Working Paper 205/21

LIFE-CYCLE RISK-TAKING WITH PERSONAL DISASTER RISK

Fabio C. Bagliano
Carolina Fugazza
Giovanna Nicodano

February 2021

Life-Cycle Risk-Taking with Personal Disaster Risk*

Fabio C. Bagliano^ Carolina Fugazza^

Giovanna Nicodano^^

^Università di Torino and CeRP (Collegio Carlo Alberto)

^^Università di Torino, CeRP (Collegio Carlo Alberto) and Netspar

December 31, 2020

Abstract

Inspired by a growing body of empirical work, this paper models a non-linear labour income process allowing for a personal disaster, such as long-term unemployment or disability, during working years. Such a disaster entails an uncertain but potentially large permanent shock to earnings. Personal disaster risk allows to match the flat investment profile in age, which is observed in the United States, when the calibration of both the disaster probability and the expected permanent loss in the disaster state is conservative.

Keywords: disaster risk, beta distribution, life-cycle portfolio choice, non-linear income process, unemployment risk, disability risk

JEL classification: D15, E21, G11

Address: Dipartimento ESOMAS, Università di Torino, Corso Unione Sovietica 218bis, 10134, Torino (Italy).

E-mails: fabio.bagliano@unito.it; carolina.fugazza@unito.it; giovanna.nicodano@unito.it

*We thank Orazio Attanasio, Scott Cederburg, Janet Currie, Kilic Mete, Dirk Krueger, Claus Munk and participants to the 2019 AFA Meetings and the 2020 Eurofidai Meetings for suggestions. We acknowledge financial support from the Italian MIUR ("Department of Excellence" 2018-2022 grant).

1 Introduction

What happens if a person's future ability to work is permanently reduced? Insurance against permanent shocks, such as disability and long-term unemployment, is well known to be incomplete (Guvenen and Smith, 2014; Low. Meghir and Pistaferri, 2010; Low and Pistaferri, 2015). Therefore, young households make provisions to cushion against a personal disaster, even if the possibility of its occurrence is quite rare. Against this background, this paper examines the pattern of self-insurance in financial markets over the life-cycle in light of the possibility of a rare personal disaster during working years. The findings show that personal disaster risk can alter lifetime *ex-ante* investment choices for the average worker, even if *ex-post* most workers will not experience a disaster. Additionally, this paper reveals that uncertainty about the size of uninsured human capital losses, which characterizes these rare disasters, enhances the precautionary behavior of young workers. This behaviour will result in lower risk-taking at the beginning of working life, with respect to a comparable deterministic human capital loss. Uncertainty about the extent of losses is crucial in order to closely match the observed age profiles of US investors from 1992 to 2016, based on the methods of Ameriks and Zeldes (2004), when the calibrations are conservative.

We contribute to the household finance literature by linking risk-taking in financial markets with the *ex-ante* uncertain, but potentially extreme permanent impact of income shocks. In contrast to the household finance literature, we go beyond the positive probability of zero labor income implied by the linear income process proposed by Cocco, Gomes and Maenhout (2005) and later adopted within this strand of literature. Inspired by a growing body of empirical work showing that earnings dynamics display non-linearities (Arellano, Blundell and Bonhomme, 2017; Guvenen, Karahan, Ozkan and Song, 2016; De Nardi, Fella and Paz-Pardo, 2020), we model the occurrence of a disaster that brings about a permanent income reduction of uncertain proportion. Specifically, the fraction of human capital lost follows a beta distribution.[1] The flexibility of the beta distribution allows us to concentrate a large probability mass on the likeliest values of proportional human capital reduction, leaving open the possibility of extremely unlikely but devastating realizations. Importantly, when careers are calibrated to broadly match observed US labor market features, optimal investment in the risky asset remains flat over the whole working life, in line with early evidence on US portfolios (Ameriks and Zeldes, 2004), which we update to 2016. This situation occurs even when we account for the large insurance coverage of permanent income shocks, as in Guvenen, Karahan, Ozkan and Song (2017) and Guvenen and Smith (2015), which may ultimately reduce the expected human capital losses due to long-term unemployment. Without disaster risk, the implied optimal stock holding still counter-factually decreases with age before retirement, unless the long-term unemployment rate is as high as that observed during the Great Recession (as in Bagliano, Fugazza and Nicodano, 2019).

Our results highlight the role of the non-linear income process in flattening the age profile of risk taking. With a linear income process, prior models resort to using additional features to explain reduced risk taking in financial markets (see Cocco, 2004; Munk and Sorensen, 2010; Kraft and Munk, 2011; Bagliano, Fugazza and Nicodano, 2014; Hubener, Maurer and Mitchell, 2016; and Chang, Hong and Karabarbounis, 2018). In the latter study, for instance, uncertainty resolves over time thanks to agents learning about their income volatil-

[1] This distribution may also characterize the damage caused by natural disaster (see Bhattacharjee, 2004, and Lallemant et al., 2015).

ity, otherwise the linearity of labor income shocks would lead to the usual high risk taking when young. In our paper, uncertainty resolves because time passes without the occurrence of disasters. This outcome explains why the young bear more labor income risk, which is the intuition pioneered in Viceira (2001) and Benzoni, Colling-Dufresne and Goldstein (2007). More precisely, we model working life careers as a three-state Markov chain driving the transitions between employment, short-term unemployment and personal disaster states. Uncertain permanent earning losses that occur in the disaster state represent productivity loss due to long-term unemployment (Arulampalam (2001); Schmieder, von Wachter and Bender, 2016), disability or both (Low and Pistaferri, 2015).

This model nests the traditional life-cycle framework within the household finance literature (Cocco, Gomes and Menhout, 2005). Indeed, when the disaster probability is zero and/or human capital erosion is compensated by full insurance, the agents optimally reduce exposure to risky assets as they approach retirement. This pattern obtains since human capital provides a hedge against shocks to stock returns, which makes bearing financial risk generally acceptable. Investment in stocks should therefore be relatively high at the beginning of working careers, when human capital is large relative to accumulated financial wealth. Risky investment then gradually declines until retirement, as human capital decreases relative to financial wealth. When personal disaster risk is instead only partially insured, the above effect is moderated by the resolution of uncertainty concerning labor and pension income as the worker safely approaches retirement age.[2] Since the risk of a personal disaster declines as an individual approaches retirement, the resolution of uncertainty compensates for the

[2] We do not model the option to change labor supply to buffer income shocks, as in Bodie, Merton and Samuelson (1992) and Gomes, Kotlikoff and Viceira (2008). This option is open to those who find a new job, while what drives our results is the *ex ante* possibility of a large loss in the disaster state.

hedge effect and the optimal investment in stocks is relatively flat over the life cycle.

Our model delivers additional implications concerning life-cycle choices in the context of incomplete insurance against personal disaster risk. First, the distribution of optimal consumption growth becomes negatively skewed, due to disasters, in line with evidence on durable consumption growth (Yang, 2011). Second, personal disaster risk changes the age profile of savings thereby shrinking the heterogeneity of optimal portfolio choices across agents characterized by different career histories. Young workers increase early precautionary savings to buffer against possible, albeit rare, future disasters. Optimal consumption consequently declines during the early years but increases during both late working years and retirement years. Third, the average implied savings to income ratio increases, as in other life-cycle models highlighting the role of earnings shocks for solving life-cycle portfolio choice puzzles (Bagliano, Fugazza and Nicodano, 2014; Chang, Hong and Karabarbounis, 2018). While the implied savings to income ratio may appear counter-factually high, our model does not incorporate the effects of means-tested welfare programs that lead to zero optimal precautionary saving for poor households (Hubbard, Skinner and Zeldes, 1995).

This model does not address non-participation in the stock market. It should otherwise allow for correlation between stock returns and labor income shocks (see Bagliano, Fugazza and Nicodano (2014) for additional conditions and Bonaparte, Korniotis and Kumar (2014) for empirical results) or correlation between stock returns and the skewness of labor income shocks (see Catherine, Sodini and Zhang, 2020). Likewise, prominent papers study consumption and labor market choices with permanent income shocks (Low, Meghir and Pistaferri, 2010 and Low and Meghir, 2015), focusing on the design of social insurance against employment and productivity risk without allowing for investments in risky assets. Our paper belongs to the household finance tradition that allows for risky investments but overlooks both moral hazard stemming from social insurance programs and the associated difference between productivity and employment risk. Finally, personal disaster risk differs from both the individual stock market disaster in Fagereng, Gottlieb and Guiso (2017) and the aggregate economic collapse in the macro-finance literature (Barro, 2006), although disasters may be correlated. As Arellano, Blundell and Bonhomme (2017) point out, macroeconomic disasters are statistically elusive events, while disasters at the micro level happen all the time.

The rest of the paper is organized as follows. In Section 2 we provide evidence on life-cycle portfolio holdings and institutional details on long-term unemployment, disability and social insurance for the United States. Section 3 presents the benchmark life-cycle model and briefly outlines the numerical solution procedure adopted. We detail the model calibration in Section 4 and discuss our main results in Section 5, where the ability of the model to match the stock-holdings observed in real data is also assessed. Section 6 concludes the paper.

