Preliminary draft

A PUBLIC GUARANTEE OF A MINIMUM RETURN TO DEFINED CONTRIBUTION PENSION SCHEME MEMBERS

by Giuseppe Grande and Ignazio Visco (*)

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ABSTRACT

The recent financial crisis has clearly demonstrated the exposure of defined contribution (DC) pension scheme members to financial market tail risks, i.e., to rare extreme drops in financial asset prices. This paper argues that the government might offer DC plan members a guaranteed minimum return equal to the growth rate of nominal GDP. The guarantee could be implemented through a swap between the Treasury and the worker nearing retirement. Option pricing formulas show that the guarantee could be quite expensive, but public provision could reduce the costs borne by workers. Such an arrangement would be sustainable for the government while also giving workers an acceptable benefit/contribution ratio in worst-case scenarios; it would allow them to keep the upside investment risk and it would be consistent with private pension provision.

Keywords: defined contribution pension schemes, Modigliani's Treasury-CFDB swap, financial market tail risks, return guarantees, exchange option.

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(*) Ignazio Visco is Deputy Director General of the Bank of Italy. Giuseppe Grande, Bank of Italy, Economic Outlook and Monetary Policy Department. The corresponding author is Giuseppe Grande: giuseppe.grande@bancaditalia.it. The authors would like to thank Riccardo Cesari, Antonio Di Cesare, Paolo Sestito and Roberto Violi for helpful comments on an earlier draft. The views expressed herein are those of the authors and do not necessarily reflect those of the Bank of Italy. The authors are solely responsible for any errors.

1. Introduction

In a typical defined contribution (DC) pension scheme the investment risk is borne entirely by the plan member. This basic characteristic of DC schemes has long been acknowledged and underscored both by researchers on social security systems and by experts of economic policy authorities.[1] Involving as it does the transfer of financial risks from firms or from the public sector to workers, this feature should be a concern for government authorities. In fact, sample surveys reveal a low level of financial education among consumers, even in countries where pension funds have been in existence the longest.[2] It is also likely that exposure to investment risk goes some way to explain the low levels of membership and contributions to these schemes in a number of countries.[3]

The management of financial investment risks is a rather complex task, even for specialized intermediaries. This is because the prices of financial assets are subject to what are known as tail risks, in other words to the tendency to generate extreme losses with a greater frequency than what "chance" would be expected to generate in normal conditions. A clear demonstration of how vulnerable any pension scheme based on the accumulation of financial assets is to stock exchange downturns was seen in the recent financial crisis. In 2008 the pension funds of the OECD countries on average registered negative returns of about 20-25 per cent.[4] In the defined benefit (DB) pension fund sector, financial balances deteriorated by between 5 and 20 percentage points.

The objective of avoiding or reducing workers' exposure to financial risks is one of the factors that determined the development of mixed pension schemes, such as "hybrid" DB funds or DC funds with forms of protection of returns. Moreover, in some countries, governments provide rate or return guarantees.[5] As argued in Visco (2009), in the wake of a dramatic financial crisis, it is vital to consider whether the forms of protection from fluctuations in the prices of financial assets now available can actually provide an effective shield against extreme investment risks. As with the unresolved problem of aggregate longevity risk,[6] it is worth considering the possible benefits of some form of public intervention.

This paper assesses the possibility of a government guarantee of a minimum return to members of DC pension funds. The proposed insurance scheme – whose underlying spirit is in many ways similar to the planned reform of the social security systems developed by Franco Modigliani and his co-authors in the first half of this decade – is based on a swap agreement between the Treasury and the worker who is nearing retirement, whereby the worker cedes his portfolio of financial assets

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[1] See, for example, Fornero (1999), Blake (2000), Campbell and Feldstein (2001), the Group of Ten (2005) and the Pensions Commission (2005).
[2] See OECD (2005), and Lusardi and Mitchell (2007). For an analysis of some of the evidence in Italy, see Cesari, Grande and Panetta (2008).
[3] For a discussion of the low level of contributions to pension funds and the high degree of exposure to financial risks of members of DC pension funds, see Visco (2009).
[4] For an assessment of the impact of the financial crisis on pension funds, see Antolín and Stewart (2009).
[5] See Turner and Rajnes (2001) and Pennacchi (2002).
[6] See Group of Ten (2005), Visco (2007), and Antolín and Blommestein (2007).

accumulated with the pension fund to the Treasury in exchange for a "notional" fund given by the contributions paid into the pension fund and capitalized at a rate equal to the growth rate of nominal GDP.

We will now briefly examine the main characteristics of mixed DB pension schemes (Section 2) and mixed DC schemes (Section 3). Section 4 examines the swap-based guarantee mechanism. Section 5 presents statistics illustrating the long-term relationship between GDP growth and trends in financial markets, followed by an analysis of the benefits and risks of the proposed guarantee scheme (Sections 6 and 7, respectively). Section 8 addresses the issue of how to determine the market value of the public guarantee. The final section concludes.

2. Mixed defined benefit pension schemes

Several DB pension schemes aim to shelter members completely from investment risks, but establish a proportional relationship between contributions paid in and benefits (a typical characteristic of DC funds) in order to favour fund solvency.

Interestingly, these mixed pension schemes (also known as "hybrid") are organized in order to guarantee plan members a fixed return on the sums accrued. Two types of mixed pension scheme are: (1) cash balance plans, which in the past decade have expanded significantly in the United States; and (2) the centralized DB pension fund proposed in the first half of this decade by Franco Modigliani and his co-authors.

CASH BALANCE PLANS. To all effects and purposes, these are DB schemes, in which the investment risk is borne entirely by the pension fund sponsor (typically the employer). Like all other defined-benefit plans cash balance plans are guaranteed, within certain limits, by the Pension Benefit Guaranty Corporation. What sets them apart from the others is the fact that the amount of pension benefits does not depend on parameters such as pay or length of service but on two other variables: the contributions paid in and a predetermined capitalization rate (which can be fixed or variable). In fact, in a cash balance plan, the contributions paid in by the worker (directly, or by the employer on his behalf) are capitalized at a previously set rate, which in turn can be indexed to another variable (for example, the yield on 1-month US Treasury bonds). The benefit the worker receives at retirement will therefore depend on the total amount of contributions and this capitalization rate. Members' contributions to cash balance plans are invested on financial markets by the employer; it is therefore the employer who is responsible for all the entire investment risk and return.[7]

THE MODEL OF MODIGLIANI ET AL. The model for a pension system proposed by Modigliani and his co-authors arose primarily from the concern that workers should not be left alone to cope with investment risks.[8] According to the proposal, the fulcrum of the retirement system should be a centralized and fully funded scheme, that is financed by

[7] The cash balance accounts differ from typical DB funds in other ways too, such as the procedure for paying out the pensions, which in cash balance accounts can take the form of a lump sum rather than an annuity.
[8] See Modigliani, Muralidhar and Ceprini (2000), Modigliani and Muralidhar (2004), and references therein.

mandatory contributions and that offers defined benefits (contributory funded defined benefit scheme, CFDB scheme). In this system, the workers (and their employers) pay in contributions to the fund, which credits them to individual accounts. These payments are managed on a collective basis (by government or by private intermediaries delegated for this purpose); once a year (or in any event periodically), the pension fund effects a swap with the Treasury, whereby the earnings accrued in the year through the management of the portfolio are swapped with those that would have been generated if the return on the portfolio's management had been equal to a previously determined amount (5 per cent, in Modigliani and Muralidhar's proposal of 2004). This swap assures all workers that the contributions to the CFDB fund are capitalized at a fixed rate and that the investment risk will be borne entirely by government.[9]

3. Defined contribution pension schemes with forms of financial protection

DC pension funds offer members three main forms of protection from investment risk[10]: (1) they can use financial market instruments, for example by circumscribing portfolio choices to fixed-income securities (in particular to those indexed to consumer prices) or by subscribing derivatives options contracts; (2) they can utilize guarantees on returns provided by third parties, such as an insurance company or other pension fund sponsor; (3) they can set up a reserve fund (at the level of the individual fund or groups of funds) or establish other forms of mutual risk transfer.[11] The reserve funds, in particular, enable the financial results to be spread out over time: in years of high returns part of these are placed in the reserve fund, to be drawn on in years when the returns fall below a given threshold.

