

# Private Wealth and Pensions across European Countries

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# Motivations

## □ The effect of pension on savings:

- An old issue in the literature (Feldstein, 1974)
- Ambiguous overall effect: displacement effect and early retirement effect

↗ public benefits=> ↗ consumption over the life-cycle => ↘ private savings

↗ public benefits=> Earlier retirement=> ↗ private savings

=>Related policy issue: Adequacy of savings to retirement needs.

## □ This paper: estimates the effect of pension wealth on private non-pension wealth for 7 euro area countries.

=> Heterogeneity in the euro zone : are there differences in households' portfolio and wealth across euro area countries due to differences in pension schemes ?

# Related literature

- ❑ **No consensus** on the magnitude of the effect. Papers differ in terms of country, time period, identification strategy, endogeneity bias, sample selection, etc.
- ❑ **Recent empirical analysis:** Individual data. Regressions derived from a simple life-cycle model of consumption, and account for the planning horizon and wealth effect of pension. e.g. Gale 1998, Engelhardt and Kumar 2011, Hurd et al. 2012, Alessie et al. 2013.
- ❑ **Identification strategies**
  - Pension reforms. Attanasio and Rohwedder 2003, Attanasio and Brugiavini 2003
  - Cross-country differences and non linearity of pensions within country. see Engelhardt and Kumar 2011, Hurd et al. 2012, Alessie et al. 2013.
  - Endogeneity issues related to individual heterogeneity in taste of saving: instrumental variable regression. See Engelhardt and Kumar 2011, Hurd et al. 2012, Alessie et al. 2013.

# This paper (1)

- ❑ Effect of mandatory pension wealth on private net wealth in **BE, DE, FR, GR, IT, LU, PT**
- ❑ Cross-section data from a cross-country harmonized wealth survey (HFCS-ECB) combined with pension wealth estimates (OECD pension models). **Reference year: 2014.**
- ❑ Standard reduced form equation of wealth accumulation based on the life-cycle (Gale, 1998)
- ❑ Due to large cross-country heterogeneities: country-by-country regressions => **Identification provided by non linearities in pension schemes**

# This paper (2)

## ❑ **Harmonized cross-country approach**

- sample selection: cross-country differences in entry into the labour market/transition from work to retirement (individuals aged 30-54)
- instrumental variable definition (based on NRA in each country)

## ❑ **Our contribution compared with previous cross-country papers for Europe** (Alessie et al. 2013, Hurd et al. 2012 based on SHARELIFE)

- Wealth accumulation during working life (30-54 instead of 54-75 or 65-75)
- New data : harmonized Wealth survey (HFCS) OECD pension wealth simulations. More observations to do country-by-country analysis
- Year 2014 (after the financial crisis)
- Only cross-section information (wage), no retrospective information on careers. Detailed control variables (education, household composition, credit constraints, gift and inheritances received)

# This paper: main results

- ❑ Need to account for **heterogeneous effects across the net wealth distribution** (quantile regressions)
- ❑ Need to account for the **endogeneity between pension wealth and non pension wealth arising from individual expectations about at what age to retire** (Instrument in the spirit of Engelhardt and Kumar (2011))
- ❑ **Substantial cross-country heterogeneity**: crowd in/crowd out effects:
  - depending on the country
  - depending on the type of assets (financial assets, housing assets)

Underlying issues: Continental versus Mediterranean welfare states? The role of housing as a store of value for old age? Financial crisis and reforms across country?

# Presentation outline

- Empirical model
- Data: wealth survey (HFCS) and OECD pension simulations
- Results
- Conclusion

# Empirical model (1)

□ Standard empirical specification derived from a simple life-cycle model, following Gale (1998) (e.g. Alessie et al. 2013).

□ We estimate :

$$W_i = \beta_0 + \beta_1 Y_i + \beta_2 Q * P_i + \gamma Z_i + u_i$$

$i$  : the individual index,

$W_i$  : non pension wealth

$Y_i$ : income

$P_i$ : pension wealth (mandatory pensions for the private sector)

$Q$ : Gale's Q factor (with  $r=2\%$ )

$X_i$  : Additional controls (age, gender, household composition, education, credit constraints, gifts and inheritances received)

$u_i$  the error term.

□ We run OLS, IV and Quantile and IV Quantile regressions

Instrumented Quantile regressions with CQIV – stata module of Chernozhukov et al.(2015) )

The error term  $u$  is defined, for  $X = (1, Y, P, Z)$  as:

- $E(u|X) = 0$  in the case of standard OLS
- $q_\tau(u_\tau|X) = 0$  with  $q_\tau$  the conditional  $\tau$ -quantile for the quantile regressions



# Empirical model (2)

## ❑ Identification: non linearities in pension schemes and differences in individuals' pension enrollment

Due to cross-country heterogeneities: country-by-country regressions

## ❑ Endogeneity issue and instrumental variable

- Unobservable factors such as preference for leisure may affect both pension and saving (See e.g. Engelhardt and Kumar 2011, Hurd et al. 2012, Alessie et al. 2013)
- Our pension wealth variable : simulated pension benefits using gender, year of birth, number of years of contribution and the mean earning histories by cohort and wage level.
- Endogeneity arising from individual expectations “at what age they will retire”. => **Pension wealth instrumental variable: considering the country specific normal retirement age instead of the individual expectations**

# DATA

## ❑ **Household Wealth survey : Household Finance and Consumption Survey - HFCS (ECB)**

- Harmonized household level information on wealth and income for European countries
- Compared to SHARE: covers the full population (not only 50+) + detailed information on wealth composition
- Detailed information on wealth composition, household composition, current income but not on wage history
- Cross section. Wave 2. Reference year : 2014 (except for Spain: 2011). 20 countries.