2 Households Portfolios and Personal Disasters

This section introduces the main stylized facts concerning financial risk-taking and personal disaster risk in the United States. The first subsection builds on the method of Ameriks and Zeldes (2004) to examine the empirical relationship between age and conditional risky shares; that is, it pertains to the fraction of financial wealth held in risky assets conditional on participation in the stock market. These life-cycle investment profiles in US data will later be matched with the model-implied profiles.

Since such profiles are calibrated to disaster risk, the second subsection summarizes some relevant features of disability and long-term unemployment.

2.1 Life-Cycle Profiles of Household Portfolios

We pool data from the independent cross-sectional surveys in the Survey of Consumer Finances (SCF), covering the years from 1992 to 2016. The SCF is nationally representative of households in the United States and collects detailed information on their characteristics and their investment decisions. We classify the households' financial assets into two categories, *safe* and *risky*, following Chang, Hong and Karabarbounis (2018). Safe assets include checking accounts, savings accounts, money market accounts, certificates of deposit, the cash value of life insurance, US government and state bonds, mutual funds invested in tax-free bonds and government-backed bonds, and trusts and annuities invested in bonds and money market accounts. Risky assets include stocks; stock brokerage accounts; mortgage-backed bonds; foreign and corporate bonds; mutual funds invested in stock funds; trusts and annuities invested in stocks or real estate; and pension plans that are thrift, profit-sharing, or stock purchase plans. In Table 1, we report the summary statistics concerning both the households' financial assets composition and their main characteristics. We restrict the sample to households with positive financial assets and with a head of household aged between 21 and 70.

Table 1: Descriptive statistics: SCF data 1992-2016

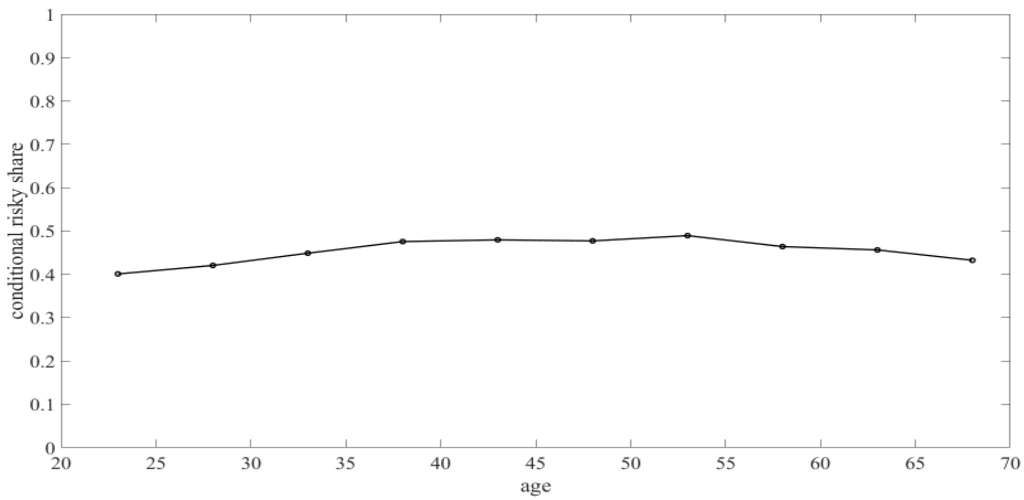
Wave	1992	1995	1998	2001	2004	2007	2010	2013	2016
Financial assets									
Amount ($)									
Safe	126,323	135,264	138,320	148,852	139,953	137,447	143,926	126,739	141,793
Risky	70,842	91,448	167,039	202,997	161,592	162,939	129,381	137,308	159,180
Total (Safe+Risky)	197,166	226,712	305,359	351,849	301,544	300,386	273,307	264,046	300,973
Conditional Share									
Safe	64.1%	59.7%	45.3%	42.3%	46.4%	45.8%	52.7%	48.0%	47.1%
Risky	35.9%	40.3%	54.7%	57.7%	53.6%	54.2%	47.3%	52.0%	52.9%
Men	78.6%	76.8%	76.9%	77.0%	75.7%	76.5%	76.6%	75.1%	74.2%
Age	45.6	46.2	46.5	46.5	47.5	48.2	47.6	48.2	48.9
No high school	12.5%	11.5%	10.6%	10.5%	9.4%	9.1%	8.8%	7.6%	11.3%
High school	30.1%	32.6%	31.6%	31.3%	29.9%	31.4%	30.7%	29.2%	24.9%
Some college	24.2%	27.0%	27.2%	26.0%	26.3%	26.3%	26.7%	27.2%	28.4%
College	33.2%	28.8%	30.6%	32.3%	34.4%	33.2%	33.9%	36.1%	35.4%
N (households)	3906	4302	4326	4475	4526	4423	6555	6026	6261

The table reports the average composition of households financial assets and demographic characteristics across various SCF waves ($1992-2016$). The sample is restricted to households with heads aged between 21 and 70 years and with a positive amount of financial assets. Nominal variables are expressed in 2015 U.S. dollars.

In Figure 1, we report the life cycle age profile of the average conditional portfolio share invested in risky assets. The dots represent five-year averages (from age group $21-25$ to age group $66-70$). The conditional risky share is flat over the life cycle, ranging from 40%

to 49%.

Figure 1: Conditional Risky Share - SCF data



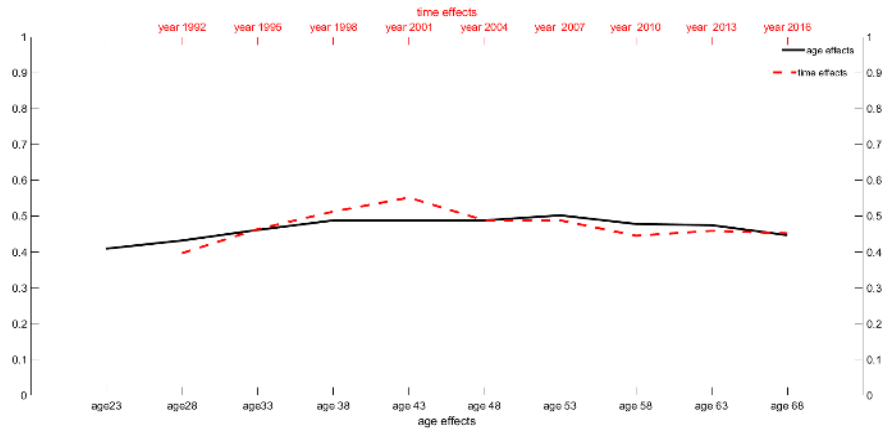
This figure displays the life cycle profile of conditional risky share of financial assets held by U.S. households grouped by five-years age classes (21 − 25,...,66 − 70). The dots represent five-year averages.

Ideally, we should distinguish the impact of age on household risk taking from that of both the calendar year and birth cohorts. However, the three effects cannot be separately identified. We therefore estimate three regression models in which one effect is held constant against the other two at a time, following Ameriks and Zeldes (2004). The age dummies are constructed on the basis of five-year age groups, from 21 to 70, and the reference age group is aged between 46 and 50 years. Similarly, the birth year cohort dummies refer to five birth-year groups (from 1924−1928 to 1989−1993), and we the cohort 1953−1958 serves as the reference group. Finally, the time effects refer to the years in which the surveys are collected, and the year 2004 represents the reference group. In Figure 2 panel a, we report the regression estimates of time and age effects based on OLS estimates with cohort effects excluded (dashed and solid lines, respectively); panel b reports the time and cohort effects based on OLS estimates with age effects excluded (dashed and solid lines, respectively); finally, panel c reports the age and cohort effects based on OLS estimates with time effects excluded (dashed and solid lines, respectively).[3] The conditional risky share is remarkably flat across ages and cohorts in all specifications. The time effects show an increase during the 1990s and a relative slowdown after 2000. Such a time-based evolution in households' behavior may reflect both the "Internet bubble" in the 1990s and higher households' background risk during the 2000s. These patterns are robust across education levels (unreported here). These results reveal that the conditional risky share is almost flat in age and across cohorts through 2016, confirming the patterns originally unveiled by Ameriks and Zeldes (2004).

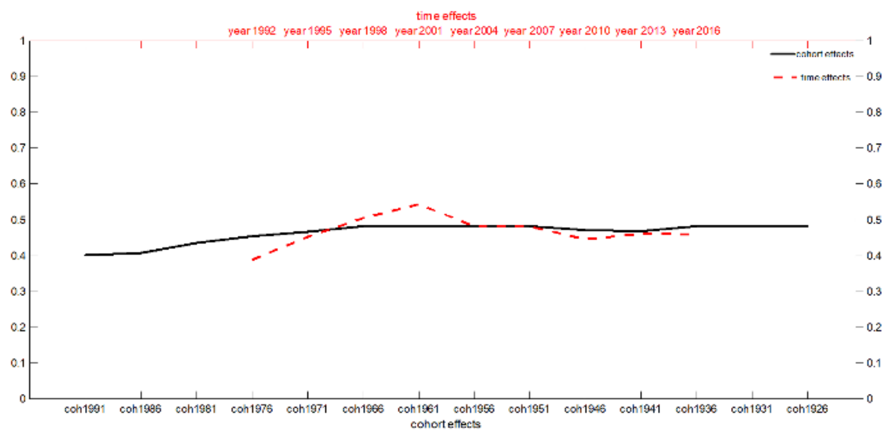
[3] We set to zero all the coefficients that are not statistically significant from zero at 5% level.