These forms of protection based on financial market instruments have two major drawbacks. First, they can entail considerable costs in the form of fees and premiums (for derivatives and insurance policies) or lower returns (when the accumulated amount is invested mainly in low-risk securities with intrinsically low returns or when the smoothing of returns over time penalizes some cohorts). If these lower net returns are not offset by higher contributions during the accumulation phase, they will result in a lower pension during the payout phase. In other words, lower net returns always imply less consumption either during one's working life or in retirement or both.

Second, and even more important, financial market instruments may not be able to insure against extreme, systemic shocks that hit a number of portfolio asset classes simultaneously. These financial crises reflect asset price tail risks, i.e. extreme declines in financial asset prices that are very rare but that can decimate the value of a worker's accumulated pension fund assets.

It could be argued that, in such extraordinary circumstances, it is always possible for the government to step in and rescue the pension fund, especially when participation

[9] Another advantage of the CFDB fund is that the pooled management of the contributions of all workers reduces the cost of managing the portfolio; also, workers with the same pension contributions and similar in all other respects are guaranteed similar rates of return.

[10] For a description of the forms of minimum return guarantees for mandatory DC schemes, see Turner and Rajnes (2001).

[11] Boeri, Bovenberg, Coeuré and Roberts (2006).

is compulsory, not voluntary. Public intervention is indeed an option, provided it is limited to workers who are close to retirement and so will be unable to enjoy the eventual recovery in financial asset prices in the medium/long term. But rescues can only be an extreme remedy, for two obvious reasons: they are costly for taxpayers and they can encourage opportunistic behaviour by pension funds in the future.

4. A public minimum return guarantee for defined contribution schemes

How can the government insure participants in a defined contribution pension scheme against tail risk? One way would be to provide a minimum return guarantee that protects future pensioners against any sharp curtailment of the amount accumulated. The minimum return should be set at a level that will (1) generate expenditure commitments for the government that can be financed from an ad hoc fund and that in any case are sustainable and (2) ensure an acceptable pension income in relation to the contributions paid in over one's working life.

One solution that appears reasonable is to set the guaranteed minimum return equal to the nominal growth rate of GDP. This choice is similar to the one adopted in the notional systems of several countries (for instance, Italy and Sweden), where the public pensions, financed through pay-as-you-go mechanisms, are indexed to the growth of the economy as a whole. The advantages of a publicly guaranteed minimum return equal to the nominal GDP growth rate will be discussed in Section 6 and the risks in Section 7. The rest of this section lays out how the government could manage an insurance programme of this type.

The government would guarantee DC plan participants that upon retirement they would receive a final accumulated amount not lower than a "notional" amount given by the level that would have been reached if the contributions had been capitalized year by year at the nominal growth rate of GDP:

$$W_T = \max(G_T, A_T) \tag{1}$$

where W_T is the final amount to which the pension fund participant is entitled at the time of retirement and after T years of contribution, G_T is the final amount of the notional fund and A_T the final amount of the pension fund.

The final amount of the notional fund would be calculated as follows:

$$G_T = \sum_{e=e_a}^{e_p-1} c_e w_e \prod_{i=e}^{e_p-1} (1 + g_i) \tag{2}$$

where T is the number of years of contribution, e_p is the participant's age at retirement, e_a is his age at the time of enrolment in the pension fund, e is his current age, c_e is the percentage contribution rate, w_e is gross earnings and g_i is the nominal rate of growth in GDP in the year in which the participant was i years old.[12]

The final amount of the pension fund can be expressed as:

$$A_T = \sum_{e=e_a}^{e_p-1} c_e w_e \prod_{i=e}^{e_p-1} (1+\rho_i) \tag{3}$$

where ρ_i is the average rate of return earned by the DC plan on its portfolio of financial assets when the participant was i years old.

Following the idea of Modigliani et al., the guarantee would take the form of a swap contract between the worker who is nearing retirement (the prospective retiree) and the Treasury: in the event that the accumulated amount of the notional fund is greater than that of the pension fund, the prospective retiree transfers the financial assets accumulated with the pension fund to the Treasury and the Treasury, in exchange, transfers the money value of the notional fund for that prospective retiree, calculated with equation (2). If the guarantee is exercised, therefore, it implies a net transfer of resources from the government to the prospective retiree, equal to the difference between the final amount of the notional fund and that of the pension fund:

$$\max(G_T - A_T, 0) = \max\left(\sum_{e=e_a}^{e_p-1} c_e w_e \left(\prod_{i=e}^{e_p-1}(1+g_i) - \prod_{i=e}^{e_p-1}(1+\rho_i)\right), 0\right). \tag{4}$$

In order to pay out this insurance cover, the Treasury could institute a special "guarantee fund" with two key characteristics:

- with prospective retirees for whom the guarantee is triggered, the fund would execute a swap involving (*i*) payment of the retiree's notional fund amount and (*ii*) acquisition of the assets that the retiree had accumulated with the pension fund;
- the fund would finance the swap by running down reserves or by means of government-guaranteed securities or government transfers. The profits would come from the premiums paid by DC plan participants to cover the costs of the guarantee and from income on financial operations.

Stylized financial statements of the guarantee fund are presented in Figure 1. The income statement shows, under expenses, the net outlays generated by the swaps (i.e. the difference between the notional funds paid to prospective retirees and the value of the financial assets transferred by them), interest payments on government securities and valuation losses on proprietary securities. Under revenues, it would show

[12] Clearly, the government-guaranteed rate of return may differ from the nominal growth rate of GDP and the guarantee may be applied even to only a part of the contributions (for example, those paid in by the worker or, in the Italian case, the severance pay provisions allocated to the pension fund).

Figure 1

Stylized financial statements of the guarantee fund
for defined contribution pension schemes (1)

BALANCE SHEET

Assets	*Liabilities and net equity*
financial assets	government transfers
	government-guaranteed securities
(deficit)	retained profits
(net loss for the year)	net profit for the year

INCOME STATEMENT

Expenses	*Revenues*
net outlays generated by swaps	premiums paid by DC plan members
interest expense	interest, dividends and other proceeds on proprietary securities
valuation losses on proprietary securities	valuation gains on proprietary securities
net profit for the year	(net loss for the year)

(1) The items "deficit" and "net loss for the year" appear in parentheses to indicate that they are alternative to the items "retained profits" and "net profit for the year", respectively.

premiums paid by DC plan participants, profits on financial operations and valuation gains on the securities portfolio.

The balance sheet shows the sources of financing of the fund. Payments to prospective retirees would be financed by running down reserves, by proceeds from issues of government-guaranteed securities or by transfers from the government. Profits would be generated by the build-up of premiums paid by pension plan participants benefiting from the guarantee and by investment of those premiums in financial assets, as well as by income and capital gains on the financial assets acquired through swaps with prospective retirees. Initially the fund's reserves will be very low, because workers will have been paying premiums only for a short period. With the passage of time, the prices of the financial assets acquired with swaps can be expected to rise, outpayments of notional funds will presumably cease and, thanks in part to income on the financial assets acquired, the guarantee fund would begin to show a profit and make good the deficits (since at a certain point it would also be able to begin set aside reserves). An assessment of the level of the premiums that workers would have to pay is provided in Section 8.