## ❑ **OECD pension model**

- Harmonized methodology and assumptions across country (inflation, growth)
- Pension wealth: discounted sum of all future pension benefits taking into account residual life expectancy and indexation of pension benefits (by country)
- **Main national basic, minimum and mandatory schemes (both public and private pensions) for private-sector workers under pension rule of 2014 .**
- Computed considering various multiple of average earnings and retirement ages

# DATA

## ❑ Matching household non pension wealth (HFCS) with individual pension wealth (OECD model)

Based on:

- gender, age, income (as a multiple of the average income of the age group)
- The age at which the individuals expect to retire
- whether the individuals declare in the HFCS to be eligible in the future to public or private pension

## ❑ Sample selection

- **Reference person aged 30-54 and in employment** (cross-country heterogeneity in entry into the labour market, transition to retirement)
- **Self-employed people excluded** (pension wealth not available in OECD simulations)
- Countries for which we have the required information (7).

Countries excluded because of too small sample size, or because some crucial information is missing (expected retirement age in the HFCS or simulation of OECD pension), or because of reference year (Spain 2011 in the HFCS)

# DATA: sample composition (mean of the main variables)

|  | Belgium    | Germany      | France       | Greece     | Italy        | Luxembourg | Portugal     |
|--|------------|--------------|--------------|------------|--------------|------------|--------------|
| Net wealth                               | 148,651    | 123,454      | 140,303      | 38,528     | 92,736       | 353,845    | 68,531       |
| Financial assets                         | 40,951     | 38,528       | 33,630       | 4,052      | 10,461       | 87,208     | 12,235       |
| Real estate properties                   | 133,615    | 108,914      | 126,408      | 36,875     | 84,715       | 343,471    | 82,282       |
| Housing wealth owners (Y/N)              | 0.78       | 0.62         | 0.72         | 0.61       | 0.66         | 0.82       | 0.86         |
| Adjusted Pension wealth                  | 107,677    | 92,848       | 115,777      | 68,387     | 73,644       | 372,605    | 51,462       |
| Adjusted and instrumented pension wealth | 97,895     | 90,314       | 140,159      | 69,409     | 72,911       | 383,034    | 58,510       |
| Wage                                     | 45,401     | 52,731       | 38,892       | 17,674     | 24,549       | 73,348     | 18,843       |
| Age                                      | 44         | 44           | 43           | 42         | 45           | 43         | 43           |
| Men (Y/N)                                | 0.65       | 0.71         | 0.63         | 0.70       | 0.68         | 0.71       | 0.59         |
| Married couples (Y/N)                    | 0.55       | 0.66         | 0.49         | 0.70       | 0.63         | 0.63       | 0.69         |
| Education                                |            |              |              |            |              |            |              |
| % Upper secondary                        | 0.34       | 0.48         | 0.37         | 0.58       | 0.48         | 0.32       | 0.22         |
| % Tertiary                               | 0.56       | 0.48         | 0.53         | 0.27       | 0.17         | 0.47       | 0.35         |
| Nber of employed people                  | 1.67       | 1.71         | 1.61         | 1.33       | 1.42         | 1.72       | 1.62         |
| % of individuals with inheritances       | 0.29       | 0.30         | 0.44         | 0.27       | 0.27         | 0.21       | 0.28         |
| % of individuals with credit constraint  | 0.03       | 0.06         | 0.09         | 0.07       | 0.03         | 0.10       | 0.08         |
| <b>Number of individuals</b>             | <b>532</b> | <b>1,260</b> | <b>3,700</b> | <b>732</b> | <b>1,852</b> | <b>714</b> | <b>1,905</b> |

=> **Wealthier people than in the country representative sample**

## Main variables definitions

**Net (non-pension) wealth**=total assets (real assets + financial assets)-total liabilities

**Financial assets**= deposits, mutual funds, bonds, non-self employment private businesses, publicly traded shares, money owned to household, private pension plans and whole life insurance policies)