Figure 2: Age, Time and Cohort effects on Conditional Risky Share

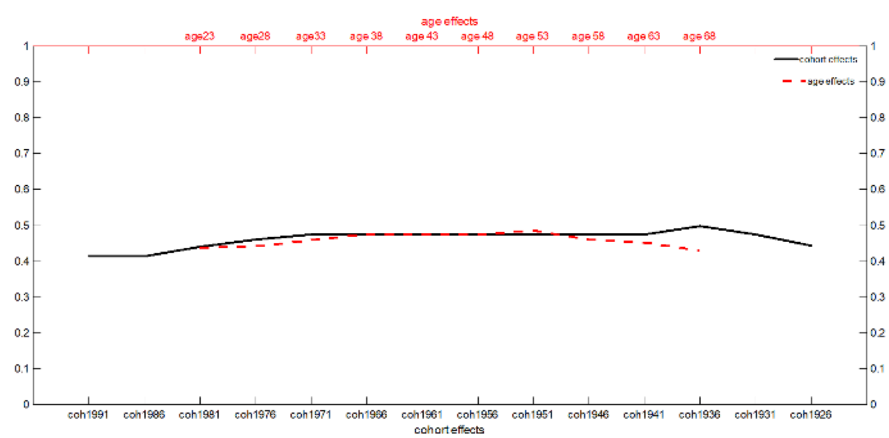
Panel (a)



Panel (b)



Panel (c)



This figure displays the estimated age, cohort and time effects on conditional risky share under different model specifications. (a) The cohort effect is assumed to be constant across ages and periods; (b) the age effect is assumed to be constant across cohorts and periods; (c) the time effect is assumed to be the same across ages and cohorts. SCF data from 1992 to 2016 on households with heads aged between 21 and 70 years. Coefficients that are not statistically significant at 5% are set to zero.

2.2 Uninsured Personal Disasters

This section provides an assessment of human capital losses deriving from personal disasters, such as layoffs or disability. It also sheds light on estimates of the fraction of permanent income shocks that remain uninsured. In so doing, it touches upon heterogeneity across households and over the sample years. We calibrate the labor income process and the insurance parameters of our model against this background.

Unemployment may lead to persistent earnings losses that increase with the duration of unemployment because of skill deterioration and diminishing chances of finding a new occupation. In the United States the share of unemployed workers who were jobless for more than one year, while historically low, doubled during the Great Recession, reaching 24% of total unemployment in 2014 and hitting all education groups (e.g., see Kroft, Lange, Notowidigdo and Katz, 2016).[4] The chances of finding a job decrease, together with unemployment benefits, as the duration of unemployment increases.[5] Early estimates of persistent earnings losses due to long-term unemployment (see Jacobson Lalonde and Sullivan, 1993a) are around 25% of average earnings six years after separation, relative to workers with similar characteristics that stayed with the same employer during the same episode. Guvenen, Karhan, Ozkan and Song (2017) measure the effects of full-year non-employment across workers with heterogeneous histories in a more recent sample (1978-2010). Earnings losses are in the 35% to 40% range after 10 years. Such losses mostly derive from lower chances of future employment, rather than lower earnings conditional on working. Indeed workers who are employed 10 years after the shock bear smaller earnings losses, around 8% to 10% percent. Given our focus on equity investment, it is important to stress that earnings losses are large not only for

[4]For instance, in 2013, the share of US unemployed workers with a high school (college) education who had been looking for work for two or more years was 12.8% (13.5%) (see Mayer, 2014).

[5]Krueger, Cramer and Cho (2014) and Kroft, Lange, Notowidigdo and Katz (2016) show that the re-employability of the long-term unemployed progressively declines over time, to the extent that they are more likely to exit the labor force than to become re-employed. The presence of more job openings does not lead to increased employment among individuals who are jobless for more than six months, and this pattern holds across all ages, industries and education levels (Ghayad and Dickens 2012).

workers with low earnings but also for those in the top 5% of the past earnings distribution.[6]

Large negative shocks associated with health are another form of personal disaster. Mental health problems have an especially large impact on labor market outcomes, possibly because they also affect prime-age workers. The onset of mental illness initially reduces earnings by as much as 24%, and negative effects can last several years. Moreover, disorders reduce the probability of employment by about 14% (Currie and Madrian, 1999).

Whether personal disaster risk arises from layoffs or individual productivity declines, it is subject to incomplete insurance. Layoffs are usually partially insured by the US unemployment insurance system, but long-term unemployment is not. Personal productivity shocks are rarely insured by social welfare programs, except from major observable health problems, because of moral hazard. When awarded, disability benefits are more generous than unemployment benefits, offering a replacement rate of about 42% to the average worker (Gruber, 2000). The replacement rates are higher for low income people and for those who do not have employer-provided health insurance (Low and Pistaferri, 2015). Of course, informal insurance mechanisms, including family support, may also exist. Guvenen and Smith (2014) infer the extent of overall partial insurance from a dynamic model of consumption and linear labor income shocks where agents learn about their income growth rates. The partial insurance parameter is estimated to be 0.45, implying that almost one-half of both permanent and transitory income shocks are smoothed away through mechanisms different from savings. Blundell, Pistaferri, and Preston (2008) also estimate the extent of partial insurance. While it varies across cohorts, their estimate on the whole sample is that about 36% of permanent shocks are insured (and almost 95% of transitory shocks).

Last but not least, the extent of the coverage is *ex-ante* uncertain, adding to the uncertainty of the losses experienced in a personal disaster state. For instance, the structure of disability insurance has an initial claim stage and an appeal process, with fluctuations over time in

[6] Jung and Kuhn (2019) find that a shock at the top of the earnings distribution, such as the loss of a particularly good job, is a relevant source of persistent earnings losses.

the award rates. Importantly, such screening may be subject to error. According to Low and Meghir (2015), the probability of being rejected while having a severe work limitation exceeds 0.5.

Against this varied background, the model will allow for residual uninsured losses in the personal disaster state that can be on average small but feature uncertainty as to their actual size. The calibrations will refer to the case of an expected human capital erosion as low as 20% of the permanent labor income component.

3 The life-cycle model

We model an investor who maximizes the expected discounted utility of consumption over her entire life and wishes to leave a bequest as well. The investor starts working at age t_0 and retires with certainty at age $t_0 + K$. The effective length of her life, which lasts at most T periods, is governed by age-dependent life expectancy. At each date t, the survival probability of being alive at date $t+1$ is p_t, the conditional survival probability at t (with $p_{t_0-1} = 1$). Investor's i preferences at date t are described by a time-separable power utility function:

$$\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0}\left[\sum_{j=1}^{T}\beta^j\left(\prod_{k=-1}^{j-2}p_{t_0+k}\right)\left(p_{t_0+j-1}\frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + (1-p_{t_0+j-1})\,b\frac{(X_{it_0+j}/b)^{1-\gamma}}{1-\gamma}\right)\right] \quad (1)$$

where C_{it} is the level of consumption at time t, X_{it} is the amount of wealth the investor leaves as a bequest to her heirs after her death, $b \geq 0$ is a parameter capturing the strength of the bequest motive, $\beta < 1$ is a utility discount factor, and γ is the constant relative risk aversion parameter.

3.1 Labor and retirement income

During working life individuals receive exogenous stochastic earnings as compensation for labor supplied inelastically. Working life careers are modelled as a three-state Markov chain considering employment (e), short-term unemployment (u_1) and a disaster state characterized by long-term (u_2) unemployment. Individual labor market dynamics are driven by the following transition matrix:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} \pi_{ee} & \pi_{eu_1} & \pi_{eu_2} \\ \pi_{u_1e} & \pi_{u_1u_1} & \pi_{u_1u_2} \\ \pi_{u_2e} & \pi_{u_2u_1} & \pi_{u_2u_2} \end{pmatrix} = \begin{pmatrix} \pi_{ee} & 1-\pi_{ee} & 0 \\ \pi_{u_1e} & 0 & 1-\pi_{u_1e} \\ \pi_{u_2e} & 0 & 1-\pi_{u_2e} \end{pmatrix} \tag{2}$$

where $\pi_{nm} = \text{Prob}\,(s_{t+1} = n | s_t = m)$ with $n, m = e, u_1, u_2$. If the worker is employed at t ($s_t = e$), she continues the employment spell at $t+1$ ($s_{t+1} = e$) with probability π_{ee}, otherwise she enters short-term unemployment ($s_{t+1} = u_1$) with probability $\pi_{eu_1} = 1 - \pi_{ee}$. For simplicity, we set the probability of directly entering the disaster state of long-term unemployment at zero, $\pi_{eu_2} = 0$. After being unemployed for one period, at t ($s_t = u_1$), she exits unemployment ($s_{t+1} = e$) with probability π_{u_1e} or becomes long-term unemployed ($s_{t+1} = u_2$) with probability $\pi_{u_1u_2} = 1 - \pi_{u_1e}$; that is, we set $\pi_{u_1u_1} = 0$. Finally, if she is long-term unemployed at t ($s_t = u_2$), she either returns to employment in the following period ($s_{t+1} = e$) with probability π_{u_2e} or remains in the disaster state with probability $\pi_{u_2u_2} = 1 - \pi_{u_2e}$.

