement the household dissaves, since income is zero and consumption is positive.

Fig.1. *Life-cycle paths of income, consumption and wealth for different values of T*



The time path of the household's wealth, implicit in the time path of saving, is described by the broken line OBL of Fig.1. This line shows that household's wealth increases continuously during the period of work and decreases continuously during retirement, as in

M.-B. However, while in M.-B. the increase of wealth during the working period is constant, here wealth increases at different rates, since the household's saving changes as the composition of the family changes.

Let us now see what happens if the timing of births increases, say, from T to T'. The time-paths of consumption and wealth of the household are now described by the dotted lines of Fig.1.

It is easy to see that the increase in the timing of births has a striking impact on the life-cycle path of wealth. In fact, while the amounts of wealth held by the household in the first and in the last part of its life are independent from the timing of births, in between the amount of wealth held by the household in the case of early timing is much lower than the amount of wealth held in the case of later timing.

The explanation of this difference is quite simple: if the household generates its children very early, at the time the children are born the amount of wealth accumulated by the household is very low and remains relatively low until the children do not leave the family, since during the children rearing period the household's propensity to save is very low. Vice-versa, if the household generates its children at a later stage, for a relatively long period its wealth grows at a high rate, since in absence of children the household's propensity to save is very high. After the children are born the rate of saving falls and remains very low until the children leave. However, during the child-rearing period, the household's wealth remains relatively high and, in any case, higher than in the case of early timing.

Let us now consider the effects of a change of the consumption weight of the children, q. Equation (2) shows that the level of consumption per-adult-equivalent is a decreasing function of q. However, by deriving C_h with respect to q, we can see that, if q rises, the consumption of the household decreases in the periods it has no dependent children and increases in the period the household has two dependent children.

Fig. 2. *Life-cycle paths of income, consumption and wealth for different values of q*

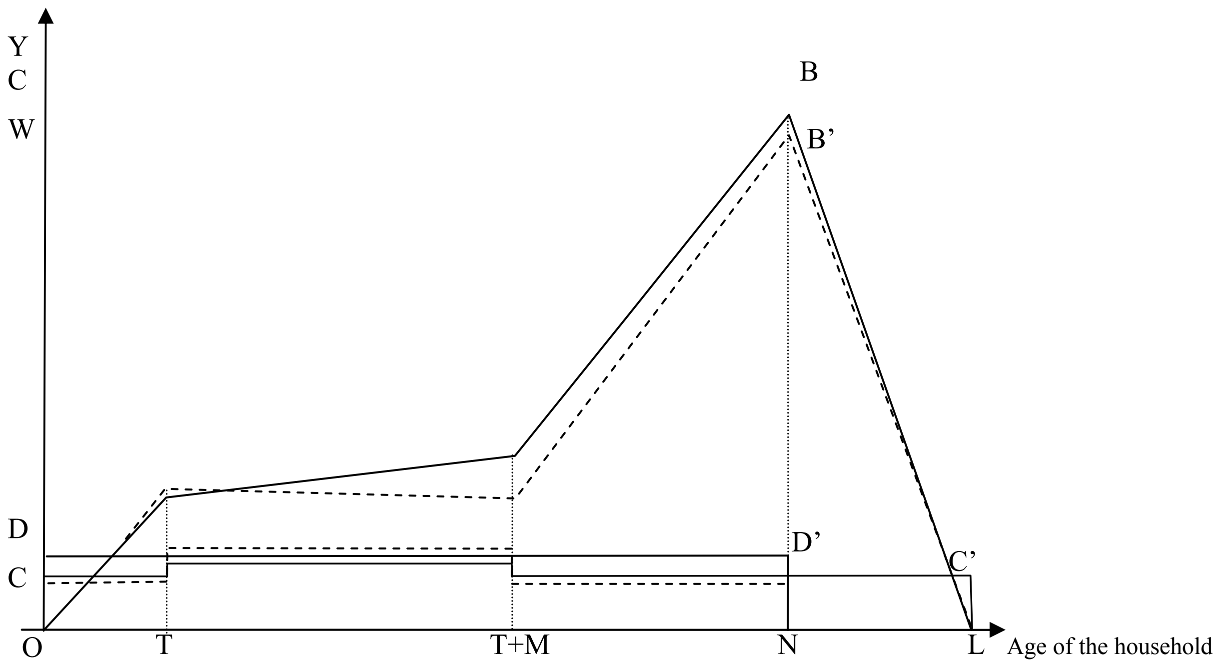
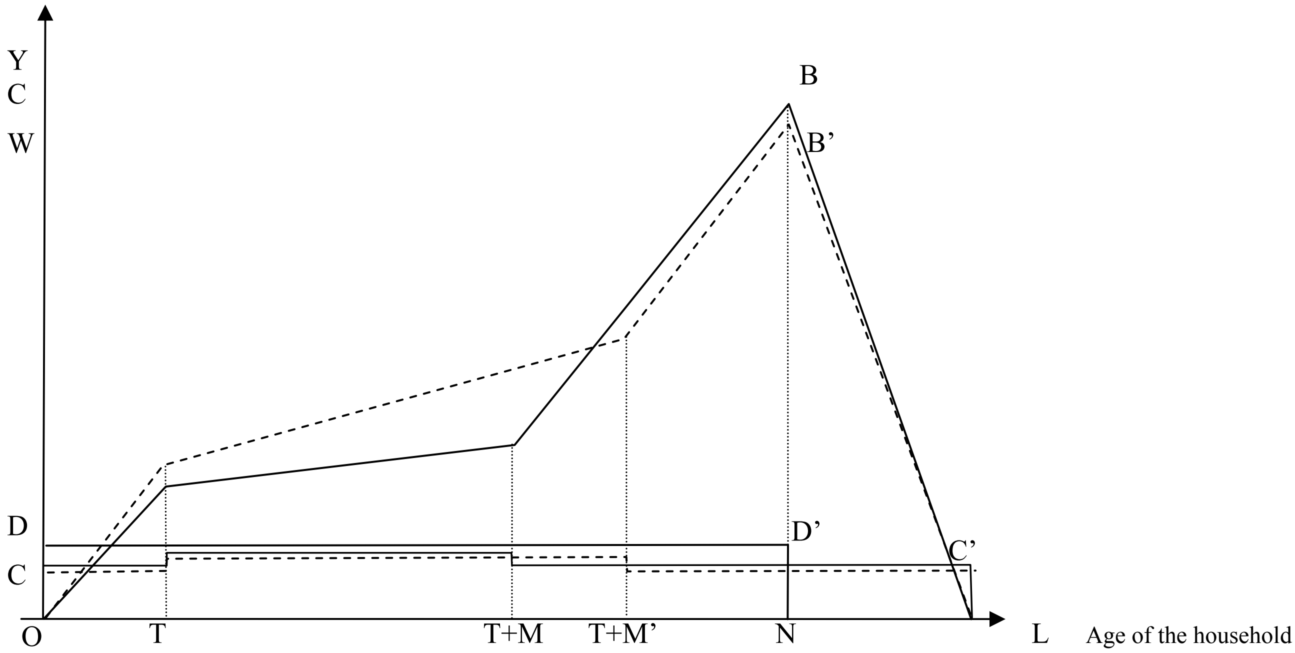