**Real estate properties**=household main residence + other real estate properties

**Adjusted pension wealth**= discounted sum of all future pension benefits multiplied by the gale's Q factor (with r=2%)

# Pension wealth on Net non-pension Wealth

|                   |       | OLS               | IV      | Q                 |                 |                 | IVQ              |                  |        |
|-------------------|-------|-------------------|---------|-------------------|-----------------|-----------------|------------------|------------------|--------|
|                   |       |                   |         | Q1                | Q2              | Q3              | Q1               | Q2               | Q3     |
| <b>Belgium</b>    | Coeff | -0.040            | -0.130  | 0.030             | -0.184          | -0.015          | 0.057            | <b>-0.378 **</b> | -0.111 |
|                   | Lower | -0.789            | -0.901  | -0.251            | -0.524          | -0.545          | -0.420           | -0.827           | -0.682 |
|                   | Upper | 0.710             | 0.641   | 0.310             | 0.156           | 0.515           | 0.303            | -0.018           | 0.442  |
| <b>Germany</b>    | Coeff | 0.048             | -0.072  | 0.030             | <b>0.210 **</b> | <b>0.139 **</b> | -0.014           | 0.037            | -0.808 |
|                   | Lower | -0.754            | -1.238  | -0.065            | 0.054           | -0.273          | -0.330           | -0.086           | -1.666 |
|                   | Upper | 0.850             | 1.093   | 0.126             | 0.366           | 0.551           | 0.076            | 0.539            | 0.960  |
| <b>France</b>     | Coeff | 0.379             | 0.234   | -0.132            | 0.078           | -0.066          | -0.293           | <b>-0.162 **</b> | -0.207 |
|                   | Lower | -0.011            | -0.386  | -0.343            | -0.128          | -0.454          | -0.540           | -0.524           | -0.729 |
|                   | Upper | 0.768             | 0.854   | 0.078             | 0.284           | 0.322           | 0.047            | -0.007           | 0.139  |
| <b>Greece</b>     | Coeff | 0.101             | 0.073   | -0.002            | -0.075          | -0.109          | -0.049           | -0.094           | 0.123  |
|                   | Lower | -0.438            | -0.518  | -0.067            | -0.234          | -0.370          | -0.109           | -0.316           | -0.084 |
|                   | Upper | 0.641             | 0.664   | 0.062             | 0.084           | 0.151           | 0.024            | 0.008            | 0.606  |
| <b>Italy</b>      | Coeff | <b>-0.581 ***</b> | -0.378  | 0.097             | 0.112           | 0.099           | 0.130            | 0.113            | 0.070  |
|                   | Lower | -0.969            | -0.782  | -0.132            | -0.184          | -0.318          | -0.061           | -0.117           | -0.309 |
|                   | Upper | -0.194            | 0.026   | 0.326             | 0.408           | 0.515           | 0.340            | 0.438            | 0.423  |
| <b>Luxembourg</b> | Coeff | -5.404            | -4.334  | 0.081             | -0.056          | -0.732          | <b>0.638 **</b>  | 0.491            | 0.170  |
|                   | Lower | -13.471           | -12.291 | -0.564            | -0.381          | -2.064          | 0.189            | -0.173           | -1.472 |
|                   | Upper | 2.664             | 3.622   | 0.726             | 0.269           | 0.600           | 0.833            | 1.200            | 0.899  |
| <b>Portugal</b>   | Coeff | -0.021            | -0.734  | <b>-0.216 ***</b> | -0.105          | -0.042          | <b>-0.797 **</b> | <b>-0.632 **</b> | 0.295  |
|                   | Lower | -0.560            | -2.914  | -0.310            | -0.322          | -0.262          | -1.167           | -0.885           | -0.608 |
|                   | Upper | 0.518             | 1.446   | -0.122            | 0.112           | 0.178           | -0.447           | -0.021           | 0.920  |

# Pension wealth on net non-pension wealth

|                   |       | OLS               | IV      | Q                 |                 |                 | IVQ              |                  |        |
|-------------------|-------|-------------------|---------|-------------------|-----------------|-----------------|------------------|------------------|--------|
|                   |       |                   |         | Q1                | Q2              | Q3              | Q1               | Q2               | Q3     |
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|                   | Lower | -0.789            | -0.901  | -0.251            | -0.524          | -0.545          | -0.420           | -0.827           | -0.682 |
|                   | Upper | 0.710             | 0.641   | 0.310             | 0.156           | 0.515           | 0.303            | -0.018           | 0.442  |
| <b>Germany</b>    | Coeff | 0.048             | -0.072  | 0.030             | <b>0.210 **</b> | <b>0.139 **</b> | -0.014           | 0.037            | -0.808 |
|                   | Lower | -0.754            | -1.238  | -0.065            | 0.054           | -0.273          | -0.330           | -0.086           | -1.666 |
|                   | Upper | 0.850             | 1.093   | 0.126             | 0.366           | 0.551           | 0.076            | 0.539            | 0.960  |
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|                   | Lower | -0.438            | -0.518  | -0.067            | -0.234          | -0.370          | -0.109           | -0.316           | -0.084 |
|                   | Upper | 0.641             | 0.664   | 0.062             | 0.084           | 0.151           | 0.024            | 0.008            | 0.606  |
| <b>Italy</b>      | Coeff | <b>-0.581 ***</b> | -0.378  | 0.097             | 0.112           | 0.099           | 0.130            | 0.113            | 0.070  |
|                   | Lower | -0.969            | -0.782  | -0.132            | -0.184          | -0.318          | -0.061           | -0.117           | -0.309 |
|                   | Upper | -0.194            | 0.026   | 0.326             | 0.408           | 0.515           | 0.340            | 0.438            | 0.423  |
| <b>Luxembourg</b> | Coeff | -5.404            | -4.334  | 0.081             | -0.056          | -0.732          | <b>0.638 **</b>  | 0.491            | 0.170  |
|                   | Lower | -13.471           | -12.291 | -0.564            | -0.381          | -2.064          | 0.189            | -0.173           | -1.472 |
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|                   | Lower | -0.560            | -2.914  | -0.310            | -0.322          | -0.262          | -1.167           | -0.885           | -0.608 |
|                   | Upper | 0.518             | 1.446   | -0.122            | 0.112           | 0.178           | -0.447           | -0.021           | 0.920  |

# Results: cross country heterogeneity

## ❑ Similar conclusions for all countries (net wealth and financial wealth)

- Heterogeneous effects along the wealth distribution (Quantile regressions)
- Large confidence intervals at the top of the distribution

## ❑ Cross-country heterogeneity: « main » cases

- ✓ « Crowd out » effect : bottom or middle of the distribution

BE (NW, FW), FR (NW), GR (FW)

BE, FR: also a negative effect of pension wealth on the probability to hold real estate property

- ✓ « Crowd in » effect : Bottom of the distribution

DE (FW), LU (NW, FW)