Stochastic labor income is driven by permanent and transitory shocks. In each working period, labor income Y_{it} is generated by the following process:

$$Y_{it} = H_{it}U_{it} \qquad t_0 \le t \le t_0 + K \tag{3}$$

where $H_{it} = F(t, \mathbf{Z}_{it}) P_{it}$ represents the permanent income component. In particular, $F(t, \mathbf{Z}_{it}) \equiv F_{it}$ denotes the deterministic trend component that depends on age (t) and a vector of individual characteristics ($\mathbf{Z}_{it}$) such as gender, marital status, household composition and education. As in Cocco, Gomes and Menhout (2005), the logarithm of the stochastic permanent component is assumed to follow a random walk process:

$$N_{it} = \log P_{it} = \log P_{it-1} + \omega_{it} \tag{4}$$

where ω_{it} is distributed as $N(0, \sigma_\omega^2)$. U_{it} denotes the transitory stochastic component and $\varepsilon_{it} = \log(U_{it})$ is distributed as $N(0, \sigma_\varepsilon^2)$ and uncorrelated with ω_{it}.

In our set-up, labor income received by the employed individual at time t depends on her past working history because we allow unemployment and its duration to affect the permanent component of labor income, H_{it}. Thus, after one-period unemployment the permanent component H_{it} is equal to H_{it-1} eroded by a fraction Ψ_1, and after a two-period unemployment spell the permanent component, H_{it-1}, is eroded by a fraction Ψ_2, with $\Psi_2 > \Psi_1$. This introduces non-linearity into the expected permanent labor income.[7] In compact form, the permanent component of labor income H_{it} evolves according to

$$H_{it} = \begin{cases} F(t, \mathbf{Z}_{it}) P_{it} & \text{if } s_t = e \text{ and } s_{t-1} = e \\ (1 - \Psi_1) H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_1 \\ (1 - \Psi_2) H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{5}$$

We now model the human capital erosion parameters, Ψ_1 and Ψ_2 as random variables that follow standard *Beta* distributions with shape parameters (a_j, b_j): thus, $\Psi_j \sim Beta(a_j, b_j)$.[8]

[7] The longer the unemployment spell the larger is the worker's human capital depreciation. See Guvenen, Karhan, Ozkan and Song (2017); and Schmieder, von Wachter and Bender (2016).

[8] This modelling compactly represents the uncertainties surrounding possible future negative earnings

This distribution allows to represent outcomes, like proportions, being defined on the continuum between 0 and 1. The standard *Beta* distribution gives the probability density of the value of Ψ_j, with $j = 1, 2$, on the interval (0,1):

$$f(\Psi_j : a_j, b_j) = \frac{\Psi_j^{a_j-1}(1-\Psi_j)^{b_j-1}}{B(a_j, b_j)} \tag{6}$$

where B is the beta function, thus $B(a_j, b_j)$ plays the role of normalization constant to ensure that the total probability is 1:

$$B(a_j, b_j) = \int_0^1 \Psi_j^{a_j-1}(1-\Psi_j)^{b_j-1} d\Psi_j \tag{7}$$

The expected value of Ψ_j, with $j = 1, 2$, is equal to:

$$E(\Psi_j) = \frac{a_j}{a_j + b_j} \tag{8}$$

and the variance is

$$Variance(\Psi_j) = \frac{a_j b_j}{(a_j + b_j)^2(a_j + b_j + 1)} \tag{9}$$

In the short-term unemployment state ($s_t = u_1$) individuals receive an unemployment benefit as a fixed proportion ξ_1 of the last working year permanent income $H_{it-1} = F_{it-1}P_{it-1}$, whereas in the long-term unemployment state ($s_t = u_2$) benefits are available in proportion ξ_2. Thus, the income received during unemployment is

$$Y_{it} = \begin{cases} \xi_1 H_{it-1} & \text{if } s_t = u_1 \\ \\ \xi_2 H_{it-2} & \text{if } s_t = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{10}$$

shocks. These include the award process of disability insurance; or the differential personal impact of crisis times, such the Great Recession or the Covid one, and of ordinary business cycle contractions.

Finally, during retirement, income is certain and equal to a fixed proportion λ of the permanent component of labor income in the last working year:

$$Y_{it} = \lambda F\left(t, \mathbf{Z}_{it_{0+l}}\right) P_{it_{0+l}} \qquad t_0 + K < t \leq T \tag{11}$$

where retirement age is t_0+K, t_0+l is the last working period and λ is level of the replacement rate.

3.2 Investment opportunities

We allow savings to be invested in a short-term riskless asset, yielding a constant gross real return R^f, and one risky asset, characterized as "stocks" yielding stochastic gross real returns R_t^s, for each period. The excess returns of stocks over the riskless asset follows

$$R_t^s - R^f = \mu^s + \nu_t^s \tag{12}$$

where μ^s is the expected stock premium and ν_t^s is a normally distributed innovation, with mean zero and variance σ_s^2. We do not allow for excess return predictability and other forms of changing investment opportunities over time, as in Michaelides and Zhang (2017).

At the beginning of each period, financial resources available to the individual for consumption and saving are given by the sum of accumulated financial wealth W_{it} and current labor income Y_{it}, which we call cash on hand $X_{it} = W_{it} + Y_{it}$. Given the chosen level of current consumption, C_{it}, next period cash on hand is given by

$$X_{it+1} = (X_{it} - C_{it}) R_{it}^P + Y_{it+1} \tag{13}$$

where R_{it}^{P} is the investor's portfolio return:

$$R_{it}^{P} = \alpha_{it}^{s} R_{t}^{s} + (1 - \alpha_{it}^{s}) R^{f} \tag{14}$$

with α_{it}^{s} and $(1 - \alpha_{it}^{s})$ denoting the shares of the investor's portfolio invested in stocks and in the riskless asset respectively. We do not allow for short sales and we assume that the investor is liquidity constrained. Consequently, the amounts invested in stocks and in the riskless asset are non negative in all periods. All simulation results presented below are derived under the assumption that the investor's asset menu is the same during working life and retirement.

3.3 Solving the life-cycle problem

In this intertemporal optimization framework, the investor maximizes the expected discounted utility over life span, by choosing the consumption and the portfolio rules given uncertain labor income and asset returns. Formally, the optimization problem is written as:

$$\max_{\{C_{it}\}_{t_0}^{T}, \{\alpha_{it}^{s}\}_{t_0}^{T}} \left(\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0} \left[\sum_{j=1}^{T} \beta^{j} \left(\prod_{k=-1}^{j-2} p_{t_0+k} \right) \left(p_{t_0+j-1} \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + \right. \right. \right.$$
$$\left. \left. \left. + (1 - p_{t_0+j-1}) \, b \frac{\left(X_{it_0+j}/b\right)^{1-\gamma}}{1-\gamma} \right) \right] \right) \tag{15}$$

$$s.t. \quad X_{it+1} = (X_{it} - C_{it}) \left(\alpha_{it}^{s} R_{t}^{s} + (1 - \alpha_{it}^{s}) R^{f} \right) + Y_{it+1} \tag{16}$$

with the labor income and retirement processes specified above and the no-short-sales and borrowing constraints imposed. Given its intertemporal nature, the problem can be restated in a recursive form, rewriting the value of the optimization problem at the beginning of period t as a function of the maximized current utility and of the value of the problem at

$t+1$ (Bellman equation):

$$V_{it}\left(X_{it,}P_{it},s_{it}\right)=\max_{C_{it},\alpha^s_{it}}\left(\frac{C_{it}^{1-\gamma}}{1-\gamma}+\beta E_t\left[p_tV_{it+1}\left(X_{it+1,}P_{it+1},s_{it+1}\right)\right.\right.$$
$$\left.\left.+\left(1-p_t\right)b\frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma}\right]\right) \tag{17}$$