Fig.2 shows the paths of consumption, saving and wealth of a household, which generates its twins with timing 6, when q=0.4 (continuous lines) and when q=0.5 (dotted lines). The consequences of a change of q on the pattern of consumption and saving of the household do not require an explanation. However, it is important to notice that, as a consequence of the saving pattern, in the first part of the working life of the household its wealth is larger when the consumption weight of the children is higher, while for the rest of its life the opposite is true.

Fig. 3. *Life-cycle paths of income, consumption and wealth for different values of M*



Finally, let us now consider the impact of M. Equations (1) and (2) show that both the per-adult-equivalent consumption and the consumption of the household are decreasing functions of M. However, the increase of M has another important influence on the behaviour of the household, since it lengthens the stay of the children in the original family and hence increases the number of years in which the household's saving is lower.

Fig.2 shows the paths of consumption, saving and wealth, of a household which generates a couple of twins with timing T when M=19 (continuous lines) and when M'=25 (dotted lines). Notice that, when M increases, in the retirement period the household's dissaving falls, since the household's consumption is lower, while in the active period the household's saving increases, with the exception of the sub-period (T+M)-(T+M'), during which it falls. As a consequence, when M increases, in the first part of the household's life its level of wealth increases, while in the second part it decreases.

4 Aggregate saving and wealth in a stationary economy

When population is constant it is possible to use Figures 1 and 2 to derive some of the macroeconomic features of the economy, by simply viewing them as cross-sections of a scaled down economic system in which not only L and N, but also T, q and M are the same for all age-cohorts and there is only one household per age-cohort.

In fact, in this context it is easy to conclude that aggregate saving and the aggregate propensity to save are zero, since the aggregate saving of the whole economy is equal to the life-time saving of the individual and we know that the latter is zero.

As for the aggregate wealth-income ratio, Fig.1 shows that it is positively correlated to the timing of births. In fact, when T increases, the amount of wealth held by a certain number of cohorts of active workers increases, while the amount of wealth held by the other cohorts and aggregate income remain constant. In turn, Figs.2 and 3 suggest that the aggregate wealth-income ratio can be either an increasing or a decreasing function of q and M, depending on the value of T. However, in order to reach more precise conclusions it is sufficient to obtain the wealth-income ratio function, which is the following:

$$(3) \qquad w \equiv \frac{W}{NY} = \frac{(L-N)L + 2qM\left[T + \left(\frac{M-N}{2}\right)\right]}{2(L+qM)}.$$

where w is the aggregate wealth-income ratio.

By deriving w with respect to T, M and q we obtain:

$$\frac{\partial w}{\partial T} = \frac{2qM}{2(L+qM)} > 0 \quad \forall q, M, L, N, T$$

$$\frac{\partial w}{\partial M} = q\frac{2TL + 2ML + qM^2 - L^2}{2(L+qM)^2} \geq 0 \Rightarrow T \geq \frac{L^2 - M(2L+qM)}{2L}$$

$$\frac{\partial w}{\partial q} = q\frac{ML(2T + M - L)}{2(L+qM)^2} \geq 0 \Rightarrow T \geq \frac{L-M}{2}$$

The derivative with respect to T does non require any comment. On the contrary, the derivatives with respect to q and M are rather interesting and qualify the intuitions we have drawn from the observation of Figures 2 and 3. In fact, they show that both derivatives are increasing functions of T and that their signs can be either positive or negative depending on the values of the parameters L, M, T and q.

4.1 Timing of births and distribution of wealth in an egalitarian stationary economy

Under the assumptions made at the beginning of the previous paragraph, the broken lines OBL of Figures 1, 2 and 3, considered as cross-sections of different economic systems, describes the scaled down distribution of wealth among the age-cohorts in perfectly egalitarian economies[5]. Therefore, on the basis of the simple observation of these Figures it is possible to see that in an egalitarian stationary economy the distribution of wealth among the age-cohorts depends not only on age, but also on q, M and, especially, on T.

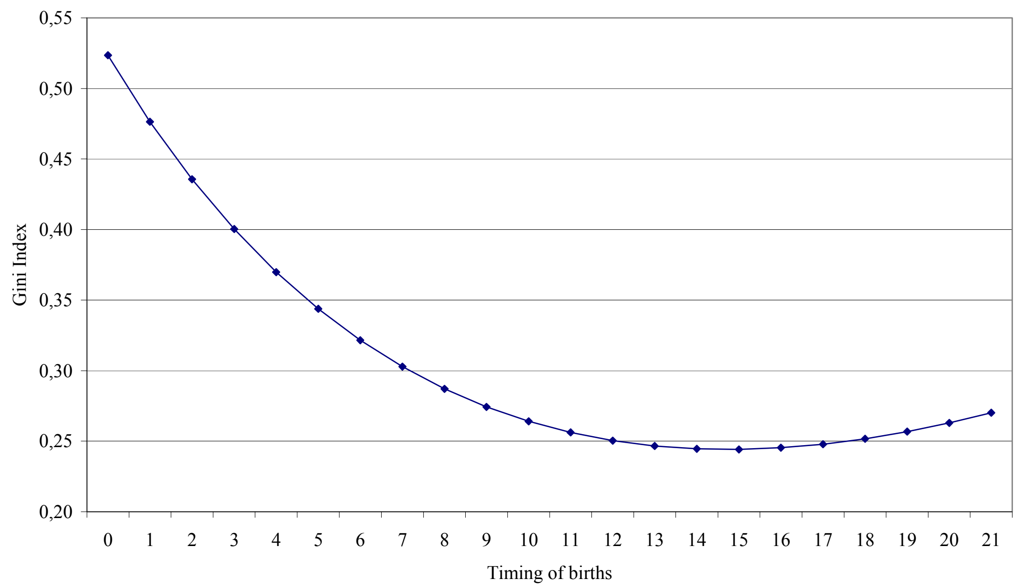
In the following, for the sake of brevity, we limit ourselves to examine the effects of the timing of births on the Gini coefficient in our *benchmark case* under the simplifying assumption that all children are twins. The relationship is reported in Fig.4, which shows that, as the timing of births increases, the Gini coefficient of inequality decreases at a decreasing rate up to a certain value of the timing, beyond which the coefficient becomes an increasing function of the timing.