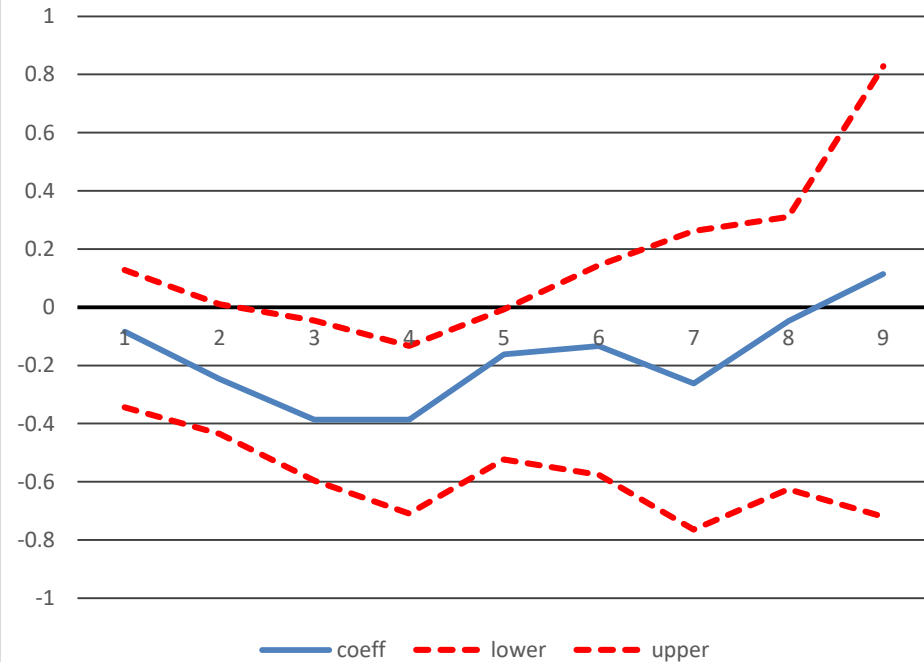
DE: also a positive effect of pension wealth on the probability to hold real estate property

Remark: when both significant effects for NW and FW: larger effect for NW than for FW (BE, LU)

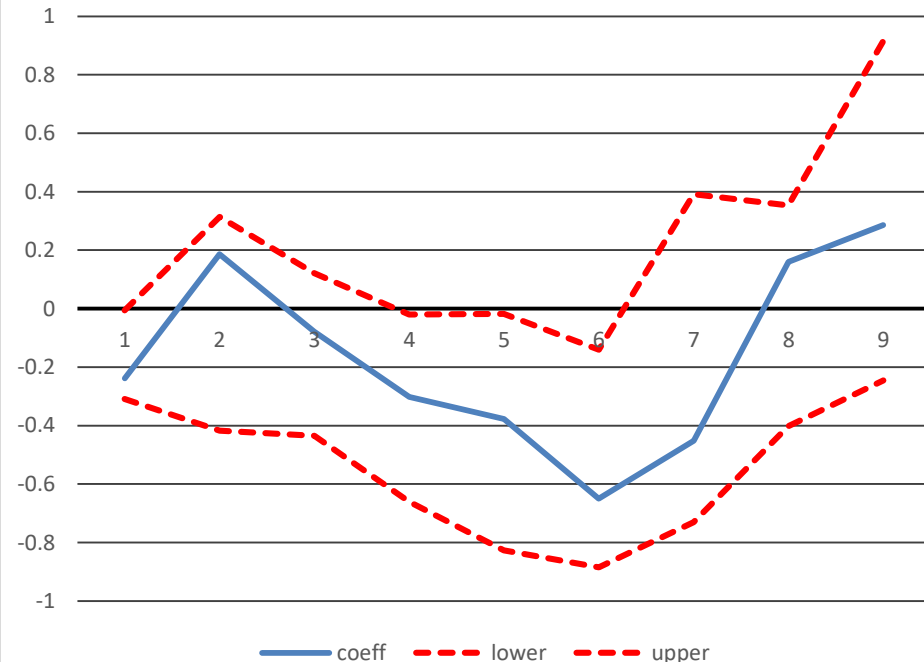
- ✓ PT: Crowd out at the bottom (NW, FW) and crowd in at the top (FW)