At each time t the value function V_{it} describes the maximized value of the problem as a function of three state variables: cash on hand at the beginning of time t (X_{it}), the stochastic permanent component of income at beginning of t (P_{it}), and the labor market state $s_{it}(=e,\, u_1,\, u_2)$. The Bellman equation can be written by making the expectation over the employment state at $t+1$ explicit:

$$V_{it}\left(X_{it,}P_{it},s_{it}\right)=\max_{C_{it},\alpha^s_{it}}\left(\frac{C_{it}^{1-\gamma}}{1-\gamma}\right.$$
$$+\beta\left[p_t\sum_{s_{it+1}=e,u_1,u_2}\pi\left(s_{it+1}|s_{it}\right)\widetilde{E_tV}_{it+1}\left(X_{it+1,}P_{it+1},s_{it+1}\right)\right.$$
$$\left.\left.+\left(1-p_t\right)b\sum_{s_{it+1}=e,u_1,u_2}\pi\left(s_{it+1}|s_{it}\right)\frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma}\right]\right) \tag{18}$$

where $\widetilde{E_tV}_{it+1}$ denotes the expectation operator taken with respect to the stochastic variables ω_{it+1}, ε_{it+1}, and ν^s_{it+1}. The history dependence that we introduce in our set-up by making unemployment affect subsequent labor income prospects prevents having to rely on the standard normalization of the problem with respect to the level of P_t. To highlight how the evolution of the permanent component of labor income depends on previous individual labor market dynamics we write the value function at t in each possible state as (dropping the term involving the bequest motive):

$$V_{it}(X_{it},P_{it},e) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} V_{it+1}(X_{it+1},P_{it+1},e) & \text{with prob. } \pi_{e,e} \\ \text{with } P_{it+1} = P_{it}e^{\omega_{it+1}} & \text{and} \\ X_{it+1} = (X_{it}-C_{it})R^p_{it} + F_{it+1}P_{it+1}e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} V_{it+1}(X_{it+1},P_{it+1},u_1) & \text{with prob. } 1-\pi_{e,e} \\ \text{with } P_{it+1} = (1-\Psi_1)P_{it} & \text{and} \\ X_{it+1} = (X_{it}-C_{it})R^p_{it} + \xi_1 F_{it}P_{it} \end{cases} \end{cases}$$

$$V_{it}(X_{it},P_{it},u_1) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} V_{it+1}(X_{it+1},P_{it+1},e) & \text{with prob. } \pi_{u_1,e} \\ \text{with } P_{it+1} = (1-\Psi_1)P_{it-1}\,e^{\omega_{it+1}} = P_{it}\,e^{\omega_{it+1}} & \text{and} \\ X_{it+1} = (X_{it}-C_{it})R^p_{it} + F_{it-1}P_{it+1}e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} V_{it+1}(X_{it+1},P_{it+1},u_2) & \text{with prob. } 1-\pi_{u_1,e} \\ \text{with } P_{it+1} = (1-\Psi_2)(1-\Psi_1)P_{it-1} = (1-\Psi_2)P_{it} & \text{and} \\ X_{it+1} = (X_{it}-C_{it})R^p_{it} \end{cases} \end{cases}$$

$$V_{it}(X_{it},P_{it},u_2) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} V_{it+1}(X_{it+1},P_{it+1},e) & \text{with prob. } \pi_{u_2,e} \\ \text{with } P_{it+1} = P_{it}e^{\omega_{it+1}} & \text{and} \\ X_{it+1} = (X_{it}-C_{it})R^p_{it} + F_{it-2}P_{it+1}e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} V_{it+1}(X_{it+1},P_{it+1},u_2) & \text{with prob. } 1-\pi_{u_2,e} \\ \text{with } P_{it+1} = (1-\Psi_2)P_{it} & \text{and} \\ X_{it+1} = (X_{it}-C_{it})R^p_{it} \end{cases} \end{cases} \tag{19}$$

This problem has no closed form solution; therefore, we obtain the optimal values for consumption and portfolio shares by means of numerical techniques. To this aim, we apply a backward induction procedure starting from the last possible period of life T and computing optimal consumption and portfolio share policy rules for each possible value of the continuous state variables (X_{it} and P_{it}) by means of the standard grid search method.[9] Going backwards, for every period $t = T-1, T-2, ..., t_0$, we use the Bellman equation (18) to obtain optimal rules for consumption and portfolio shares.

[9] The problem is solved over a grid of values covering the space of both the state variables and the controls in order to ensure that the obtained solution is a global optimum.

4 Calibration

Parameter calibration concerns investor's preferences, the features of the labor income process during working life and retirement, and the moments of the risky asset returns. For reference, we initially solve the model by abstracting from the unemployment risk as in Cocco, Gomes and Maenhout (2005). Then, we introduce unemployment risk and consider two scenarios: (i) unemployment spells cause permanent deterministic income losses, as in Bagliano, Fugazza and Nicodano (2019), and (ii) unemployment implies a disaster risk, since it has permanent but uncertain and possibly large consequences on the worker's earnings ability.

Across all scenarios, the agent begins her working life at the age of 20 and works for (a maximum of) 45 periods (K) before retiring at the age of 65. After retirement, she can live for a maximum of 35 periods until the age of 100. In each period, we take the conditional probability of being alive in the next period p_t from the life expectancy tables of the US National Center for Health Statistics. We set the utility discount factor $\beta = 0.96$, and the parameter capturing the strength of the bequest motive $b = 2.5$. Finally, the benchmark value for the relative risk aversion is $\gamma = 5$. This value is relatively standard in the literature (Gomes and Michaelides 2005; Gomes, Kotlikoff and Viceira 2008) and captures an intermediate degree of risk aversion. However, Cocco, Gomes and Maenhout (2005) choose a value as high as 10 in their benchmark setting. The riskless interest rate is set at 0.02, with an expected equity premium μ^s=0.04 with standard deviation $\sigma_s = 0.157$. Finally, we impose a zero correlation between stock return innovations and aggregate permanent labor income disturbances ($\rho_{sY} = 0$). Table 2 summarizes the benchmark values of relevant parameters.

Table 2: Calibration parameters

Description	Parameter	Value
Working life (max)	T	20 -65
Retirement (max)	$t_0 + K$	65 -100
Discount factor	β	0.96
Risk aversion	γ	5
Replacement ratio	λ	0.68
Variance of permanent shocks to labor income	σ_ω^2	0.0106
Variance of transitory shocks to labor income	σ_ϵ^2	0.0738
Riskless rate	r	0.02
Excess returns on stocks	μ^s	0.04
Variance of stock returns innovations	σ_s^2	0.025
Stock ret./permanent lab. income shock correlation	ρ_{sY}	0

	Unemployment no disaster	**Unemployment with disaster risk**
Unemployment benefits		
Short-term unemployed (ξ_1)	0.3	0.3
Long-term unemployed (ξ_2)	0.1	0.1
Human capital erosion		
Short-term unemployed (Ψ_1)	0	0
Long-term unemployed (Ψ_2)	0.20	(expected) 0.20
—*Beta* distribution a_2	-	*0.01*
—*Beta* distribution b_2	-	*0.04*

This table reports benchmark values of relevant parameters.

4.1 Labor income, unemployment and disaster risk

The labor income process is calibrated using the estimated parameters for US households with high school education (but not a college degree) in Cocco et al. (2005). For the high school group, the variances of the permanent and transitory shocks (ω_{it} and ε_{it} respectively) are equal to $\sigma_\omega^2 = 0.0106$ and $\sigma_\varepsilon^2 = 0.0738$. After retirement, income is a constant proportion λ of the final (permanent) labor income, with $\lambda = 0.68$. The parameter values assumed above are maintained across all scenarios.

We use data from the Current Population Survey (CPS) to calibrate the transition probabilities from employment to unemployment to reflect the risk of entering unemployment along with the observed average unemployment rates at different duration. According to the evidence based on CPS reported in Kroft, Lange, Notowidigdo and Katz (2016), the annual transition probability from employment to unemployment is 4%. Given the duration dependence and the steady decline in the annual outflow rate from unemployment to employment during the first year of unemployment (Kroft, Lange, Notowidigdo and Katz, 2016), we set the probability of leaving unemployment after the first year at 85%. The annual transition probabilities between labor market states are chosen to match the average annual unemployment rate in the United States:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} 0.96 & 0.04 & 0 \\ 0.85 & 0 & 0.15 \\ 0.85 & 0 & 0.15 \end{pmatrix} \tag{20}$$

Our calibration appears quite conservative, since the chance of being employed 15 months later for those who had been unemployed 27 weeks or more is only 36% (see, by comparison, the evidence on CPS data in Krueger, Cramer and Cho, 2014). Indeed, the assumed transition matrix (20) yields unconditional probabilities of being short-run (3.8%) and long-run unemployed (0.7%) in line with historical unemployment rates in the US over our sample period.