The intuitive explanation of the shape of the relationship is the following: as we have already noticed in the previous paragraph, when the timing of births increases, the amount of wealth held by a certain number of cohorts of active workers increases, while the amount of

[5] The pioneering contribution on wealth inequality in a life-cycle model is the work by Atkinson (1971). A number of authors have extended this analysis in many directions: see Davies and Shorrocks (2000) for a survey.

wealth held by all other cohorts remains constant. Therefore, the cohorts whose wealth has increased obtain a larger share of total wealth, while all other cohorts obtain a smaller share. The impact of these changes on the Gini coefficient depends, obviously, on the position of the winners and on the loosers in the distribution of wealth shares. In this perspective we first notice that, whatever the timing of births, the share of wealth held by the cohorts of pensioners is a decreasing function of the timing of births. In fact, as the timing of births increases, total wealth increases too, while the amount of wealth held by each cohort of pensioners remains constant. However, the losses of the cohorts of pensioners are likely to have a very small impact on the Gini coefficient, since these cohorts are distributed more or less evenly over all wealth classes. Therefore, for an explanation of the relationship between the timing of births and the Gini coefficient we must look at what happens to the shares of the cohorts of active workers.

Fig. 4. *Gini coefficient as timing of births varies*



For the present purpose the cohorts of active workers can be divided into two subsets: according to whether, as a consequence of an increased timing of births, their wealth increases (and therefore their share of total wealth increases) or remains constant (and therefore their share of total wealth falls). Now, when the timing of births is very low, let us say nil, the cohorts which obtain a wealth increase, as a consequence of an increase in the timing of births, are mostly cohorts of very young workers, which belong to the poor end of the distribution, while the cohorts belonging to the other subset of active workers are cohorts of middle age and old workers, which are mostly well-off. Therefore, it is easy to understand why, when the timing of births is very low, an increase of the timing of births brings about a substantial reduction of the Gini coefficient.

However, as the timing of births increases, the subset of cohorts which loose from an increase of the timing of births include an increasing number of very young and therefore "poor" cohorts, while the subset of cohorts which gain include an increasing number of cohorts which are relatively rich. Therefore, the improvement in the distribution of wealth, due to the reduction of the wealth shares of the elder (and richer) cohorts of active workers, is increasingly counteracted by a worsening of the distribution caused by the reduction of the wealth shares of the younger (and poorer) cohorts and by the gain of some cohorts which are relatively well-off. Hence, the convexity of the curve of Fig. 4. At some value of the timing the two opposite forces balance themselves and beyond that value the Gini coefficient increases, since the improvement due to the loss of wealth shares of the richest cohorts of active workers is more than compensated by the worsening due to the losses of the poorest cohorts and by the gains of relatively rich cohorts.

5 Aggregate saving in an economy with a steadily growing population

5.1 The determinants of the rate of growth of population

If the length of life is given, population can grow steadily if and only if the households of each cohort generate, on average, more than two children. However, when households generate, on average, more than two children the rate of growth of population is not only an increasing function of the average number of children per-household, but also a decreasing function of the natural age of the parents at the time children are born. The steady state relationship between the rate of growth of population, on one side, and the average number of children generated by the households and the natural age of the parents, on the other, can be easily derived if we assume, for the sake of simplicity, that all households generate their children at the same natural age, X, and that all age-cohorts generate the same average number of children per-household. In fact, if we call R_X the number of parents of natural age X and R_0 the number of newborns, under our assumptions we have: $R_0 = \frac{f}{2} R_X$, where f is the average number of children generated by each household. Now, since in steady state $R_0 = (1+n)^X R_X$, it follows that the steady state annual rate of growth of population is given by $n = \left(\frac{f}{2}\right)^{\frac{1}{X}} - 1$. In our model X=M+T and therefore we can say that *the rate of growth of population is an increasing function of the average number of children generated by the households and a decreasing function of the timing of births and of the child-rearing period.*

If agents chose their consumption and saving paths independently of family composition, the way *n* changes would not influence the time path of consumption and saving and, hence, the aggregate propensity to save. On the contrary, in a model like ours, in which the paths of consumption and saving of the decision unit, the household, depend on the dynamics of its membership, the way the change of *n* is brought about is crucial for the aggregate propensity to save. To show this, in the following we derive, first, the aggregate propensity to save function and then examine how it is influenced by changes of the number of children generated by the households and of the timing of births. For the sake of brevity we do not deal with the effects of changes of the rearing period, which in any case appear to be less relevant.

5.2 The aggregate propensity to save function

Aggregate saving is given by:

$$S = s_a YA - (1 - s_{wc})YP = s_{wc} YA_{wc} + s_c YA_c - (1 - s_{wc})YP$$

where s_a is the aggregate propensity to save of the households of active workers, s_{wc} the propensity to save of the households of active workers without dependent children, s_c

the propensity to save of the households of active workers with dependent children, A, A_{wc} and A_c, the total number of active workers, the number of workers without children and the number of workers with children.

Therefore the aggregate propensity to save can be written as:

(4) $$s = s_a - (1 - s_{wc})p = s_{wc}l_{wc} + s_c l_c - (1 - s_{wc})p = s_{wc} - (s_{wc} - s_c)l_c - (1 - s_{wc})p$$

where l_c and $l_{wc} = 1 - l_c$ are the shares of active workers with and without dependent children, with $l_c = \frac{\sum_{r=0}^{M-1}(1+n)^{L-(M+T)+r}}{\sum_{j=0}^{N-1}(1+n)^{L-N+j}}$, and p is the ratio between the number of pensioners and the number of active workers, with $p = \frac{\sum_{i=0}^{L-N-1}(1+n)^i}{\sum_{j=0}^{N-1}(1+n)^{L-N+j}}$.

As for s_{wc} and s_c, if all households generate the same number of children, we have:

$$(a) \quad s_{wc} = \frac{2(L-N) + fqM}{2L + fqM} \qquad \text{and} \qquad (b) \quad s_c = \frac{2(L-N) - fqN + fqM}{2L + fqM}$$

where, clearly, $s_{wc} > s_w$.

In the following, for the sake of simplicity, we assume that all households generate the same number of children even when the latter are not an integer. As a consequence, the propensities to save of the two subsets of households will be given by functions (a) and (b).