5. Long-run growth in GDP and share prices: a comparison

The public insurance scheme described above has two main factors of risk: the nominal growth rate of the economy and the returns of the financial markets. Before examining the advantages (for the workers) and risks (for the government) of this form of guarantee, let us examine the empirical regularities in the relative performance of GDP and financial asset prices. Our analysis of portfolio returns focuses on equities, for which long time series are more available.

Some statistics on the long-run relation between GDP rates and equity returns are presented in Table 1. Panel (*a*) of the table considers data on the United States covering 80 years, from 1929 to 2008. For four long time horizons (10, 20, 30 and 40 years), the table shows the years in which the average growth rate of GDP was higher than the average rate of return on equities for at least one of the four horizons. The table highlights that, for long time horizons, the cases in which nominal GDP outpaces the stock index:

- are not very frequent. This happens in 20 of the 70 years for the 10-year horizon, 11 of 60 for the 20-year horizon and never for the 30- and 40-year horizons (for which 50 and 40 observations, respectively, are available);[13]
- are rarer, the longer the investment horizon;
- show some degree of persistence. In the 80 years examined, the cases in which the growth rate of GDP exceeds that of share prices are concentrated in two sub-periods: the years just after the Second World War and the 1970s-early 1980s.

Panel (*b*) of the table compares data for 10-year periods starting from 1970 for 15 countries. The main results are:

- the relative performance of GDP and equities shows marked similarity across countries. Almost everywhere the GDP growth rate exceeds that of equity prices in the 1970s, while the opposite occurs in the two subsequent decades;
- for the current decade, the data now available indicate that in all 15 countries the equity indices have gained less than GDP, owing to the collapse of the stock markets in 2008.

Broadly similar results (not reported) are obtained from a comparison of GDP growth with a portfolio consisting entirely of bonds and with a portfolio divided equally between equities and bonds (the comparison is limited to the three sub-periods 1970-79, 1980-89 and 1990-2000).

In summary, this section provides some evidence on the risk that over long-term horizons financial market returns turn out to be lower than the nominal growth rate of GDP. On the basis of past experience and available statistics, that risk has two main characteristics: (*i*) it is relatively low and tends to diminish as the investment horizon lengthens; (*ii*) it is, however, also poorly diversifiable, as it is highly concentrated in time and widely dispersed by geographical area.

[13] As the time series began in 1929, the rates of return on periods of 10, 20, 30 and 40 years are available starting from 1939, 1949, 1959 and 1969, respectively.

Table 1

A comparison of long-run growth rates in nominal GDP and equity indexes (1)

(*per cent*)

(*a*) Years in which the long-run annual growth rate in nominal GDP exceeded that of the stock price index in the US (from 1929 to 2008)

Year →	42	43	44	45	46	47	48	49	71	73	74	75	76	77	78	79	80	81	82	83	84	85	08
Excess of GDP over equity:																							
10-year growth rates	1.62	6.80	3.38	3.50	6.32	2.06	4.37	2.15	0.17	2.87	6.92	4.77	3.13	6.18	6.04	3.84	2.47	4.97	3.40				6.80
20-year growth rates								0.55			0.01			0.16	1.73	1.45	0.81	2.56	0.87	1.07	1.66	0.95	
30-year growth rates																							
40-year growth rates																							

(*b*) Average annual growth rates in nominal GDP and equity indexes since 1970 in 15 countries (2)

	Australia	Belgium	Canada	Denmark	France	Germany	Ireland	Italy	Japan	Netherl.	Spain	Sweden	Switzerl.	UK	US
GDP [*a*]															
1970-1979	13.0	10.4	12.6	11.7	12.6	7.9	17.6	17.0	12.3	10.7	17.9	11.0	6.2	14.9	9.6
1980-1989	11.0	6.4	8.6	7.9	9.1	4.9	11.3	13.6	6.1	4.2	12.3	9.9	5.6	9.7	7.6
1990-2000	5.0	4.2	4.5	4.3	3.5	4.2	10.2	5.7	2.0	5.4	7.2	4.9	2.9	5.6	5.3
2001-2007	7.1	4.1	5.1	3.8	3.9	2.3	8.5	3.7	0.4	4.4	7.3	4.4	3.0	5.2	4.9
2001-2008	7.3	3.9	5.0	3.7	3.8	2.4	7.2	3.5	0.1	4.4	6.9	4.2	3.1	4.9	4.7
1970-2000	9.6	6.9	8.4	7.9	8.2	5.6	12.9	11.8	6.5	6.7	12.2	8.4	4.8	9.9	7.6
1970-2008	9.1	6.3	7.6	7.0	7.3	4.9	11.7	10.0	5.2	6.2	11.1	7.5	4.5	8.8	7.0
Equity [*b*]															
1970-1979	5.5	6.6	10.0	9.6	8.3	2.6	13.6	0.7	12.4	5.2	-0.4	6.9	2.5	11.7	6.7
1980-1989	16.9	19.9	11.7	23.6	22.3	16.8	22.5	25.5	20.5	19.6	28.2	30.7	10.4	22.3	16.1
1990-2000	10.7	10.2	10.0	10.0	12.1	9.9	14.2	10.3	-6.2	18.0	13.3	15.2	15.6	12.7	14.4
2001-2007	13.1	8.3	9.0	9.6	4.4	4.7	5.0	3.2	3.0	2.5	10.5	5.3	3.4	4.7	1.5
2001-2008	5.4	-2.7	3.0	0.2	-2.6	-2.1	-8.5	-4.8	-3.8	-7.3	3.5	-1.6	-2.1	-0.5	-4.7
1970-2000	10.8	12.0	10.5	14.0	14.0	9.5	16.4	11.4	7.8	14.0	12.7	17.0	9.4	15.2	12.3
1970-2008	9.7	8.8	8.9	11.0	10.4	7.0	10.8	7.9	5.3	9.3	10.7	12.9	6.9	11.8	8.6
GDP vs. equity [*a-b*]															
1970-1979	7.5	3.8	2.6	2.1	4.3	5.3	4.0	16.3	-0.1	5.5	18.3	4.1	3.7	3.2	2.9
1980-1989	-5.9	-13.5	-3.1	-15.7	-13.2	-11.9	-11.2	-11.9	-14.4	-15.4	-15.9	-20.8	-4.8	-12.6	-8.5
1990-2000	-5.7	-6.0	-5.5	-5.7	-8.6	-5.7	-4.0	-4.6	8.2	-12.6	-6.1	-10.3	-12.7	-7.1	-9.1
2001-2007	-6.0	-4.2	-3.9	-5.8	-0.5	-2.4	3.5	0.5	-2.6	1.9	-3.2	-0.9	-0.4	0.4	3.4
2001-2008	1.9	6.7	2.0	3.5	6.4	4.5	15.6	8.3	4.0	11.7	3.4	5.8	5.2	5.3	9.4
1970-2000	-1.2	-5.1	-2.1	-6.1	-5.8	-3.9	-3.5	0.4	-1.3	-7.3	-0.5	-8.6	-4.6	-5.3	-4.7
1970-2008	-0.6	-2.5	-1.3	-4.0	-3.1	-2.1	0.9	2.1	-0.2	-3.1	0.3	-5.4	-2.5	-3.0	-1.6

Sources: Based on Bureau of Economic Analysis, Bloomberg, OECD.Stat (http://stats.oecd.org/Index.aspx), Thomson Financial Datastream, Shiller (2005)'s, and Dimson, Marsh and Staunton (2002)'s data.