# Crowd out Net wealth

FRANCE Net Wealth



BELGIUM Net Wealth



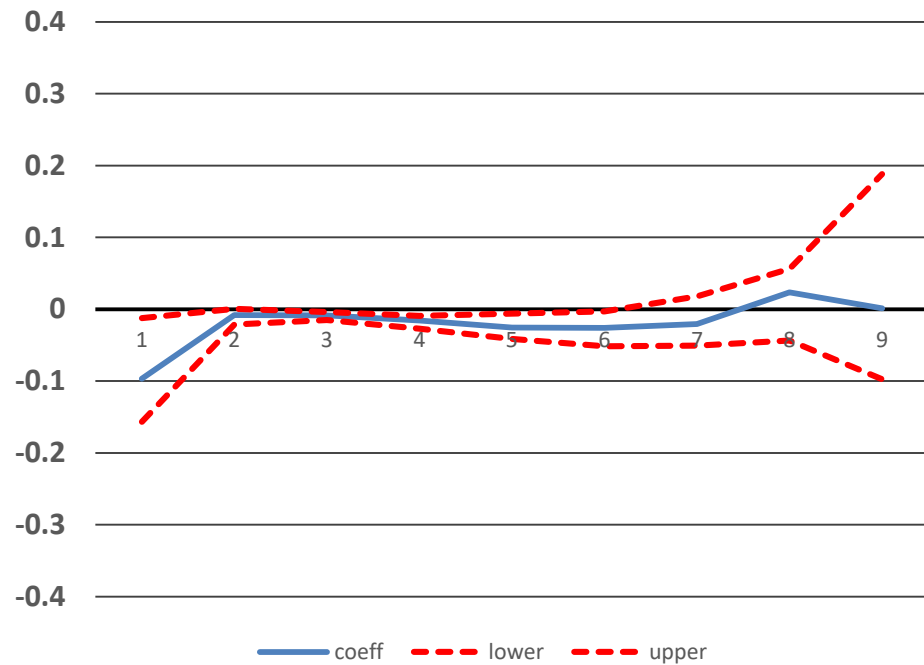
Additional results with housing wealth

BE and FR : also a **negative effect of pension wealth on the probability to hold real estate properties** (IV Probit) : real estate property as a store of value for old ages.