5.3 Number of children per-household and aggregate propensity to save

From what we have said in sections 5.1 and 5.2 it is evident that an increase of the number of children generated by the households affects the aggregate propensity to save, since it affects both the propensities to save of the three classes of households (active workers without children, active workers with children and pensioners) and the weights of such classes. More precisely, when the number of children generated by the households increases, the rate of per-adult equivalent consumption decreases and, therefore, the propensity to save of the households of active workers without children increases while both the negative savings of the households of pensioners and the propensity to save of the households of active workers with children decrease. As for the weights of the three classes of households, we can say that, as the rate of growth of population increases, because f increases, the weight of the pensioners falls, as in M.-B.'s model, while the weights of the households of active workers with children, l_c, and without children, l_{wc}, can move either way, depending on several parameters and, in particular, on the timing of births, the length of the rearing period and the rate of growth of population.

Therefore, when the rate of growth of population increases because of an increase of f, the rate of dissaving of the pensioners falls, as in M.-B., and at a greater extent, since in our model, in addition to the loss of weight of the pensioners we have a decrease of their rate of dissaving. However, the aggregate propensity to save of the households of the active workers, s_a, which is constant in M.-B., may move either way depending on the values of the parameters of the model. As a consequence, while in M.-B. the aggregate propensity to save is an increasing function of the rate of growth of population, within the context of our model it is not possible to say, a priori, whether an increase of n, caused by an increase of f, causes an increases or a decrease of the aggregate propensity to save.

To clarify the problem let us derive s with respect to f:

$$\frac{ds}{df}=\frac{ds_a}{df}-\frac{d[(1-s_{wc})p]}{df}=\left[(1-l_c)\frac{ds_{wc}}{df}+l_c\frac{ds_c}{df}-(s_{wc}-s_c)\frac{dl_c}{df}\right]-\left[(1-s_{wc})\frac{dp}{df}-p\frac{ds_{wc}}{df}\right]$$

Now, while the sign of the second term on the right hand side is certainly negative, the sign of the first, ds_a/df, is uncertain since the first term of ds_a/df is positive, the second negative and the third positive or negative according to whether dl_c/df is negative or positive. As a consequence, the sign of ds/df is uncertain. However, since for realistic values of f, let us say 2<f<5, both l_c and dl_c/df are decreasing functions of T, we can say that, the lower the timing of births the higher the probability of an inverse correlation between s_a and f and, hence, between s and f.

To show that, indeed, for sufficiently low values of T, the aggregate propensity to save can be a decreasing function of f, we present some numerical simulations based on our *benchmark case*. Figures 5a and 5b show the relationships between f, on the one side, and s and ds/df, on the other, under different values of T: 0, 3 and 6. More precisely, the Figures show that the aggregate propensity to save is an increasing function of f, up to a critical value, f*, beyond which s becomes a decreasing function of f. Table 1 reports the critical values of f and the corresponding values of the rate of growth of population for different values of T.

Fig. 5a: *s-f locus for different values of T (T=0, 3, 6) .*

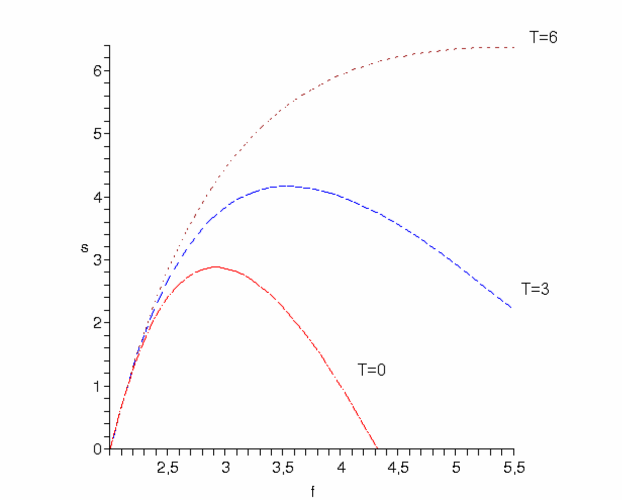
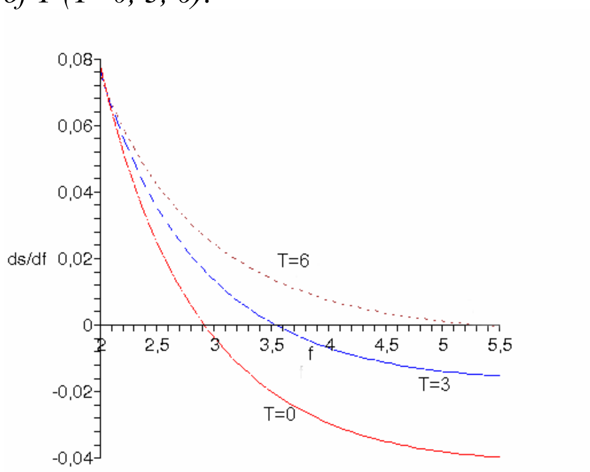


Fig. 5b: *ds/df locus for different values of T (T=0, 3, 6).*



Tab.1. Critical value of f and n in the benchmark case
(T=0 q=0.4, N=40, L=50, M=19).

Timing	T=0	T=1	T=2	T=3	T=4	T=5	T=6
f*	2.916	3.080	3.283	3.545	3.900	4.428	5.410
***n*(f*) (%)**	2.005	2.182	2.389	2.636	2.946	3.367	4.061

It should be noticed that both the Figures and the Table show that f* and n(f*) are increasing functions of T, in the interval 0≤T≤6, while, for T>6, f* becomes unrealistically large.

The explanation of the shape of the s-f function and of the relationship between f* and T is the following.

If T≤6 and 2<f<5, the aggregate propensity to save of the households of active workers is a decreasing function of f essentially because, as f increases, the share of households of active workers with children increases too. However, when f is very close to 2 (and, hence, n is very close to zero) the reduction of the rate of dissaving of the pensioners, caused by an increase of f (and, consequently, of n), is larger than the reduction of the aggregate propensity to save of the households of active workers. Therefore, for f sufficiently close to 2 the aggregate propensity to save is an increasing function of f, as shown in Fig. 5[6]. As f increases, both the rate of dissaving of the pensioners and the aggregate propensity to save of the households of active workers get smaller and smaller. However, the former decreases more slowly than the latter, essentially because the increase of the weight of the households of active workers with children, caused by the increase of f (and, hence, of n), is larger than the loss of weight of the pensioners. Therefore, as f increases, the increase of the aggregate propensity to save becomes smaller and smaller until at some critical value, f*, it becomes zero and then negative.