(1) All growth rates are annual averages in the period indicated. Equity indexes all incorporate reinvested dividends. – (2) For Belgium, Canada, Germany, Ireland, Italy, Japan, Spain, Switzerland and the UK, GDP growth rates are available since 1971.

6. The advantages for the worker of a government-guaranteed minimum return

Members of DC pension plans could benefit in several ways from a publicly guaranteed minimum return pegged to GDP, via the exchange of a notional fund for the final accrued balance of the pension fund.

In the first place, this would provide a guarantee for DC pension plan members that the contributions made during their working lives will be capitalized at a rate that, at least, takes account not only of the cost of living but also of the real growth of the economy. Therefore, for amounts saved under the DC plan, workers would not be exposed to inflation shocks or real shocks affecting single industries or groups of workers.[14] More importantly, they would also be sheltered from the risk of becoming poorer in relation to the rest of the economy.

Second, a government guarantee scheme would also provide cover against macroeconomic risk factors that would otherwise be hard to neutralize. The guarantee allows intertemporal transfers of resources to benefit generations of workers who make their retirement savings at times when returns from the financial markets are unsatisfactory over long periods, as in the 1940s and the 1970s (see Section 5). In such periods, diversifying investments over financial asset classes and geographical areas does not significantly reduce risks.

Third, for workers, DC plans with a government-guaranteed minimum return would be better than either mixed DB plans or DC plans with financial protection (see Sections 2 and 3, respectively). Compared with mixed DB plans, DC schemes with a guaranteed minimum return give members the chance to benefit when the financial markets are performing well. If, at the end of the savings period, the return on the pension fund is greater than the accumulated growth of nominal GDP, the worker has the right to the sum accrued in the pension fund, whereas mixed DB plans have a pre-determined capitalization rate for the contributions, and any surplus financial market returns go to the entities providing the guarantee.[15] Compared with DC plans with investment risk protection based on financial market instruments or on mutual assistance schemes, a publicly guaranteed minimum return may be less costly and more effective in addressing extreme financial risks.

Lastly, the possibility of swapping notional funds with the accrued amount in the pension fund would have the advantage of maintaining a high degree of flexibility in DC retirement schemes, above all in terms of differentiating accumulation plans according to members' risk preferences and to different options for disbursing pension benefits.

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[14] The real rate of return of the notional fund could also be uncoupled from the real rate of growth of GDP, thus protecting workers from aggregate real shocks. However, this would have implications for the sustainability of the fund guarantee, as it would weaken the link between wealth accumulated under the pension scheme and overall growth in the economy (see Section 7).

[15] The asymmetry of returns received by members of DC plans with a guaranteed minimum return (i.e. the fact that they benefit from positive trends in the financial markets without being exposed to the risk of falling asset prices) creates the risk of opportunistic behaviour by DC plan members. This is discussed in the next section.

7. The risks of a guarantee fund

A guarantee fund exposes government to two risks in particular: the uncertain sustainability of the insurance scheme and the possibility of opportunistic behaviour by the insured. Specific portfolio management problems can also arise.

As regards the sustainability of the insurance scheme,[16] the first risk factor is financial market trends: the greater the volatility of the return on the pension fund portfolio, the greater the likelihood that the guarantee will be triggered and the greater the government's potential outlay. The second risk factor is the rate of GDP growth, which has direct and indirect effects working through the growth in gross earnings. The higher the rate of GDP growth, the higher the probability that the guarantee will be triggered and, in that case, the larger the amount of the notional fund that the government would have to pay to members when they retire.[17]

The sustainability of the government guarantee scheme depends essentially on three conditions:

1. in the long term, the returns on financial assets tend to be at least equal to the nominal rate of growth of the economy (see the evidence in Section 5);
2. the size of the outlay is correlated with economy's growth rate, which in turn is the main factor of budget revenue growth;
3. government is able to finance even large deficits in the guarantee fund because it has a long time horizon, thereby taking advantage of the fact that over long investment periods it should be possible to make good the losses with the proceeds from financial management.

A broad indication of how long it takes for the guarantee fund to recover the net outlays originated by the swaps can be gained from long time series on the United States. As we saw in Panel (*a*) of Table 1 in Section 5, over the period 1929-2008 there were some years in which, for accumulation periods of 10 and 20 years, nominal GDP outpaced the stock market. Those years are considered in Figure 2, which shows, for each of the two investment horizons, the distribution of the number of years it took for the share price index to regain the level of nominal GDP after that the swap was executed.[18] The estimated recovery periods are quite short, no more than 6 and 3 years for the 10- and 20-year horizons, respectively. If one also takes into account possible funding costs incurred by the guarantee fund (proxied by the annual nominal growth rate of GDP), the

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[16] The state outlay produced by the swap with pension fund members would be positively correlated with three variables (see equation 4 above): (*i*) the rate of GDP growth; (*ii*) gross earnings; and (*iii*) the percentage contribution rate. It would be negatively correlated with: (*iv*) the return on the DC plan. In the event the swaps were to be funded by issuing government securities, the overall cost of the guarantee would also be affected by the rate of interest on state-backed securities.

[17] The cost of the guarantee is also affected by the pension fund contribution rate: even a variation of just a few tenths of a percentage point may have a significant impact, because it is applied to the entire amount of annual earnings.

[18] The indicator shown in Figure 2 does not take into account contributions, which have an impact on the time to recovery, while it does take into account very few cases of a second drop of the share price index below the value of the guarantee. With respect to the cases shown in Panel (*a*) of Table 1, the left-hand panel of Figure 2 does not include 2008, for which post-shock returns are not yet available.

recovery period naturally lengthens, reaching a maximum of 14 and 7 years for the 10- and 20-year horizons, respectively. Over such maturities, it should be perfectly possible for a government-backed entity such as the guarantee fund to issue bonds at convenient prices.

The indicator reported in Figure 2 suggests that the time that is necessary to recover the net outlays of the swap is quite short. This is because the chosen level of minimum return – the nominal growth rate of GDP – is such that the guarantee is triggered only when share returns have been very disappointing. This means that the swaps would allow the guarantee fund to buy a portfolio of financial assets at a time when financial asset prices are presumably relatively low. This is shown in Figure 3, which plots the combinations of the differential between GDP growth and equity returns, on the one hand, and the ratio between share prices and 10-year average earnings of listed companies, on the other. The price/earnings ratio indicator, due to Shiller (2005), can be regarded as an indicator of the general level of equity prices. Figure 3 makes it clear that, for both the 10-year and 20-year accumulation periods, when the GDP-equity growth rate differential is positive (i.e., when swaps are executed) the price/earnings ratio tends to be very low, indicating the distinct possibility of a rise in share prices.

The three conditions outlined above form the basis for the financial sustainability of the guarantee fund. However, it is clear that the fund must have its own reserves in order to offset the impact of prolonged crises in the markets. Therefore, the premium paid by workers for the guarantee is crucial. An assessment of the amount of this premium is given in the next section.