Well-established empirical evidence on job displacement shows that job losses affect earnings far beyond the unemployment spell, though the range of the estimated effects varies considerably. For example, the estimates for immediate losses following displacement may range from 30% (Couch and Placzek, 2010) to 40% of earnings (Jacobson, Lalond and Sullivan, 1993b). Earnings losses are shown to be persistent in a range from 15% (Couch and Placzek, 2010) to about 25% (Jacobson, LaLonde and Sullivan, 1993a) of their pre-displacement levels. These estimates abstract from the effect of unemployment duration, while Cooper (2013) finds that earnings losses are larger the longer unemployment lasts. Also, based on administrative data, Jacobson, LaLonde and Sullivan (2005) estimate that average earnings losses for displaced workers amount to 43-66% of their pre-displacement wage. While these studies document the effects of layoffs on average future labor income, Guvenen, Karahan, Ozkan and Song (2017) find that income losses after one-year non-employment are highly heterogeneous. This body of evidence, combined with a probability of finding a job after being unemployed for 24 months as low as 40% (Kroft, Lange, Notowidigdo and Katz, 2016), leads us to calibrate an expected drop in human capital, following a long term unemployment spell, of about 20% leaving open the possibility for rare but larger losses. Thus, while Ψ_1 is kept equal to 0,[10]

[10]More precisely, for $a_1/b_1 \to 0$, the *Beta* distribution has a spike at the left end. Thus, $\Psi_1 = 0$ with probability 1.

Ψ_2 follows a *Beta* distribution with expected value of 20% standard deviation of 33%. The calibrated distribution for Ψ_2 implies a median value for the proportional human capital erosion lower than 1%.

Unemployment benefits are calibrated according to the US unemployment insurance system. In particular, considering that the replacement rate with respect to last labor income is on average low and state benefits are paid for a maximum of 26 weeks, we set $\xi_1 = 0.3$ in case of short-term unemployment spells and set a value of $\xi_2 = 0.1$ for the long-term unemployed. No additional weeks of federal benefits are available in any state: the temporary Emergency Unemployment Compensation (EUC) program expired at the end of 2013, and no state currently qualifies to offer more weeks under the permanent Extended Benefits (EB) program.[11]

For comparison, we consider two other calibrations of the model. The "*no unemployment risk*" scenario corresponds to the standard life-cycle set up with $\pi_{ee} = 1$ and all other entries equal to zero in the transition probability matrix (2). In the "*unemployment with no disaster*" scenario, long-term unemployment has deterministic permanent consequences on future earnings (as in the set-up studied by Bagliano et al. (2019)). We therefore allow for the unemployment risk embedded in the transition probability matrix (20) together with a human capital loss of 20% (i.e. $\Psi_1 = 0$ and $\Psi_2 = 0.20$).

[11]Low, Meghir and Pistaferri (2010) acknowledge that layoffs are partially insured by the unemployment insurance system, while individual productivity shocks, other than major observable health shocks, are rarely insured in any formal way. As for other welfare programs, we do not model basic consumption needs and therefore overlook basic consumption insurance.

5 Results

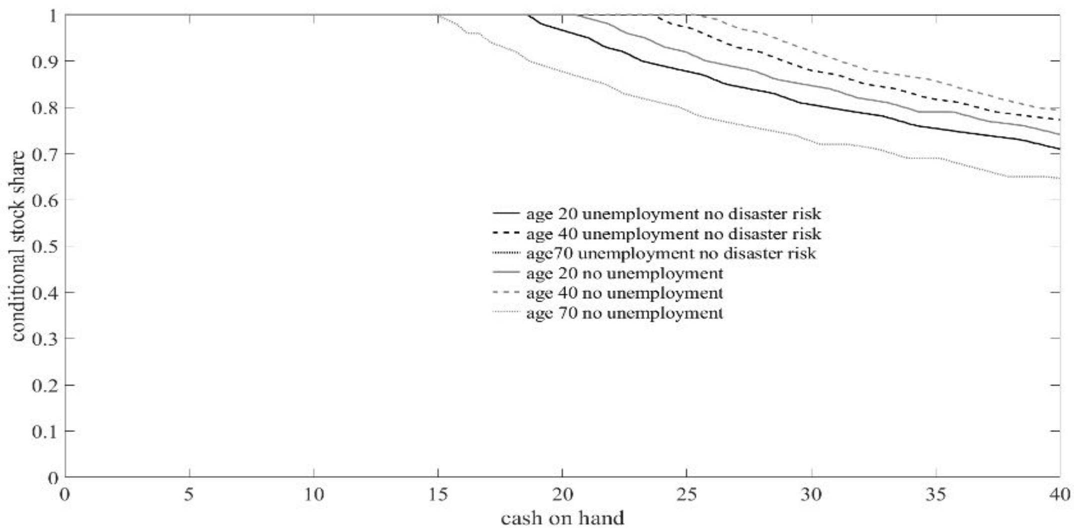
5.1 Optimal policies

Figure 3 compares investors' optimal stock shares in the case of *no unemployment* and in the case of unemployment with deterministic human capital erosion, (i.e. *unemployment without disaster risk*), reported in panel (a), to optimal shares implied by our preferred scenario of unemployment with uncertain human capital erosion, i.e. *unemployment with disaster risk*, reported in panel (b). In particular, the figure plots the optimal stock share as a function of cash on hand at three different ages (20, 40, and 70). In both the case of no unemployment and unemployment without disaster risk, the standard life-cycle results are obtained. Labor income is (close to) a risk-free return and affects the optimal portfolio composition depending on an investor's age and wealth. For example, at age 20 the sizable implicit holding of the risk-free asset (through human capital) makes it optimal for less-wealthy investors to tilt their portfolio towards the risky financial asset. Indeed, for a wide range of wealth levels, agents optimally choose to be fully invested in stocks. The optimal stock holding decreases with financial wealth because of the relatively lower implicit investment in (risk-free) human capital.

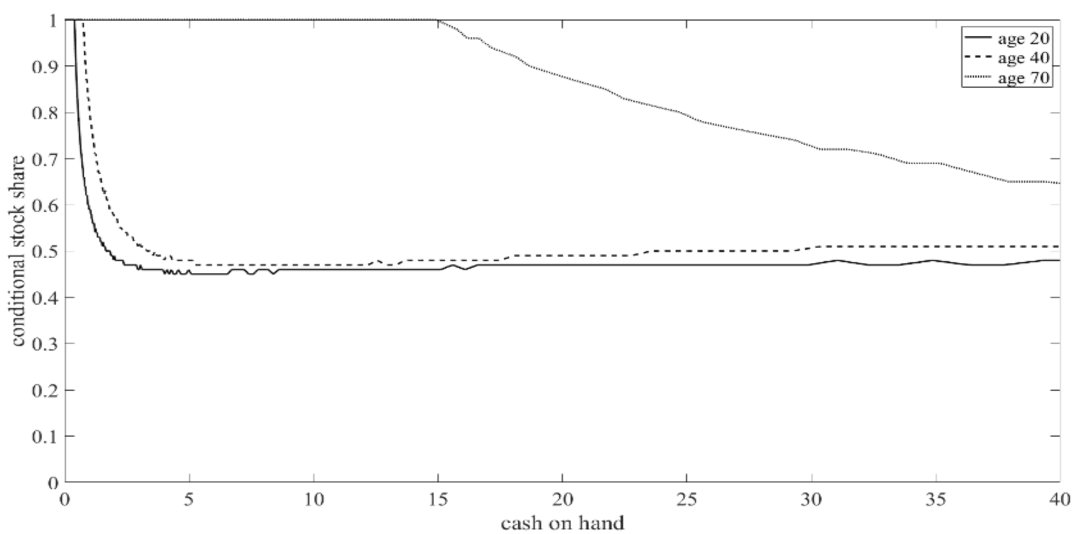
When the model is extended to allow for uncertain permanent effects of unemployment spells on labor income prospects at re-employment ("*unemployment with disaster risk*"), with the parameters governing the proportional erosion of permanent labor income set at $\Psi_1 = 0$ after one year of unemployment and at an expected $\Psi_2 = 0.2$ after 2 years, the resulting policy functions are shifted abruptly leftward. The optimal stock share still declines with financial wealth but a 100% share of investment in stocks is optimal only at very low levels of wealth. In this case, long-term unemployment implies an uncertain loss of future labor income which severely reduces the level of human capital and increases its risk at any age. Thus, for almost all levels of financial wealth, stock investment is considerably lower than in the case of no disaster risk.

Figure 3: Policy functions

(a) No unemployment - Unemployment without disaster risk



(b) Unemployment with disaster risk



This figure shows the portfolio rules for stocks as a function of cash on hand for an average level of the stochastic permanent labor income component. The policies refer to selected ages: 20, 40, and 70. Panel (a) and (b) refer respectively to the cases of no unemployment -unemployment without disaster risk and of unemployment with disaster risk. In the case of unemployment without disaster risk, the human capital loss is 20%. In the case of unemployment with disaster risk, the expected human capital loss is 20%. Cash on hand is expressed in ten thousands of U.S. dollars.

5.2 Life-Cycle Profiles

On the basis of the optimal policy functions, we simulate the whole life-cycle consumption and investment decisions for 10,000 agents. Figure 4, panel (a), shows the average optimal equity portfolio shares plotted against age. In the case of no unemployment risk (dotted line), the well-known downward sloping pattern emerges. Over the life cycle the proportion of overall wealth implicitly invested in the (close to) risk-free asset through human capital declines with age. Consequently, at early stages of the life cycle, optimal stock investment is about 100% and decreases with age to reach around 80% at retirement. When we consider unemployment risk with deterministic human capital erosion of 20% (dashed line), the optimal portfolio share of stocks still declines with age, though being only slightly lower at all ages, with a 100% optimal stock share only for very young investors.