As for the positive relationship between f* and T, it depends on the fact that, if f is not too close to 2, both ds_a/df and ds/df are increasing functions of T, essentially because l_c and dl_c/df are decreasing functions of T. It follows that, as T increases f* increases too.

Tab.2. Critical value of f and corresponding values of n (in parentheses).

	T=0	T=1	T=2	T=3	T=4	T=5	T=6
Benchmark case	2.916 (2.005)	3.080 (2.299)	3.328 (2.329)	3.545 (2.636)	3.900 (2.946)	4.428 (3.367)	5.410 (4.061)
q=0.44	2.823 (1.832)	2.977 (2.009)	3.020 (2.214)	3.244 (2.459)	3.745 (2.765)	4.238 (3.178)	5.139 (3.847)
M=20	2.944 (1.952)	3.117 (2.136)	3.335 (2.351)	3.619 (2.612)	4.014 (2.945)	4.630 (3.415)	5.956 (4.286)
N=41	2.774 (1.737)	2.918 (1.907)	3.095 (2.102)	3.322 (2.333)	3.624 (2.618)	4.060 (2.994)	4.804 (3.568)
L=51	3.014 (2.183)	3.187 (2.357)	3.400 (2.560)	3.674 (2.803)	4.042 (3.107)	4.587 (3.519)	5.581 (4.191)
N=41,L=51	2.879 (1.937)	3.033 (2.104)	3.221 (2.296)	3.461 (2.524)	3.778 (2.805)	4.235 (3.175)	5.007 (3.739)

Let us now see how the critical value of f is affected by changes of the parameters. The numerical results of the exercise are reported in Table 2, which shows that the critical value of f is an increasing function of M and L and a decreasing function of q and N. Finally, the last row of the Table shows that, if N and L increase at the same extent, so as to leave the period of retirement constant, the critical value of f falls.

5.4 Number of children per-household, rate of growth of population and aggregate propensity to save

In section 5.1 we have seen that the rate of growth of population is an increasing function of the number of children per-household, while in section 5.3 we have examined the relationship between the aggregate propensity to save and the average number of

[6] In fact, it can be shown that $\lim_{f \to 2} \frac{\partial s}{\partial f} = w_2 \frac{\partial n}{\partial f}$, which is positive, provided that w_2>0, since $\frac{\partial n}{\partial f} > 0$, where w_2 is the value of w when f=2.

children per household. In this section we use the results of these two paragraphs in order to show the co-movements of the aggregate propensity to save and of the rate of growth of population when the change of the rate of growth of population is brought about by changes in the number of children per-household.

Fig. 6a. *n-f locus for different values of T (T=0…6).*

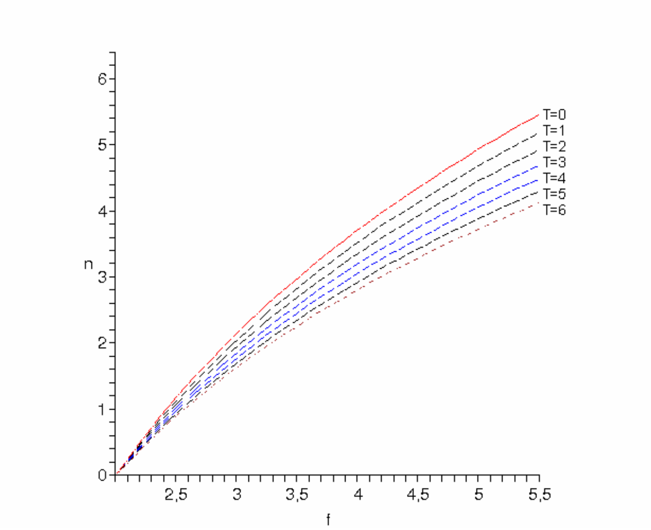
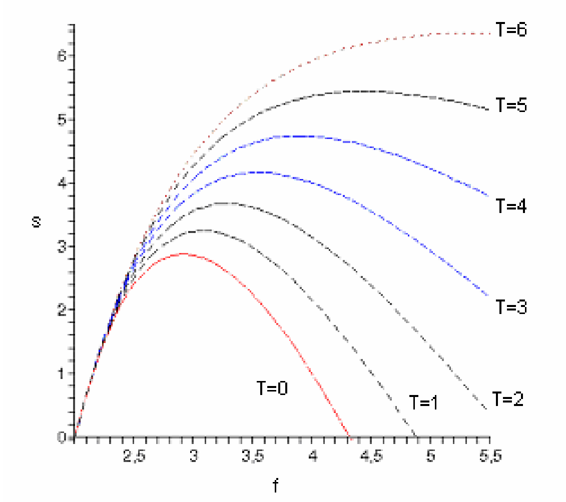


Fig. 6b. *s-f locus for different values of T (T=0…6).*



Let us consider Figures 6a and 6b which show, for our *benchmark case*, the relationships between f and n and between f and s under alternative values of T. By simply looking at the two Figures we can state that, if the timing of births is less than 6, up to a critical value of f, n and s are increasing functions of f. Viceversa, beyond the critical value of f, n increases with f, but s decreases. This means that, beyond the critical value of f, the rate of growth of population and the aggregate propensity to save move in opposite directions.

If the timing of births is 6 the critical value of f is outside the realistic range of values of this variable. Therefore we can say that, if T=6, in the realistic range of values of f, both n and s are increasing functions of f. Obviously the same can be said for T>6, since we know that the critical value of f is an increasing function of T.

A similar discussion could obviously be made for different sets of values of the parameters, such as those of Table 2, and it is not difficult to see that, at least in the neighbourhood of the benchmark case, things would not change too much. At any rate, in the perspective of this paper it is not necessary to belabour in detail on the point. In fact, the results obtained in the *benchmark case* allow us to conclude that, when population grows because the number of children per household increases, there are sets of values of the parameters such that the aggregate propensity to save and the rate of growth of population move in opposite directions. However, if the timing of births is sufficiently high, the aggregate propensity to save and the rate of growth of population move in the same direction.

5.5 Timing of births and aggregate propensity to save

Let us now examine the consequences, on the aggregate propensity to save, of changes in the timing of births.

From the formulas (a) and (b) of section 5.2 it is evident that s_{wc} and s_c are independent from T, while l_c depends on T both directly and indirectly, through n, and p depends on T indirectly, again through n. Therefore, when T changes we have both a change in the relative number of pensioners, due to the change of n, as in M.-B., and a change of the aggregate propensity to save of the households of active workers, due to a change of the relative weight of the households of active workers with children, l_c.