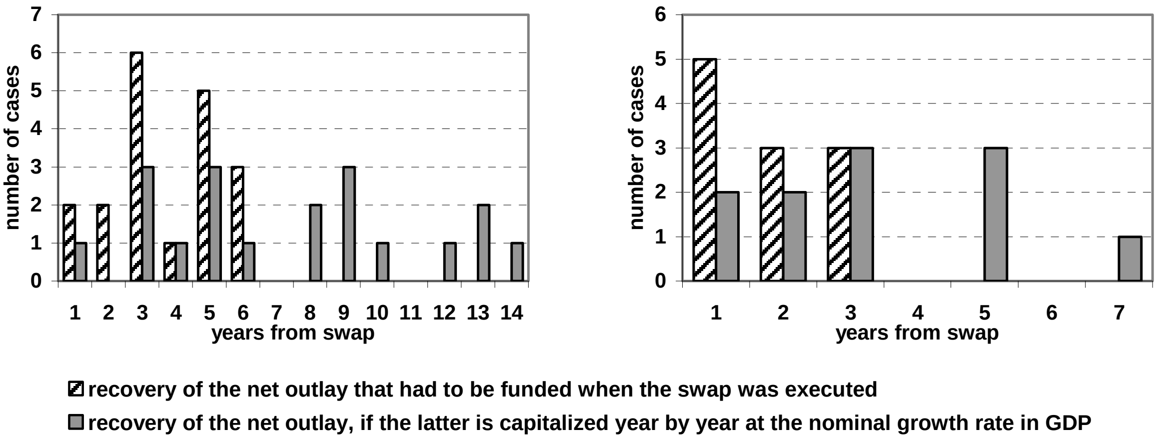
If, despite the accrual of premiums and financial proceeds, the guarantee fund should begin to show a sizeable and persistent deficit, various countermeasures could be envisaged: (*i*) an increase in the insurance premiums paid by members of DC plans; (*ii*) a reduction in the guaranteed return; (*iii*) deferred retirement. Each of these options could be appropriately modulated among the different cohorts of DC plan members.

The existence of a government-guaranteed minimum return could encourage DC plan members to over-expose themselves to financial risks. This risk could be a significant one especially for workers who, as they near retirement, are dissatisfied with the returns achieved by their pension fund. Opportunistic portfolio choices could be discouraged in several ways. First, the guarantee scheme could be made compulsory, especially if members have to pay regular premiums to the guarantee fund. This would reduce the impact of adverse selection on the guarantee fund's balance sheet. Second, a ceiling on the percentage of risky securities in a pension fund's portfolio could be set. Third, the allocation of securities within a portfolio could be required to have an individual "life cycle"; that is, as retirement approaches, the portfolio would be redistributed away from risky securities to safer assets (e.g. from shares to government securities). Last, criteria could be tightened to ensure the diversification of risk within the portfolio and the quality of eligible portfolio assets.

Figure 2

Indicator of the time to recover net outlays of swaps (1)

Swaps at the end of a 10-year accumulation period | Swaps at the end of a 20-year accumulation period

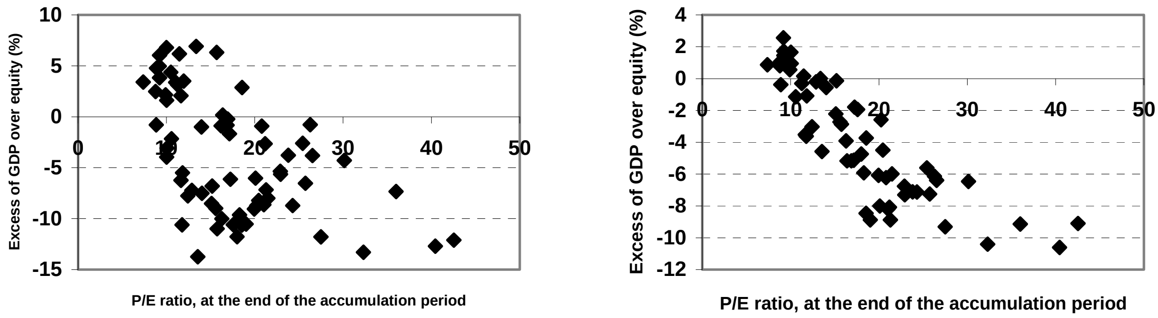


Sources: based on Bureau of Economic Analysis and Bloomberg data.
(1) Data refer to the United States and cover 80 years, from 1929 to 2008. The two panels relate to 10-year or 20-year accumulation periods and show the distribution of the number of years after the swap in which the share price index was below the value it should have had in order for the guarantee not to be exercised. Two different hypotheses are made on the amount that has to be recovered: only the net outlay at the time of the swap and the net outlay capitalized year by year at the nominal growth rate of GDP.

Figure 3

Excess of GDP over equity and price/earning ratio (1)

10-year accumulation periods | 20-year accumulation periods

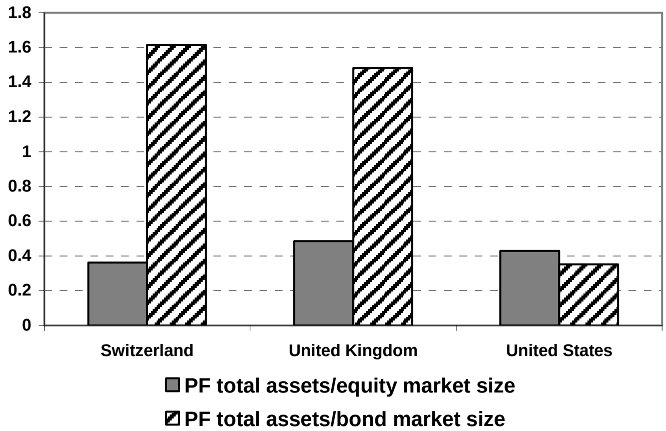


Sources: based on Bureau of Economic Analysis, Bloomberg and Shiller (2005)'s data.
(1) Data refer to the United States and cover 80 years, from 1929 to 2008. The two panels show the combinations of price/earnings ratio and excess of GDP versus equity over 10-year and 20-year accumulation periods. The excess of GDP versus equity is equal to the difference between the annual average nominal growth rate of GDP and the annual average share returns. The price/earnings ratio, computed by Shiller (2005), is defined as the ratio between the stock price index and the average earnings of listed companies over the previous 10 years.

Specific problems concerning the government guarantee fund could arise from the management of the financial assets portfolio. A solution could be to entrust the management to specialized intermediaries, appointed under transparent and competitive procedures. The investment objective could be a long-term return at least equal to that on government securities (or the rate guaranteed with the swap), plus a spread of one or two percentage points in order to quickly recoup the net outlays entailed in the swap. Voting rights at the meetings of shareholders of listed companies should be adequately regulated. The size of the guarantee fund compared with those of the main investment markets should be monitored to make sure that financial asset prices are not excessively influenced by the fund's portfolio choices. Figure 4 compares the total assets of pension funds with the size of the domestic bond and equity markets in three countries where retirement saving schemes have long been developed. While it would be necessary to take into account the geographical diversification of pension fund holdings, this simple comparison suggests that pension funds may well get large enough to have a substantial price impact on many financial assets.

Figure 4

Pension fund holdings compared with the size of domestic market, 2005
(*pension fund total assets/market capitalization*)



8. An assessment of the premium payable by the pension scheme members

The final amount to which the member is entitled at the time of retirement (equation (1)) can be rewritten as:

$$W_T = A_T + \max(G_T - A_T, 0). \quad (5)$$

The second term on the right-hand side of equation (5) is the payment generated by a put option whose strike price is G_T. Consequently, at retirement the worker is entitled to the amount accumulated with the pension fund (A_T), supplemented by the payment generated by the put option. In each year t the worker holds the portfolio of financial assets accumulated with the pension fund up to that time (A_t) and the European put option offered to him by the government. Denoting the value of the option at time *t* by f_t , the total value of the position accrued by the member at that date is:

$$W_t = A_t + f_t \,. \tag{6}$$

Option value theory makes it possible to determine the price f_t of this option. Table 2 shows the results obtained with this purely financial approach.[19] In order to better take into account the possibility of large drops in share prices, the latter are assumed to follow a jump-diffusion process, estimated over a period of high stock market volatility.[20] Three different types of guarantee are considered in the simulations:[21] two of them guarantee the repayment of the contributions paid in, in nominal and real terms (minimum rates of return of 0 and 2.5 per cent respectively).[22] The third type of guarantee is that discussed in Section 4, in which the minimum rate of return of the pension fund is not predetermined but put equal to the nominal GDP growth rate in each year.[23] As regards the riskiness of the portfolio, four investment strategies are considered: (I) 100% equities; (II) 50% equities and 50% government securities; (III) 100% government securities; and (IV) a simple life-cycle strategy, under which the portfolio is initially invested 100% in equities and then, in the last ten years of the accumulation period, gradually shifts, on a linear basis, into government securities until, at the time of retirement, the equity share is nil.[24] Lastly, four different investment durations are considered (10, 20, 30 and 40 years). GDP volatility is taken to be 2 per cent, while the correlation between GDP and the pension fund portfolio is assumed to be 0.4.[25]

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19 The fair value of the option is obtained under the assumption of investor risk neutrality. The methodology, and the underlying hypotheses, are presented in Appendix A. The solution can be obtained numerically, as the use of closed formulas *à la* Black and Scholes is hindered by the presence of periodic investments (see, for example, Zurita, 1994, and Pennacchi, 1999).