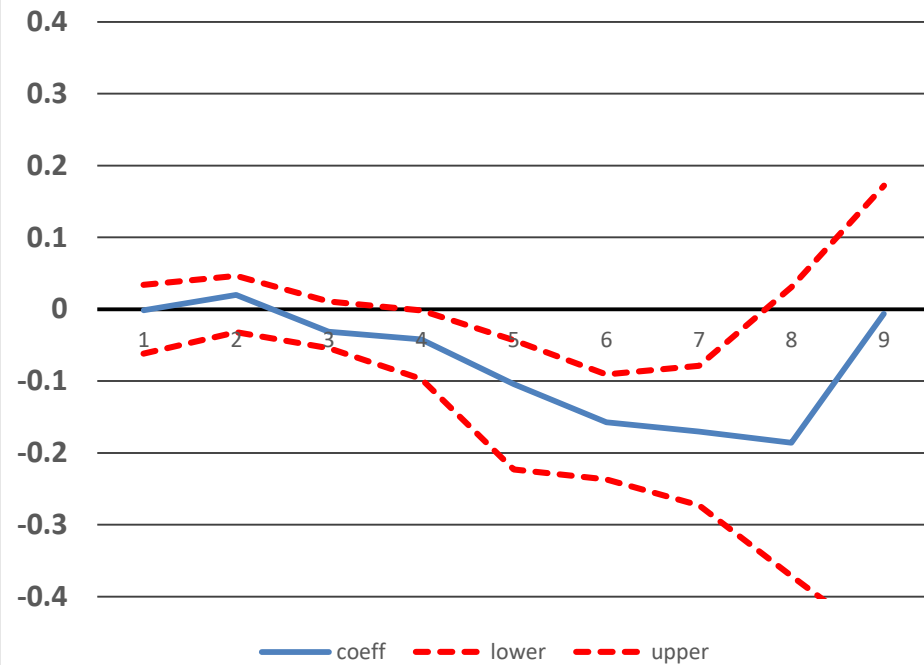


# Crowd out (Financial wealth)

## GREECE Financial Wealth

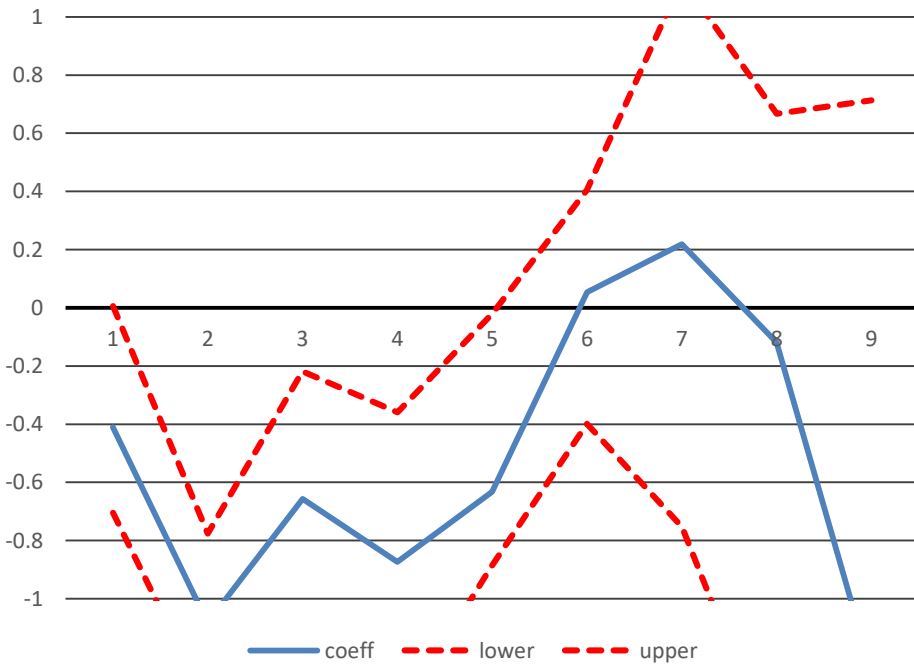


## BELGIUM Financial Wealth

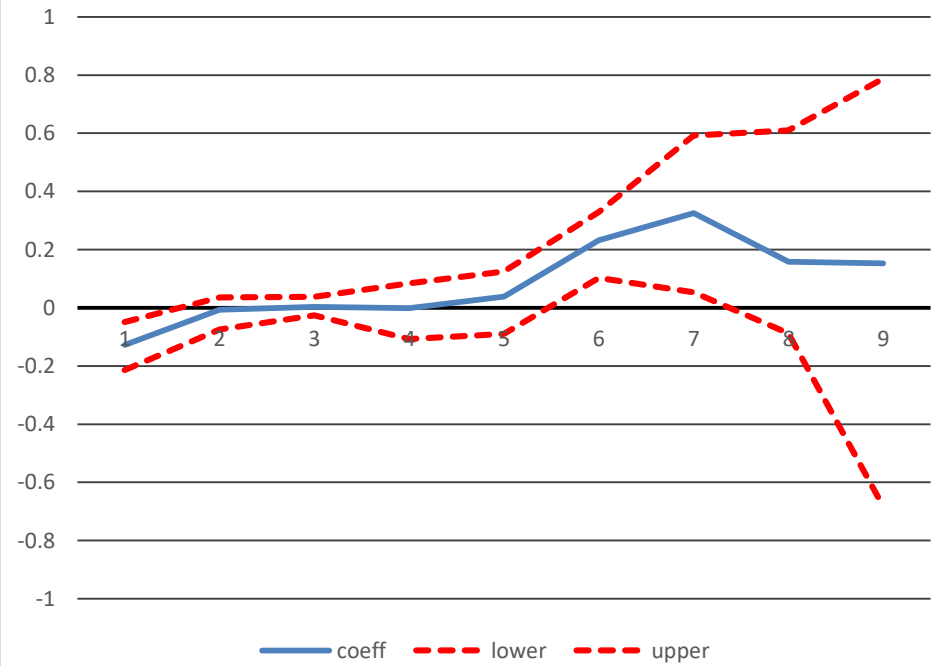


# Crowd out Portugal

PT NW

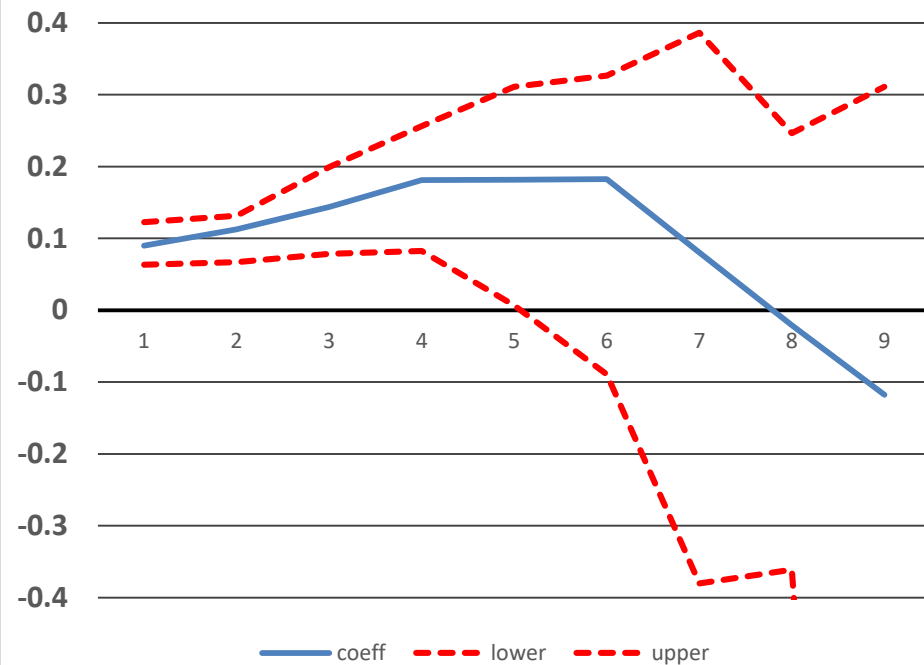


PT FW

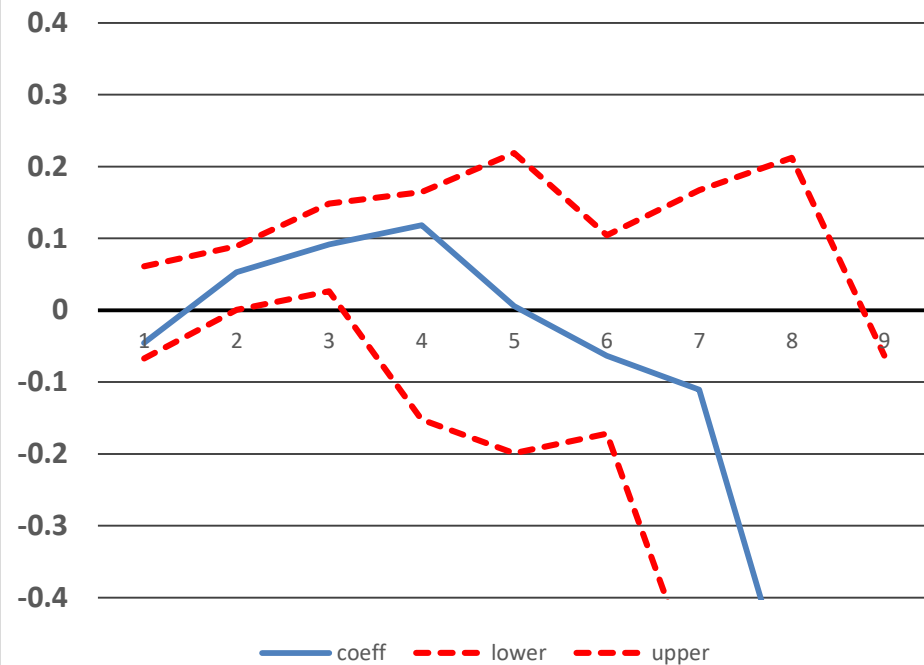


# Crowd in (Financial wealth)

LUXEMBOURG Financial Wealth



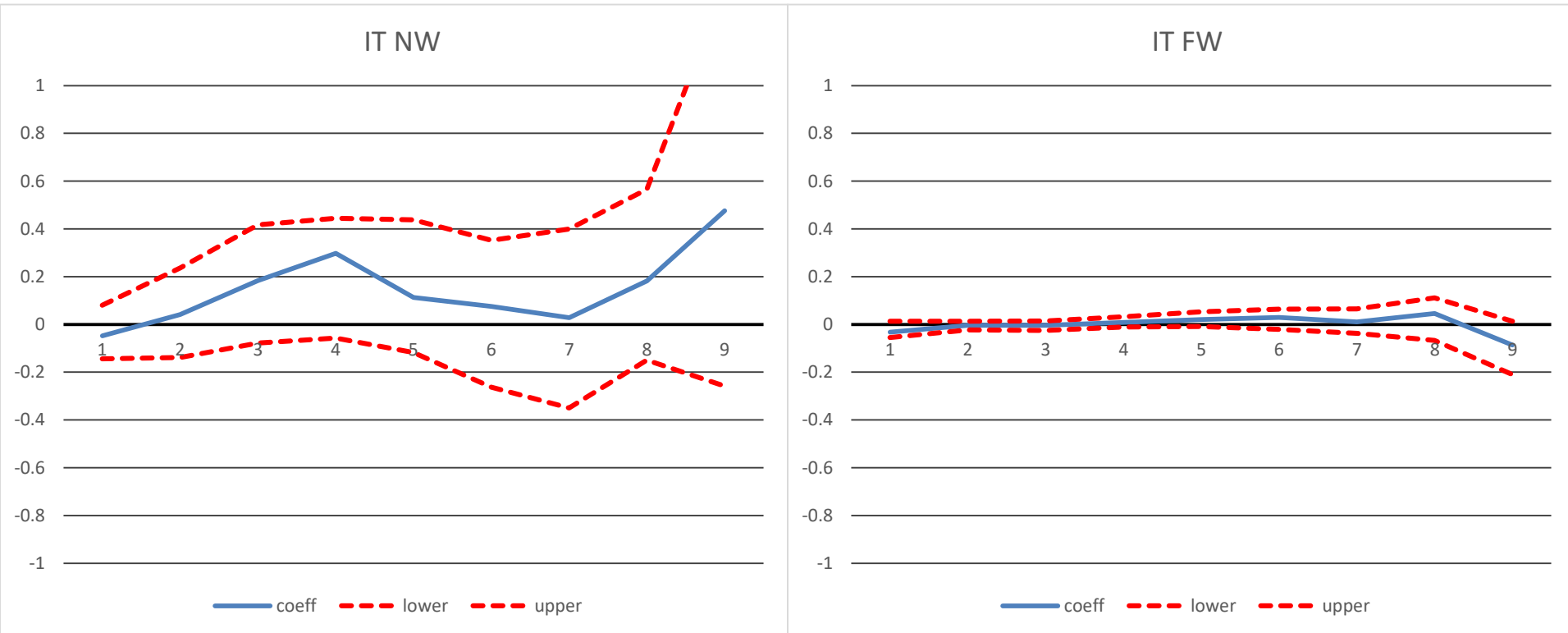
GERMANY Financial Wealth



Additional results with housing wealth

DE: also a **positive effect of pension wealth on the probability to hold real estate properties** (IV Probit)

# Results for Italy



- No significant estimates with IV Quantile regression
- While Attanasio and Brugiavini (2003) were able to find a substitutability effect between pension wealth and saving. Differences in the methodology (1992 reforms), but also in the time period?

# CONCLUSION

- ❑ **Crowd out/crowd in estimates** of pension wealth on non-pension wealth for 7 European countries
- ❑ Focus on population in employment – Year 2014
- ❑ **Cross-country heterogeneity**
  - ✓ **Crowd out effects** in the bottom or middle of the distribution in BE (NW, FW), FR (NW), GR (FW), PT (NW, FW)
  - ✓ **Crowd in effects** in LU (NW, FW), DE (FW)
  - ✓ **No significant effect in IT** [large confidence intervals]
- ❑ **How to interpret the cross-country heterogeneity?**
  - Welfare states (Mediterranean versus Continental countries)? Our results do not match with the standard Esping-Andersen classification.
  - Interaction with housing markets ? Housing as a store of value for old age in some countries