More precisely, by deriving s with respect to T we have:

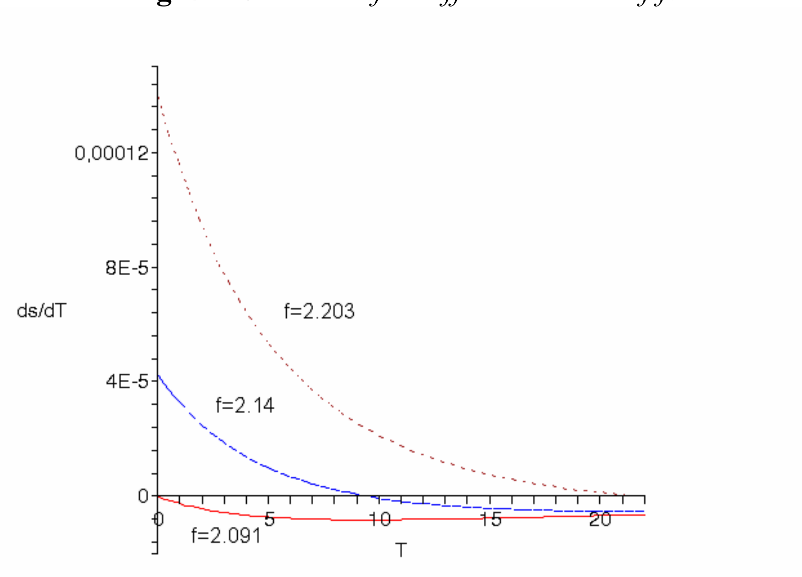
$$(5)\qquad \frac{ds}{dT}=\frac{ds_a}{dT}-(1-s_{wc})\frac{dp}{dT}=-(s_{wc}-s_c)\frac{dl_c}{dT}-(1-s_{wc})\frac{dp}{dT}.$$

where $dl_c/dT=[\partial l_c/\partial T+(\partial l_c/\partial n)(\partial n/\partial T)]$.

On the form of dp/dT there is no ambiguity since, as the timing of births increases and the rate of growth of population decreases, the ratio between the number of pensioners and the number of active workers increases.

Things are more complicated with ds_a/dT. In fact, while $\partial l_c/\partial T<0$, the sign of $\partial l_c/\partial n$ depends on T and, more precisely, is positive for T relatively low and negative for T relatively high. Therefore we can say that, when T is relatively low, dl_c/dT is certainly negative, while when T is relatively high its sign is, in principle, ambiguous. Actually, it can be shown (see Appendix) that dl_c/dT is always negative. Therefore, we can state that ds_a/dT is always positive essentially because a change of the timing of births causes a change in the opposite direction of the relative weight of the households of active workers with children, that are the group with lower propensity to save.

Fig. 7. *ds/dT locus for different values of f.*



Now, since the first term on the right hand side of eq. (5) is positive and the second negative, the sign of ds/dT is ambiguous. In fact, there are sets of values of the parameters, L, N, M, q, f and T, for which ds/dT is positive and others for which ds/dT is negative. However, it is possible to show that, for a wide range of "reasonable" values of L, N, M, q, f and T, $ds/dT>0$.

Let us start from our *benchmark case* and consider the three curves of Fig.7, which represents the relationship between *ds/dT* and T under three different values of f.

The highest curve shows that, if f=2.203, the aggregate propensity to save is an increasing function of the timing of births in the interval of possible timings, in our case 0-

21, and reaches its maximum at the maximum possible timing, 21. The lowest curve shows that, if f=2.091, at timing zero *ds/dT*=0, while for T>0 the aggregate propensity to save is a decreasing function of the timing of births. Finally, the intermediate curve shows that, if f=2.14, the aggregate propensity to save is an increasing function of the timing of births up to T=9, where it reaches its maximum, and then becomes a decreasing function of the timing.

On the basis of the numerical results underlying the curves of Fig.7 and of the fact that *ds/dT* is an increasing function of f, we can state that in our *benchmark case*, over the interval of possible timings of births, the aggregate propensity to save is

a) a decreasing function of the timing of births for f≤2.091;
b) an increasing function of the timing of birth for f≥2.203;
c) an increasing function of the timing of births up to a certain value of the timing of births that we define T* and a then a decreasing function of the timing for 2.091<f<2.203.

We call f=2.091 the upper boundary of the MB (Modigliani-Brumberg) zone and f=2.203 the lower boundary of the CS (Casarosa-Spataro) zone, since the former is the maximum value of f which guarantees a negative relationship between the aggregate propensity to save and the timing of births and the latter is the minimum value of f which guarantees a positive relationship between the aggregate propensity to save and the timing of births.

Let us now see how these boundary values are influenced by changes of q, M, L and N. The results of the computations for set of values close to our *benchmark case* are presented in Table 3, which shows that both boundary values are decreasing functions of M, q and N and increasing functions of L.

Tab.3. Boundary values of f for the CS and MB regions.

	Benchmark case	**q=0.44**	**M=20**	**N=41**	**L=51**	**N=41, L=51**
f_{CS}	2.198	2.023	2.071	ds/dT>0 for any f>2	2.434	2.201
f_{MB}	2.091	2.011	2.035	ds/dT>0 for any f>2	2.190	2.093

5.6 Timing of births, rate of growth of population and aggregate propensity to save

In section 5.1 we have seen that the rate of growth of population is a decreasing function of the timing of births, while in section 5.5 we have examined the relationship between the aggregate propensity to save and the timing of births. We now use the results of these two paragraphs to derive the co-movements of the aggregate propensity to save and of the rate of growth of population when the timing of births changes.

First, in our *benchmark case*, we can state the following:

1) if f≤2.091, the aggregate propensity to save and the rate of growth of populations move in the same direction and therefore the M.-B. Proposition is confirmed;
2) if f≥2.203, the aggregate propensity to save and the rate of growth of population move in opposite directions and therefore the M.-B. Proposition does not hold;
3) if 2.091<f<2.203, in the interval 0-T* the aggregate propensity to save and the rate of growth of population move in opposite directions, while in the interval T*-T_{max} the aggregate propensity to save and the rate of growth of population move in the same direction.