20 The parameters driving the jump component of equity returns (expected number of jumps per year and average jump size) are taken from Di Cesare (2005) and are estimated from the prices of the options on S&P 500 futures contracts listed on the Chicago Mercantile Exchange (see Appendix A). The estimation period runs from August 2000 to July 2002 and is characterized by quite a high share price volatility, associated with the collapse of the tech bubble, a recession, 9/11 and accounting frauds.

21 For a thorough analysis of the main determinants of the cost of guaranteeing a minimum rate of return for a pension fund, see Lachance and Mitchell (2003a, 2003b).

22 In regimes of low inflation a rate of return of 2.5 per cent is sufficient to ensure repayment of the principal in real terms.

23 It is important to note that, owing to the assumption of investor risk neutrality required to determine the price of the option, for all the risky assets, including nominal GDP, the rate of return expected by investors coincides with the risk-free interest rate. This means that no assumption is ever made in the simulations regarding the equity premium, the term premium or expected nominal GDP growth. On the other hand, the assumptions regarding the volatilities of the various stochastic processes considered and their correlations are of crucial importance.

24 The example of life-cycle investing is taken from Blake, Cairns and Dowd (2008).

25 The historical data on nominal GDP provide estimates of its volatility that are very low. On the basis of quarterly data for the United States covering the period from 1988:Q1 to 2008:Q4, it can be estimated that the volatility of nominal GDP is on the order of 1 per cent on an annual basis. The correlation between nominal GDP and the S&P 500 index (with dividends reinvested) would not exceed 0.4.

Table 2

Cost Estimates of Minimum Return Guarantees for Defined Contribution Pension Schemes (1)
(*annual charges as a percentage of total assets*)

Years with Individual Account	Minimum Rate of Return		
	0 percent (i.e. return of the principal)	2.5 percent (i.e. about long-run inflation)	Yearly nominal GDP growth rate (2)
		I. Portfolio invested 100% in equities	
10	1.17	2.13	2.85
20	0.47	1.22	1.99
30	0.24	0.84	1.61
40	0.14	0.62	1.37
		II. Portfolio invested 50% in equities and 50% in 10-year Treasury bonds	
10	0.23	0.81	1.43
20	0.05	0.42	1.01
30	0.02	0.26	0.82
40	0.01	0.18	0.71
		III. Portfolio invested 100% in 10-year Treasury bonds	
10	0.00	0.00	0.38
20	0.00	0.00	0.28
30	0.00	0.00	0.23
40	0.00	0.00	0.20
		IV. Life-cycle investing (3)	
10	0.25	0.86	1.52
20	0.20	0.80	1.51
30	0.11	0.62	1.34
40	0.13	0.60	1.33

Source: Authors' calculations.
(1) Estimate of the cost of a European put option with the maturity shown in the first column. Estimates are based on Monte Carlo simulations (10,000 trials) and are carried out for four different portfolios. As for the definition of minimum return, three different formulas are considered: two fixed-rate guarantees (0 and 2.5 percent per annum) and a guarantee yielding the nominal GDP growth rate each year. The methodology and underlying hypotheses are presented in Appendix A. - (2) The volatility of the nominal GDP growth rate is set equal to 2 percent per annum; the correlation between the GDP growth rate and stock returns is set equal to 0.4. – (3) The portfolio is initially invested in equities and then linearly switches into bonds in the 10 years prior to retirement.

The estimates show that the type of guarantee provided has a substantial effect on the cost of the option. Considering a 40-year accumulation period and a balanced portfolio divided equally between equities and 10-year government bonds, the cost of the option – calculated on an annual basis and as a percentage of the assets under management – is practically nil if the option guarantees only the repayment of the contributions paid in; it rises to 0.18 per cent when the option guarantees an annual nominal rate of return of 2.5 per cent and 0.71 per cent when it guarantees the nominal GDP growth rate.

Another factor that has a considerable influence on the cost of the option is the volatility of the portfolio. If the portfolio is fully invested in equities, even just guaranteeing repayment of the contributions paid in requires, over a 40-year time horizon, an annual premium equal to 0.14 per cent of the assets under management, which rises to 1.37 per cent if the strike price of the option is linked to the GDP growth rate. Conversely, the cost of the option falls drastically if the percentage of equities in the portfolio is reduced to zero; it is nil in the case of repayment of the capital paid in (in nominal and real terms), and even where the amount guaranteed is linked to the nominal GDP growth rate it is still not greater than 0.20 per cent. A life-cycle strategy is equivalent to the balanced strategy (50% equities and 50% securities) for the 10-year maturity but riskier for the longer durations (because of the larger weight of the period in which the portfolio is fully invested in equities).

Another variable that has a significant influence on the value of the guarantee is the duration of the accumulation period. With a relatively short retirement investment horizon like 10 years, in the case of a guarantee linked to the nominal GDP growth rate, the annual cost of the option well exceeds 1 per cent of assets under management if the portfolio is equally divided between equities and government securities and rises to 2.9 per cent when it is fully invested in equities.[26]

As for the guarantee linked to the nominal GDP growth rate, an important property that needs to be highlighted is that the amount insured (i.e. the strike price of the put option represented as the right-hand element of equation (5)) is itself a random variable, because it depends on the performance of GDP. This risk factor is an additional source of volatility of the derivative contract and as such should increase the value of the option. However, it can also reduce its value if the risk factor shows a significant positive correlation with the portfolio. It is therefore important to assess the sensitivity of the price of the option both to the volatility of GDP and to the latter's correlation with the portfolio. The estimates are shown in Table 3.[27] The simulations refer to the case of a balanced portfolio divided equally between equities and government securities, with a

[26] For investment horizons limited to just a few years, the effect of the duration on the value of the option is highly non-linear. See Figure 4 in Cesari, Grande and Panetta (2008), in relation to a guarantee of repayment of the contributions paid in.

[27] For the derivation of these effects in closed form, under the simplifying hypotheses that there is a single contribution to the pension fund at the beginning of the accumulation period and that the share prices follow a Brownian geometric motion without jumps, see Appendix B.