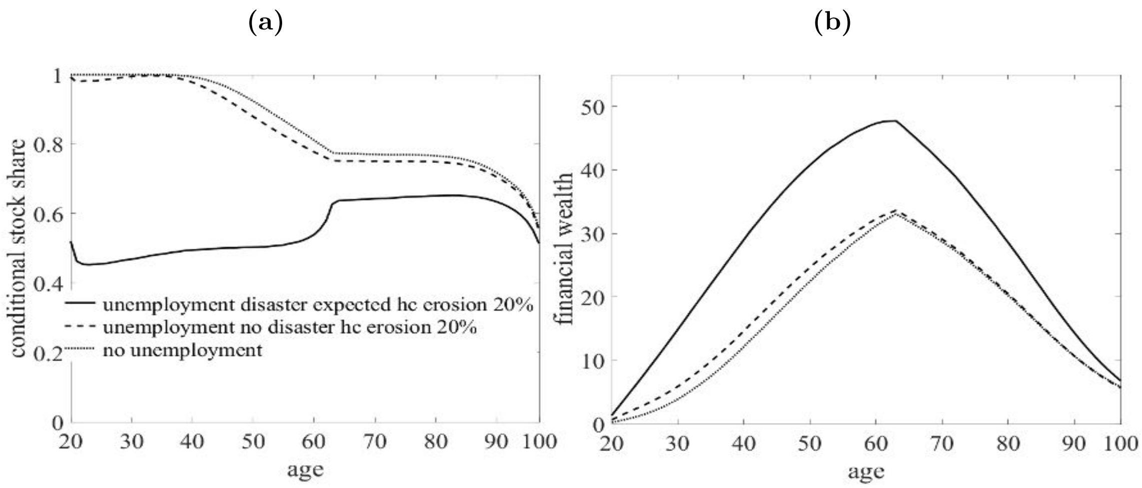
However, when we account for the disaster risk (solid line), the optimal stock investment is reduced at any age and almost flat, at around 45-55%. The risk of potentially losing a substantial portion of future labor income prospects reduces the level of human capital and increases its riskiness. Because this effect is more relevant for younger workers, it induces a lower optimal stock investment conditional on financial wealth especially when young (see Figure 3, panel b). Consequently, the age profile remains flat over the whole working life.[12] This result portrays the effects on risk-taking of the ”unusual” negative shocks that explain the consumption dynamics of US households in Blundell, Arellano and Bonhomme (2017) throughout the earnings distribution.

The reduction in the optimal portfolio share allocated to stocks is due to higher wealth

[12] The relatively low investment in stocks during retirement is due to the presence of a positive bequest motive.

accumulation, in turn induced by larger precautionary savings. Panel (b) of Figure 4 displays the average financial wealth over the life cycle for the three scenarios considered. In the face of possible, albeit rare, human capital depreciation, individuals accumulate substantially more financial wealth during working life to buffer possible disastrous labor market outcomes. Optimal average consumption when young will consequently fall, but it will be much higher during both late working years and retirement years.

Figure 4: Life-cycle average profiles



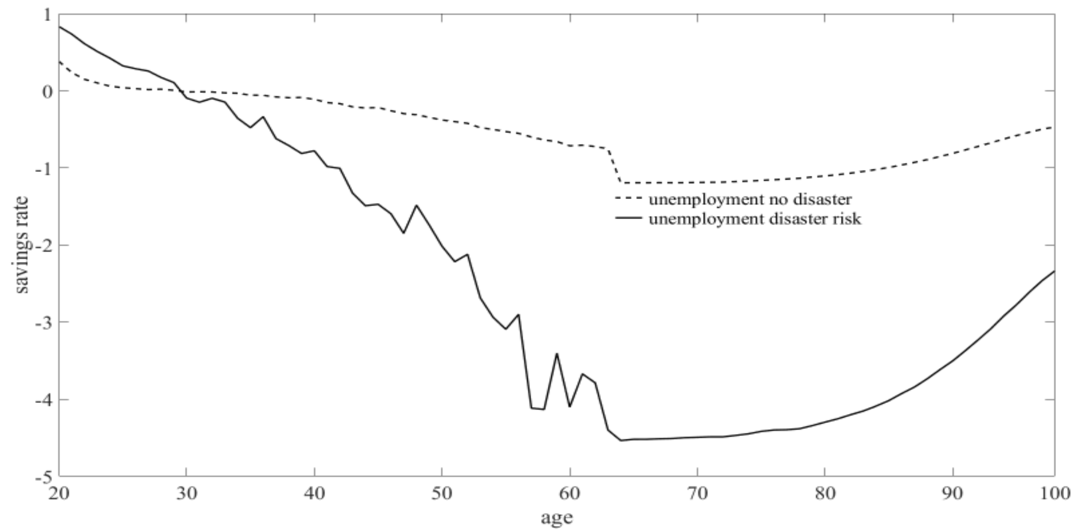
This figure displays the mean simulated stock investment and financial wealth accumulation life-cycle profiles. Age ranges from 20 to 100. The three lines correspond to no unemployment risk (dotted line); unemployment without disaster risk and a human capital erosion equal to 20% (dashed line); unemployment with disaster risk (solid line) and an expected human capital loss equal to 20%. Financial wealth is expressed in ten thousands of U.S. dollars.

Figure 5 displays the life-cycle profile of the ratio between savings and total (financial plus labor) income[13]. When the worker is 20 years old, the average propensity to save is especially high in the disaster risk case, reaching 0.8 compared with less than 0.2 when disaster risks are

[13] In the case of unemployment without disaster risk, the optimal decisions are very similar to the case of no unemployment, thus we report the savings ratio for the first case only

absent. Such propensity monotonically decreases in age, reaching the level achieved in the no disaster scenario when the worker is in her mid-thirties. The figure clearly depicts the impact on savings of the resolution of uncertainty concerning a possible disaster as individuals age.[14]

Figure 5: Life-cycle profiles of savings rate



This figure displays the savings rate dynamics for individuals of age 20 to 100, relative to total income (i.e. labor income plus financial income). The two lines correspond to unemployment without disaster risk (dotted line) and unemployment with disaster risk (solid line). In the case of unemployment without disaster risk, the human capital loss is 20%. In the case of unemployment with disaster risk, the expected human capital loss is 20%.

These results also suggest that the expectation of a higher welfare benefit, cushioning the disastrous outcomes, mitigate the adverse impact of long term unemployment on human capital, reducing the need for cautious investing and saving which characterizes early working life in our calibrations. The variation of institutions across countries may thus generate

[14]Data on Norwegian households show that they engage in additional saving, shifting portfolio composition towards safe assets, in the years prior to unemployment. There is depletion of savings after the job loss (see Basten, Fagereng and Telle, 2016).

different life-cycle patterns in equity investing. In this light, the decreasing stock holdings in Norwegian data (appearing in Fagereng, Gottlieb and Guiso, 2017) may be a consequence of higher long-term unemployment benefits with respect to the US.

Higher average savings when young obviously implies lower average consumption when young. What is perhaps less obvious is whether higher wealth shields consumption from the negative skewness of labor income shocks that may arise from both unemployment and personal disaster risk. Table 3 and 4, respectively, report the mean and the standard deviation of the skewness of labor income shocks and of consumption growth.

Table 3: Skewness of labor income shocks

The table reports the mean and the standard deviation of the skewness of labor income shocks (permanent plus transitory) faced by the simulated $10,000$ investors between age t and age $t-1$.

	mean	stdev
disaster risk	-2.817	0.814
no disaster risk	-0.008	0.026

Table 3 shows that, without disaster risk, the average skewness of labor income shocks is not statistically different from zero over the working life (-0.008), while it turns negative(-2.817) with disaster risk. Consequently, the possibility of incurring a personal disaster is able to capture the negative skewness of earnings growth uncovered by Guvenen, Karahan,Ozkan and Song (2016) in US administrative data. The calibrated scenario with unemployment risk does not instead deliver such observed feature of labour income dynamics even though the process is nonlinear.

Table 4: Skewness of consumption growth

The table reports the mean and the standard deviation of the skewness of consumption growth rates, between age t and age $t-1$, for the simulated $10,000$ investors.

	mean	stdev
disaster risk	-0.32	0.19
no disaster risk	0.02	0.27

Table 4 shows that investments and wealth accumulation are able to partially shield consumption from negative shocks to labor income. In particular, the average skewness of consumption growth is negative but closer to zero than the skewness of labor income (-0.32 versus -2.82) and displays a much lower standard deviation.

Overall, these results reveal that the remote possibility of encountering a potentially large negative shock to human capital, albeit in expectation small, originates negative skewness in both labor income shocks and consumption growth. Personal disaster risk also delivers a flat risk taking pattern over the life-cycle.