# APPENDIX

# Financial wealth

|                   |       | OLS        | IV         | Q         |           |           | IVQ       |           |           |
|-------------------|-------|------------|------------|-----------|-----------|-----------|-----------|-----------|-----------|
|                   |       |            |            | Q1        | Q2        | Q3        | Q1        | Q2        | Q3        |
| <b>Belgium</b>    | Coeff | -0.100     | -0.154     | 0.011     | -0.030    | -0.121 ** | -0.011    | -0.104 ** | -0.148 ** |
|                   | Lower | -0.434     | -0.467     | -0.037    | -0.087    | -0.233    | -0.058    | -0.223    | -0.279    |
|                   | Upper | 0.234      | 0.158      | 0.058     | 0.026     | -0.009    | 0.032     | -0.043    | -0.015    |
| <b>Germany</b>    | Coeff | -0.229     | -0.454     | 0.043 **  | 0.066 *   | 0.123 **  | 0.099 **  | 0.006     | -0.349    |
|                   | Lower | -1.148     | -1.813     | 0.003     | -0.001    | 0.024     | 0.007     | -0.199    | -0.729    |
|                   | Upper | 0.691      | 0.906      | 0.084     | 0.132     | 0.223     | 0.128     | 0.219     | 0.263     |
| <b>France</b>     | Coeff | 0.174      | 0.246      | -0.051    | -0.115 ** | -0.109    | -0.014    | -0.094    | -0.089    |
|                   | Lower | -0.137     | -0.249     | -0.102    | -0.190    | -0.249    | -0.076    | -0.171    | -0.210    |
|                   | Upper | 0.485      | 0.741      | 0.001     | -0.039    | 0.031     | 0.045     | 0.044     | 0.194     |
| <b>Greece</b>     | Coeff | 0.096      | 0.052      | 0.000     | -0.002    | 0.001     | -0.009 ** | -0.025 ** | -0.007    |
|                   | Lower | -0.179     | -0.198     | -0.003    | -0.015    | -0.020    | -0.018    | -0.041    | -0.049    |
|                   | Upper | 0.372      | 0.301      | 0.003     | 0.011     | 0.021     | -0.002    | -0.006    | 0.034     |
| <b>Italy</b>      | Coeff | -0.318 *** | -0.276 *** | -0.003    | 0.017     | 0.019     | -0.002    | 0.020     | 0.039     |
|                   | Lower | -0.447     | -0.411     | -0.016    | -0.015    | -0.056    | -0.022    | -0.008    | -0.050    |
|                   | Upper | -0.189     | -0.142     | 0.010     | 0.049     | 0.093     | 0.015     | 0.053     | 0.103     |
| <b>Luxembourg</b> | Coeff | -4.347     | -3.770     | 0.116 *** | 0.078 *** | -0.050    | 0.130 **  | 0.182 **  | 0.031     |
|                   | Lower | -10.508    | -9.778     | 0.073     | 0.011     | -0.276    | 0.068     | 0.007     | -0.376    |
|                   | Upper | 1.814      | 2.238      | 0.160     | 0.146     | 0.176     | 0.163     | 0.311     | 0.352     |
| <b>Portugal</b>   | Coeff | 0.178      | -0.100     | 0.031 **  | 0.093 **  | 0.259 **  | 0.004     | 0.038     | 0.336 **  |
|                   | Lower | -0.050     | -1.295     | 0.020     | 0.072     | 0.188     | -0.070    | -0.090    | 0.151     |
|                   | Upper | 0.406      | 1.096      | 0.043     | 0.113     | 0.330     | 0.050     | 0.125     | 0.532     |

# □ Endogeneity issue (pension wealth)

- In our case: pension wealth computed accounting for the expected retirement age (elicited through the HFCS)
- Instrumental variable: pension wealth computed using the country specific NRA

## Instrumented Pension Wealth : Retirement age

| BE | DE | FR | GR | IT | LU | PT |
|----|----|----|----|----|----|----|
| 67 | 65 | 67 | 67 | 67 | 65 | 66 |



# The background model

Following Alessie & al. (2013), we derive the empirical equation from a discrete time simple life cycle model with no uncertainty and liquidity constraint. The within period utility function is assumed to have constant relative risk aversion. We assume also perfect capital market with a constant real interest rate  $r$ .

The consumer maximisation program :

$$\begin{aligned} & \max_{c_t} \sum_{t=1}^T (1 + \rho)^{1-t} \frac{c_t^{1-\gamma}}{1-\gamma} \\ \text{s. t. } & \sum_{t=1}^T (1 + r)^{1-t} c_t = \sum_{t=1}^R (1 + r)^{1-t} E_t + \sum_{t=R}^T (1 + r)^{1-t} B_t \end{aligned}$$

With  $c_t$  the instantaneous consumption at age  $t$ ,  $E_t$  the income at age  $t$ ,  $B_t$  the pension benefit at age  $t$ ,  $R$  the retirement age,  $T$  the maximum age,  $\rho$  is the discount rate and  $\gamma$  the coefficient of relative risk aversion.

The wealth  $W_t$  at a given age  $t$  is defined as:

$$W_t = \sum_{\tau=1}^t (1 + r)^{t-\tau} (y_\tau - c_\tau) \quad (1)$$

with  $y_t$  the income at age  $t$ , corresponding to wage before retirement and pension after retirement. We set the value of the discount rate at the interest rate level, i.e.  $\rho=r$ . The consumption at age  $t$  is equal to:

$$c_t = \left( \sum_{\tau=1}^T \left( \frac{1}{1+r} \right)^{\tau-1} \right)^{-1} \left( \sum_{\tau=1}^R (1 + r)^{1-\tau} E_\tau + \sum_{\tau=R}^T (1 + r)^{1-\tau} B_\tau \right) \quad (2)$$

## The background model

Substitution of (2) in (1) provides the value of wealth at age  $t$

$$W_t = \sum_{\tau=1}^t (1+r)^{t-\tau} y_t - Q(t) \sum_{\tau=1}^R (1+r)^{t-\tau} E_t - Q(t) \sum_{\tau=R+1}^T (1+r)^{t-\tau} B_t \quad (3)$$

With Q-factor:

$$Q(t) = \frac{\sum_{\tau=1}^t \left(\frac{1}{1+r}\right)^{\tau-1}}{\sum_{\tau=1}^T \left(\frac{1}{1+r}\right)^{\tau-1}}$$