Table 3

Cost estimates of minimum return guarantees: sensitivity to different assumptions on GDP volatility (1)

(*annual charges as a percentage of total assets*)

Volatility of nominal GDP	Correlation between nominal GDP and pension fund assets				
	0.2	0.4	0.6	0.8	0.99
			10 years		
1	1.24	1.19	1.20	1.16	1.15
2	1.24	1.20	1.14	1.06	1.00
4	1.27	1.14	1.07	0.90	0.77
6	1.34	1.17	1.03	0.80	0.54
8	1.44	1.26	1.04	0.77	0.33
10	1.58	1.38	1.14	0.79	0.24
			20 years		
1	0.90	0.85	0.82	0.82	0.81
2	0.89	0.84	0.81	0.76	0.71
4	0.89	0.81	0.76	0.64	0.55
6	0.96	0.84	0.72	0.57	0.39
8	1.02	0.90	0.74	0.55	0.24
10	1.09	0.98	0.82	0.56	0.18
			30 years		
1	0.74	0.69	0.68	0.68	0.66
2	0.73	0.69	0.65	0.61	0.58
4	0.74	0.67	0.62	0.53	0.45
6	0.78	0.69	0.59	0.47	0.32
8	0.84	0.73	0.61	0.45	0.20
10	0.92	0.81	0.67	0.47	0.15
			40 years		
1	0.64	0.60	0.59	0.59	0.58
2	0.63	0.60	0.56	0.53	0.51
4	0.64	0.58	0.53	0.45	0.39
6	0.68	0.61	0.52	0.41	0.28
8	0.73	0.64	0.53	0.39	0.18
10	0.80	0.71	0.59	0.40	0.13

Source: Authors' calculations.

(1) Estimate of the cost of a European put option with a strike price linked to the yearly nominal GDP growth rate. The estimates are carried out for a number of combinations of GDP volatility and GDP-pension fund asset correlations, as well as for four different investment horizons (10, 20, 30 and 40 years). Risk-neutral valuation is assumed. The pension fund portfolio is evenly split between stocks and 10-year Treasury bonds and its volatility is equal to about 10.5 percent per annum. The estimates are based on Monte Carlo simulations (10,000 trials). For simplicity, share prices are assumed to follow a Brownian geometric motion without jumps. The methodology and the other underlying hypotheses are presented in Appendix A. The shaded areas show the combinations of GDP volatility/correlation for which, according to the pricing model in Margrabe (1978), in the case of a single contribution paid into the pension fund an increase in GDP volatility would lead to a rise in the value of the option (see Appendix B).

volatility on the order of 10.5 per cent.[28] For the volatility of nominal GDP the values considered are between 1 and 10 per cent, while for the correlation between nominal GDP and the portfolio, it is assumed to be positive and capable of taking on values ranging from 0.2 to 0.99 (almost perfect correlation).

The simulations show that a higher correlation between the value of the pension fund and that of GDP reduces the value of the option, both because it decreases the portion of cases in which the guarantee is triggered (i.e. in which the option gives rise to positive payoffs) and because it diminishes the size of the payoffs. The effect of the correlation depends, however, on the level of volatility of the two stochastic processes and, under standard assumptions regarding the volatility of GDP, it is relatively small. For example, an increase in the correlation from 0.2 to 0.4 reduces the value of the option by between 0.06 and 0.13 per cent if the volatility of GDP is 4 per cent and by between 0.07 and 0.17 per cent if it is 6 per cent.

As for the effect of an increase in the volatility of GDP on the value of the option, it is not unambiguous because it depends on the degree of correlation between GDP and the portfolio of the pension fund. An increase in the volatility of GDP has two effects of opposite sign: it amplifies the correlation effect referred to above, which lowers the value of the option, and has a direct effect on the overall volatility of the option, which increases the value of the derivative contract. As long as the volatility of GDP is relatively low, an increase in it will have little impact on the overall volatility of the option, while it strengthens the effect, of the opposite sign, of the correlation. Beyond a certain threshold, the direct effect exceeds the correlation effect and a further increase in the volatility of GDP produces an increase in the value of the option.

It is important to highlight also that the cost estimates reported in Table 2 assume that the guarantee's cash flows can be exactly replicated by trading financial assets. This is quite a restrictive assumption, as GDP-linked securities usually are not available and financial markets are likely to provide only an imperfect hedge against fluctuations of aggregate economic activity.[29] An assessment of how much the fair value of the option is affected by the lack of GDP-linked securities can be carried out under the hypothesis that the residual GDP risk (i.e. the risk that cannot be hedged by traded assets) is not priced by financial markets. Some estimates indicate that the costs of the nominal GDP guarantee might be slightly above the level shown in the last column of Table 2 (of about 0.1 percentage points).[30]

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[28] As a result of quite standard assumptions on the volatilities of stock returns and short-term interest rates; see Appendix A.

[29] Shiller (1995)'s macro markets are meant to overcome the difficulties faced by capital markets in hedging aggregate income risks. For estimates of bounds on prices of nontraded assets see, for example, Cochrane and Saá-Requejo (2000) and Kaido and White (2009).

[30] The methodology has two stages (see Pennacchi 1999, 2002): (*i*) first, estimate the difference between the expected growth rate of GDP and the equilibrium expected return on a hypothetical security that has the same volatility of GDP; (*ii*) when running Monte Carlo simulations, in the drift of the GDP process add that difference to the risk-free rate. In the estimates, the expected GDP growth rate is set equal to its annual average. The expected return on a hypothetical GDP-linked security is estimated through an asset pricing model including three factors (stock market returns, 10-year Treasury bond yields and consumer price inflation). Estimates are carried out recursively on annual data from 1980-2006 to 1991-2006 and, in each iteration, the difference between the sample average of the GDP growth rate and the expected return is

Overall, these simulations indicate that, if a purely financial approach is adopted (assuming risk neutrality), then guaranteeing a minimum rate of return equal to the nominal GDP growth rate may be rather costly if the portfolio mostly include equities, even in the case of investment horizons stretching over decades.

However, for any given type of guarantee, the cost of the guarantee may be limited by introducing restrictions on portfolio risk, in the form, say, of limits on the amount invested in equities held (possibly applied in particular to the years preceding retirement). Alternatively, for any given level of portfolio risk, the cost of the guarantee may be lowered by reducing the guaranteed amount, by, for instance, ensuring the return of the principal only (in nominal or real terms).

Finally, the opportunity to introduce a public guarantee of a minimum return to defined contribution pension scheme members rests critically on the design of the overall retirement-income system, and specifically on the relative size of defined contribution schemes within that system, the riskiness of the other components, whether enrollment to DC private schemes is mandatory rather than voluntary and the standards of pension fund investment regulation.

9. Conclusion

The blow to pension fund assets from the recent financial crisis has underscored how severely members of defined contribution schemes are exposed to financial market tail risk, i.e. to exceptionally large, albeit rare, drops in financial asset prices that may significantly reduce the accumulated value of workers' pension plans. Protecting returns against these extreme financial risks by means of systems based on market instruments or mutualistic mechanisms may prove ineffective, as well as very costly.

Accordingly, it is worth considering the possible benefits of some form of government-guaranteed minimum return. The public sector's long time horizon and nationwide sphere of action make government best placed to bear the consequences of a collapse in the prices of financial assets. Clearly, such a guarantee system must be structured so as to ensure it is sustainable for the public finances.

In the proposal put forward in this article, the government would guarantee a minimum return equal to the economy's nominal rate of growth. Taking up an idea formulated by Franco Modigliani and his co-authors some years ago, such a guarantee, should it come into operation, could take the form of a swap in which the Treasury pays the future pensioner the notional value of the guarantee (equal to the amount obtained from capitalizing contributions paid at the minimum rate of return) in exchange for the portfolio of financial assets accumulated in the pension fund.

Such a guarantee scheme would ensure an acceptable return on the contributions paid by workers, safeguarding future pensioners from inflation, real shocks in individual sectors of the economy, and a fall in living standards compared with the rest of the population. Workers would also be protected against macroeconomic risks that are

computed. The average value of this difference is then used in Monte Carlo simulations, as explained above.

difficult to diversify otherwise and that could weight down financial market returns for substantial periods of time. At the same time, this type of guarantee would allow workers to continue enjoying net returns on their financial investments if these are higher than the nominal growth of the economy over the accumulation period. Finally, the swap between the Treasury and the worker shortly before retirement would not entail any significant alteration in the operation of supplementary defined contribution pension plans, particularly as regards the availability of different investment lines during the accumulation period or the methods of disbursement of pension benefits.