If the values of the parameters are different from the *benchmark case*, the values of f which define the limits of validity of M.-B. Proposition change, as it is shown by the results of the simulations of Table 3. However, on the basis of these simulations we can conclude that, in the neighbourhood of our *benchmark case*, when the rate of growth of population changes because the timing of births changes, the aggregate propensity to save and the rate of growth of population move in opposite directions, unless the rate of growth of population is very low. The fact that, in spite of what happens in general, M.-B. Proposition maintains its validity when the rate of growth of population is very low is not surprising. In fact, since for n=0 the aggregate propensity to save is zero and for n>0, but sufficiently low, the aggregate propensity to save is positive, for the principle of continuity it must be true that at least in some neighbourhood of n=0 the rate of growth of population and the aggregate propensity to save move in the same direction, as stated by M.-B. Proposition.

6 *Wealth-income ratio, distribution of wealth and rate of growth of population*

6.1 Wealth-income ratio and rate of growth of population

As we know, in an economy in which labour productivity is constant and population grows at a constant rate, n, the steady-state wealth-income ratio is equal to s/n. Now, while in M.-B.'s individualistic model n is an independent variable and therefore it makes sense to examine the relationship between w and n, in our model n depends on f, T and M, which belong to the set of parameters which determine s. Therefore, we can only look for the relationship between the parameters of the model, on one side, and n and w, on the other. In this paragraph we'll limit our attention to the role of f and T.

In our model the wealth-income ratio corresponding to any given rate of growth of population depends on the values of f and T which bring about such a rate of growth of population. This is clearly shown by Table 4, which reports the values of the wealth-income ratio corresponding to different couples of values of f and T which yield a rate of growth of population of 1%. In particular, Table 4 shows that, as the number of children per household and the timing of births increase, the wealth-income ratio increases too. The increase is essentially due to the impact, on the distribution of wealth, of changes of the timing of births, which we have already examined if par. 4.

Tab.4. Wealth-income ratio (w) and demographic variables in the M.-B .and C-S. model (with N=40, L=50, q=0.4, M=19).

	M.-B. model	C.-S. model			
f	-	2.416	2.539	2.669	2.805
T	-	0	5	10	15
n	1%	1%	1%	1%	1%
w	4.5	2.181	2.949	3.743	4.564

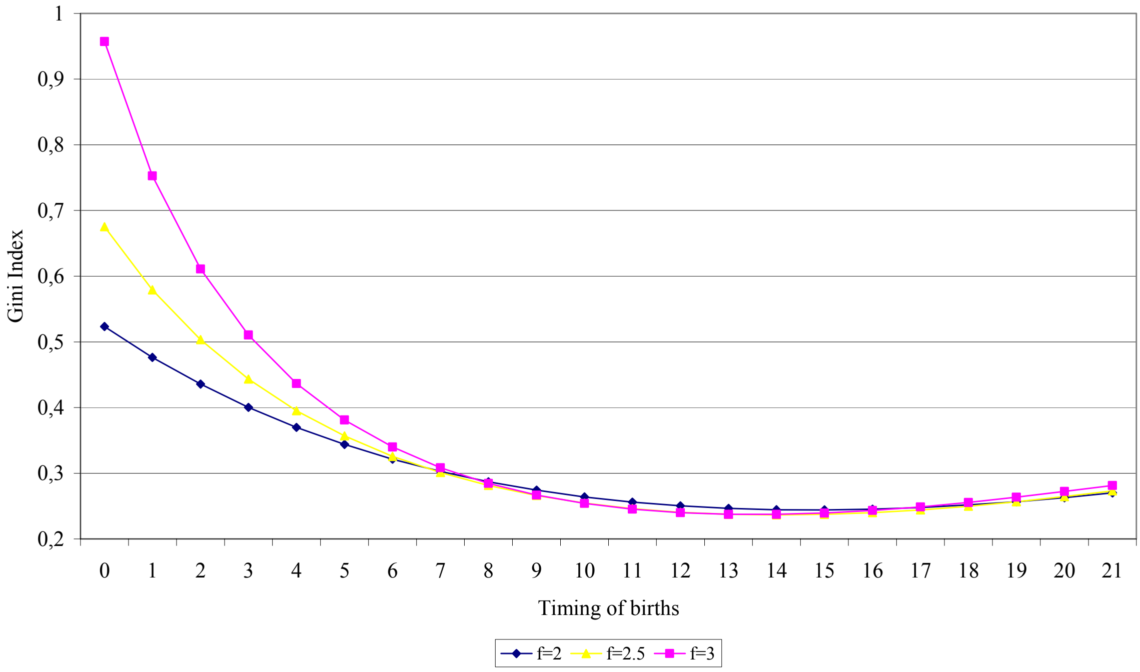
As for the co-movements of n and w, we must distinguish between the case that the parameters of the model are such as to invalidate the M.-B. Proposition and the case in which the Proposition holds. In fact, in the former case the propensity to save and the rate of growth of population move in opposite directions and, therefore, it is certain that the rate of growth of population and the wealth-income ratio move in opposite directions. On the contrary, when M.-B.'s proposition holds, the co-movements of the rate of growth of population and of the wealth-income ratio can be either in opposite directions or in the same direction. However, on the basis of our simulations, centred on the *benchmark case*, we can say that the rate of growth of population and the wealth-income ratio move in the same direction only if the change of the rate of growth of population is brought about by a change in the number of children per household and, in addition, the number of children per household and the timing of births are exceptionally high. This means that, in any realistic case, the traditional M.-B.'s proposition, that the rate of growth of population and the wealth-income ratio move in opposite directions, holds.

6.2 Distribution of wealth in a growing egalitarian economy

Let us now discuss very briefly the matter of the distribution of wealth in an egalitarian economy which grows because population grows at a constant rate.

In view of what we have said on the time path of the household's accumulation of wealth, it should be obvious that the distribution of wealth among the age cohorts of an egalitarian economy depends on all the parameters of the model. In particular, it depends, both directly and indirectly, on f and T. As a consequence, in order to analyse the co-movements of the Gini coefficient and of the rate of growth of population, we have to distinguish according to whether the change of the rate of growth of population is brought about by a change of f or by a change of T.

Fig. 8: *Gini coefficient as timing of births varies, for f=2, 2.5 and 3*



Let us consider the curves of Fig.8, which show the relationship between the Gini coefficient and the timing of births, in our benchmark case, under three different hypotheses about the number of children per household: 2, 2.5 and 3. The curves show that, whatever the number of children per household, the relationship between the Gini coefficient and the timing of births is very similar to that of a stationary economy. As a consequence, when the rate of growth of population falls because the timing of births increases, the Gini coefficient and the rate of growth of population move in the same direction up to some value of T, beyond which the two variables move in opposite directions, although at a limited extent.

As for the influence of changes of f, Fig.8 shows that, when f increases, the Gini coefficient increases substantially for low values of T, falls slightly for intermediate values of T and rises again, slightly, for high values of T. Therefore we can say that, if the timing of births is relatively low and the rate of growth of population changes because f changes, the rate of growth of population and the Gini coefficient move in the same direction and that, the lower the timing of births the stronger the impact of an increase of f on the Gini coefficient. Vice versa, when T is sufficiently high, the influence of f on the Gini coefficient is almost nil.

7 Conclusions

In this paper we have explored, in the framework of a simple life-cycle model à la Modigliani-Brumberg, the micro and macro implications of the hypothesis that the relevant decision unit is the household, rather than the individual.

At the micro level we have shown that the time paths of consumption, saving and wealth of the household are affected by the number of children generated by the household, f, by the consumption weight of the latter, q, by the timing of births, T, and by the rearing period, M.

At the macro level we have first shown that in a stationary egalitarian economy the aggregate wealth-income ratio and the degree of inequality in the distribution of wealth among the age-cohorts, as measured by the Gini coefficient, depend not only on the parameters of the original life-cycle model, but also on T, M, and q. In particular, we have shown that the Gini coefficient is a decreasing function of the timings of births up to a high value of the latter and then an increasing function.

Then we have considered an economy in which population grows at a constant rate while income per head remains constant. We have shown that both the rate of growth of population and the aggregate propensity to save depend on f, T and M and we have examined the co-movements of the aggregate propensity to save and of the rate of growth of population (a) in the case of a change of f and (b) in the case of a change of T.

For case (a) we have shown that, if T is sufficiently low and f sufficiently high, the aggregate propensity to save and the rate of growth of population move in opposite directions. On the contrary, if T is sufficiently high or f sufficiently low, the aggregate propensity to save and the rate of growth of population are positively related.

As for case (b) we have shown that the aggregate propensity to save and the rate of growth of population move in the same direction only if the number of children per-household is very close to two and, therefore, the rate of growth of population close to zero. In all other cases the two variables move in opposite directions.

Further, we have shown that in an economy with a steadily growing population the aggregate wealth-income ratio depends strongly on the timing of births and that, unless the number of children and the timing of births are exceptionally (and unrealistically) high, the wealth-income ratio and the rate of growth of population move in opposite directions.

Finally, we have shown that, whatever the number of children per household, the Gini coefficient is a decreasing function of the timing of births up to a high value of the latter and that, if the timing of births is relatively low, the Gini coefficient is an increasing function of the number of children per-household while, if the timing of births is relatively high, the impact of the number of children per-household on the Gini coefficient is almost nil.

We are fully aware that the model presented in this paper is overly simplified. However, we are convinced that the qualitative results we have obtained are rather robust and, consequently, relevant both for theoretical and empirical research. In particular, our analysis, besides enriching the original formulation of the M.-B. model with new and more realistic assumptions, shows that the co-movements of the rate of growth of population, on one hand, and the aggregate propensity to save, the wealth-income ratio and the Gini coefficient, on the other, can be, under many realistic circumstances, of opposite signs to the ones stemming from the traditional formulation of the M.-B. model.

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Appendix: The demographic weights

In this Appendix we show that **a)** $\frac{dl_c}{dT} < 0$ and **b)** $\frac{dp}{dT} > 0$.

a) As for $l_c = (1+n)^{N-X} \frac{(1+n)^M - 1}{(1+n)^N - 1}$, the derivative with respect to T is the following: $\frac{dl_c}{dT} = \frac{\partial l_c}{\partial T} + \frac{\partial l_c}{\partial n}\frac{\partial n}{\partial T}$. Since $\frac{\partial l_c}{\partial T} = -l_c \log(1+n)$ and recalling that $(1+n) = \left(\frac{f}{2}\right)^{\frac{1}{M+T}}$, we have that $\frac{\partial n}{\partial T} = -\frac{1+n}{M+T}\log(1+n) \leq 0$ for $n \geq 0$. Moreover,

$$\frac{\partial l_c}{\partial n} = \frac{\left[(1+n)^M - 1\right](1+n)^{N-X}}{\left[(1+n)^N - 1\right](1+n)}\left[N - X + \frac{M(1+n)^M}{(1+n)^M - 1} - \frac{N(1+n)^N}{(1+n)^N - 1}\right] = \frac{l_c}{(1+n)}\left[N - X + \frac{M(1+n)^M}{(1+n)^M - 1} - \frac{N(1+n)^N}{(1+n)^N - 1}\right].$$

Finally, one obtains:

$$\frac{dl_c}{dT} = -\frac{l_c \log(1+n)}{X}\left[N + \frac{M(1+n)^M}{(1+n)^M - 1} - \frac{N(1+n)^N}{(1+n)^N - 1}\right] = -\frac{l_c \log(1+n)}{X\left[(1+n)^M - 1\right]\left[(1+n)^N - 1\right]}\underbrace{\left\{M(1+n)^{M+N} - (M+N)(1+n)^M + N\right\}}_{H}.$$

Since all other terms are positive if *n>0*, we can focus on the sign of *H*. First, note that if *n=0 H=0*. Next, by deriving *H* with respect to *n*, one gets:
$\frac{dH}{dn} = (M+N)(1+n)^{M-1}\left[(1+n)^N - 1\right] > 0.\ \forall n > 0.$ Hence, *H* is positive, and $\frac{dl_c}{dT}$ negative if *n>0*.

b) Recalling that $p = \frac{1-(1+n)^{L-N}}{(1+n)^{L-N}\left[1-(1+n)^N\right]}$, then $\frac{dp}{dT} = \frac{\partial p}{\partial n}\frac{\partial n}{\partial T} =$

$$= -\frac{p\log(1+n)}{(M+T)}\left\{-N + \frac{L-N}{(1+n)^{L-N} - 1} - \frac{N}{(1+n)^N - 1}\right\}$$

$$= \frac{p\log(1+n)}{(M+T)}\frac{1}{\left[(1+n)^{L-N} - 1\right]\left[(1+n)^N - 1\right]}\underbrace{\left\{N\left[(1+n)^L - 1\right] - L\left[(1+n)^N - 1\right]\right\}}_{H'}.$$

Since all other terms are positive if *n>0*, we can focus on the sign of H'. First, note that if *n=0* $H'=0$. Next, by deriving H' with respect to *n*, one gets: $\frac{dH'}{dn} = LN(1+n)^{N-1}\left[(1+n)^{L-N} - 1\right] > 0.\ \forall n > 0.$ Consequently, H' is positive if *n>0* and, thus, $\frac{dp}{dT} > 0$.

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