For the insurance scheme to be financially sustainable, workers must pay a premium. For it also to be economically convenient, funds must limit the percentage of risky securities in their portfolio. Government would in any case be able to charge lower premiums than any equivalent private insurance scheme.

The government would manage the swap through an ad hoc guarantee fund financed out of the reserves generated by workers' premiums and possibly also by issues of government-backed securities or direct government transfers. The guarantee fund would be financially sound, as the Treasury's payments would be positively correlated with the nominal rate of GDP growth, the variable that is the main determinant of the increase in government revenues. Moreover, in the swaps the Treasury would acquire portfolios of financial assets while the securities were most likely to be undervalued and therefore likely to rise significantly in the future. If the insurance scheme were to begin building up large deficits, appropriate counter-measures could be taken.

The existence of a safety net against stock market collapse might encourage members of defined contribution pension plans to hold excessively risky portfolios, as in the case of workers close to retirement age who are dissatisfied with the returns on their pension plans. This risk of opportunistic behaviour can be handled, for instance by placing a limit on the proportion of risky securities in a portfolio as the date of pension entitlement approaches. On the other hand, a public guarantee of minimum return on pension funds would unquestionably give workers a further incentive to join defined contribution plans and increase their contribution rate.

Appendix A: The method of estimating the option value

The method for calculating the value of the put option of a DC plan member is based on risk-neutral valuation (see, for example, sections 11.6 and 16.6 in Hull, 2000) and basically follows Zurita (1994), Pennacchi (1999) and Lachance and Mitchell (2003a, 2003b). In order to better model the probability of large changes in stock market indices, share prices are assumed to have jumps superimposed upon a geometric Brownian motion (Merton, 1976).

For comparison with previous studies, the parameter assumptions are largely those of Lachance and Mitchell (2003a, 2003b). The rate of growth of annual earnings and the percentage contribution rate are assumed to be 4 per cent and 2 per cent respectively. For the risk-free rate, the estimate follows the model in Vasicek (1977). The initial value of the rate, its stationary value, the adjustment factor and the rate volatility are set at 2 per cent, 4 per cent, 80 per cent and 2 per cent respectively. The returns on 10-year bonds are estimated with the closed-form model calculated by Vasicek (1977). Share returns in continuous time follow a jump diffusion model in which (assuming risk neutrality) the drift is set equal to the risk-free rate adjusted for the average growth rate from the jumps. The parameters of the stochastic process are taken from Di Cesare (2005) and are estimated from the prices of the options on S&P 500 futures contracts listed on the Chicago Mercantile Exchange over the period August 2000-July 2002. The volatility when a jump does not occur is set equal to 16 per cent, the expected number of jumps per year is 1.8 and the average jump size is -12.8 per cent (that is, the stock market index falls by almost 13 per cent). In the simulations presented in Table 3 share prices follow a geometric Brownian motion without jumps; in that case, share price volatility is set equal to 20 per cent. Nominal GDP is assumed to follow a Brownian geometric motion, with different values for volatility and correlation with share returns.

The final amounts of the notional fund and the pension fund are calculated on the basis of (2) and (3), respectively; the payment generated by the option at the time of retirement is then calculated on the basis of (4). This calculation is made for each of the 10,000 Monte Carlo simulations. The average value of the payments generated by the option in all the simulations is then calculated, and finally its present value assuming risk neutrality is determined. The value obtained is expressed as a percentage of the present value (also calculated assuming risk neutrality) of the final amount of the pension fund.

Appendix B: The value of the option with a lump-sum investment

It is much easier to calculate the value of the option if we posit a single investment in the pension fund; on this basis, a closed-form calculation is possible. Let us consider this case under the further simplifying assumption that share prices follow a Brownian geometric motion without jumps. The minimum return guarantee linked to the nominal GDP growth rate, which is described in Section 4, corresponds to an option in which one risky asset (the amount accumulated in the pension fund) is swapped with another (the notional fund). This derivative contract, sometimes known as an exchange option, was studied for the first time by Margrabe (1978). It represents a more general case of the classic option *à la* Black-Scholes-Merton, since the strike price is itself a stochastic variable.[31] Assuming the pension fund member makes a single lump-sum payment (represented by A_0) at the beginning of the accumulation period, and taking into account that the initial value of the notional fund (G_0) is the same as that of the pension fund, the value of the option at the beginning of the accumulation period (f_0) is given by the following formula:

$$f_0 = A_0\left(N(d_1) - N(d_2)\right) \tag{B.1}$$

where

$$d_1 = \frac{1}{2}\sqrt{T}\hat{\sigma} \tag{B.2}$$

$$d_2 = d_1 - \sqrt{T}\hat{\sigma} = -\frac{1}{2}\sqrt{T}\hat{\sigma} \tag{B.3}$$

[31] See also Hull (2000), Chapter 18.

$$\hat{\sigma} = \sqrt{\sigma_A^2 + \sigma_G^2 - 2\rho_{AG}\sigma_A\sigma_G}\ . \qquad \text{(B.4)}$$

and where $N(x)$ is the probability of a standard normal random variable being smaller than x, while $\hat{\sigma}$, σ_A^2, σ_G^2 and ρ_{AG} are, respectively, the overall volatility of the payments generated by the option, the volatility of the pension fund, the volatility of the notional fund, and the correlation between the two stochastic variables.

In equations (B.1-B.4) the price of the option is evidently a function of the overall volatility $\hat{\sigma}$, and therefore of the volatility of the two underlying stochastic processes (the pension fund portfolio and nominal GDP) and their correlation. It also obviously reflects the initial value of the insured capital and the duration of the contract.

Given (B.4), the effect of the correlation ρ_{AG} on total volatility $\hat{\sigma}$ is monotonic and negative. In fact, it depends on the following condition:

$$\frac{\partial\hat{\sigma}}{\partial\rho_{AG}} \leq 0 \;\Leftrightarrow\; -2\sigma_A\sigma_G \leq 0$$

which always holds. The effect of the correlation on $\hat{\sigma}$ is annulled if the exercise price of the option is not stochastic ($\sigma_G = 0$), which is the standard Black-Scholes-Merton case.

On the other hand, the effect of the volatility of the strike price (σ_G) on total volatility $\hat{\sigma}$ is not monotonic. It depends on the following condition:

$$\frac{\partial\hat{\sigma}}{\partial\sigma_G} \geq 0 \;\Leftrightarrow\; +2\sigma_G - 2\rho_{AG}\sigma_A \geq 0 \;\Leftrightarrow\; \rho_{\mathrm{AG}} \leq \min\left(1, \frac{\sigma_G}{\sigma_A}\right) \qquad \text{(B.5)}$$

where in (B.5) it is also taken into account that the correlation cannot be greater than 1. Since in the simulations $\sigma_A \cong 0.105$, (B.5) implies that

$$\frac{\partial\hat{\sigma}}{\partial\sigma_G} \geq 0 \;\Leftrightarrow\; \rho_{\mathrm{AG}} \leq \min(1, 9.52\,\sigma_G)$$

(where the last inequality holds with a degree of approximation). Finally, in the extreme case of perfect positive correlation between the value of the portfolio and the option strike price ($\rho_{AG} = 1$), total volatility $\hat{\sigma}$ is equal to:

$$\hat{\sigma} = \left|\sigma_A - \sigma_G\right| \qquad \text{(B.4)}$$

and the price of the option is led by the differential between the two volatilities. The value of the exchange option is annulled if two stochastic processes with perfect positive correlation also have the same volatility.

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