5.2.1 Heterogeneity

The above results imply that the optimal stock investment is flat in age, even for a moderately risk averse worker, due to personal disaster risk. Moreover, workers on average invest about 50% of their financial wealth in stocks. Average patterns may however hide considerable differences across agents. The present section investigates the distribution across agents of both conditional optimal stock share and accumulated wealth.

The case of unemployment without personal disaster risk is displayed in panels (a) and (b) of Figure 6, which show the 25^{th}, 50^{th} and 75^{th} percentiles of the distributions[15]. Both the optimal stock share and the stock of accumulated financial wealth are highly heterogeneous across workers as well as retirees. The exception is young workers as they tilt their entire portfolio towards stocks given the relatively risk-free nature of their human capital. Heterogeneity of portfolio shares depends on the shape and movements through age of the policy functions displayed in Figure 3, relating optimal stock shares to the amount of available cash on hand, and on the level of cash on hand itself. Relatively steep policy functions imply that even small differences in the level of accumulated wealth result in remarkably different asset allocation choices. At the early stage of the life cycle, when accumulated financial wealth is modest, it is optimal for everybody to be fully invested in stocks. As investors grow older, different realizations of background risk induce large differences in savings and wealth accumulation. This situation pushes investors on the steeper portion of their policy functions and determines a gradual increase in the heterogeneity of optimal risky portfolio shares during their working life. After retirement, investors decumulate their financial wealth relatively slowly, due to the bequest motive, and still move along the steeper portion of their relevant policy functions; as a consequence, the dispersion of optimal shares tends to persist.

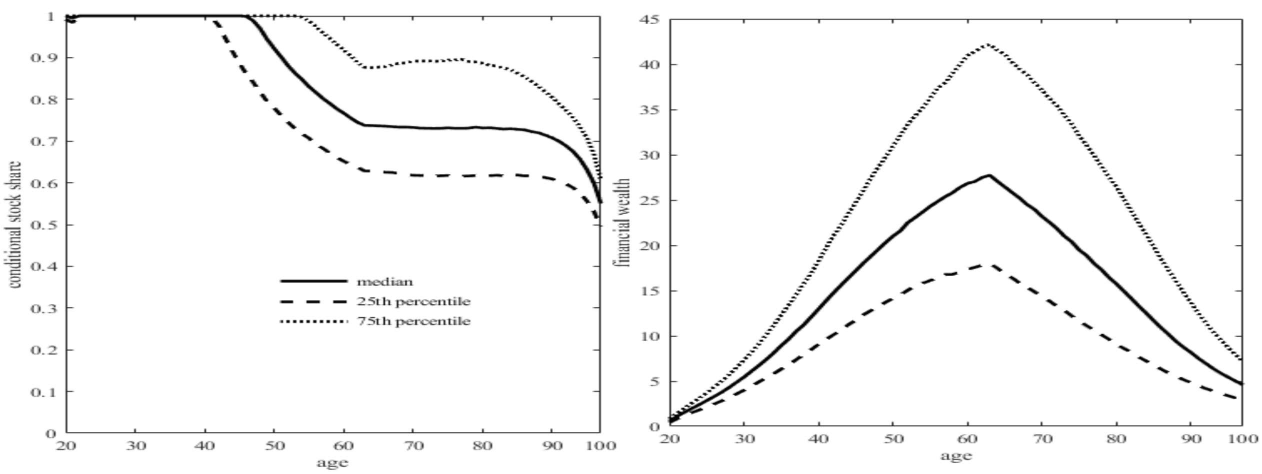
Panels (c) and (d) of Figure 6 display the life-cycle distribution of stock share and financial wealth for the disaster risk scenario. Compared with the case of unemployment without disaster risk, the distribution of optimal stock shares is much less heterogeneous.

[15]In the unreported case of no unemployment risk, the optimal decisions are very similar to the case of unemployment without disaster risk

Figure 6: Life-cycle percentile profiles

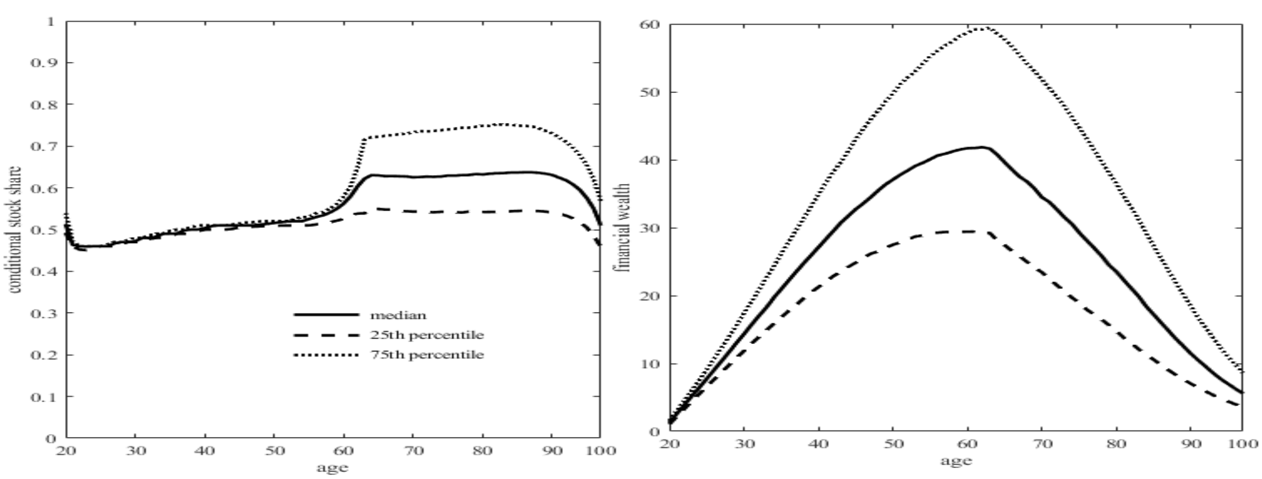
Unemployment without disaster risk

(a) **(b)**



Unemployment with disaster risk

(c) **(d)**



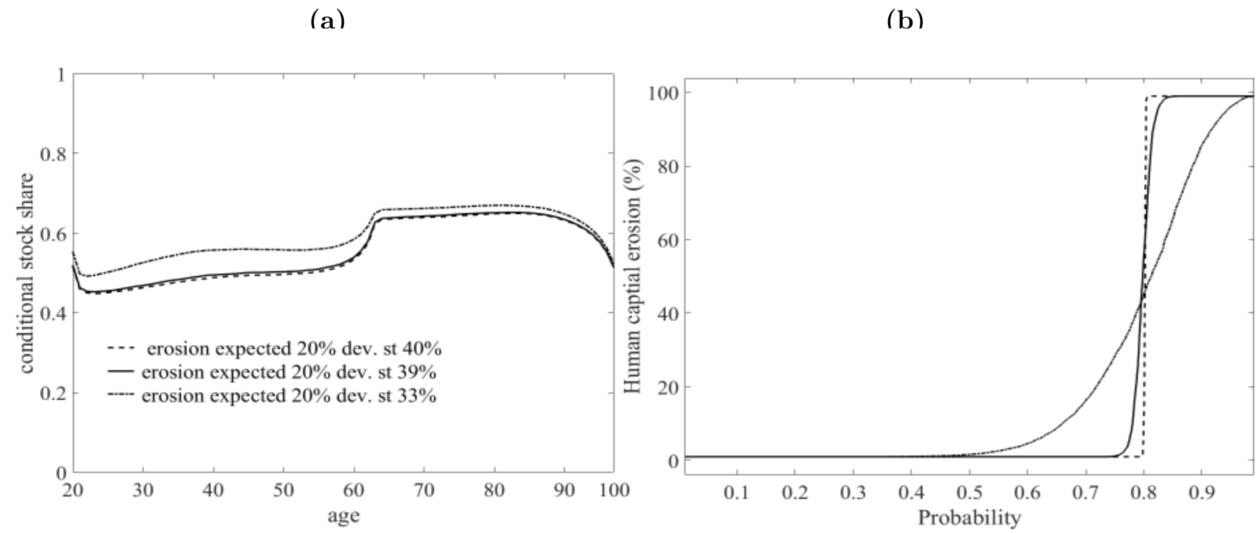
This figure displays the distribution of simulated equity share investment and financial wealth accumulation for individuals of age 20 to 100 in the case of unemployment without disaster risk (panels (a) and (b)) and with disaster risk (panels (c) and (d)). In the case of unemployment without disaster risk, the human capital loss is 20%. In the case of unemployment with disaster risk, the expected human capital loss is 20%. Financial wealth is expressed in ten thousands of U.S. dollars.

In particular, heterogeneity shrinks during working life even for young workers, given the high human capital risk they bear at the beginning of their careers. Indeed, policy functions are relatively flat when long-term unemployment is uninsured (see panel (b) of Figure 3) implying that even large differences in the level of accumulated wealth result in homogeneous asset allocation choices.

5.3 Uncertainty around the disaster risk

In this section we check the sensitivity of life-cycle profiles with respect to the uncertainty around the potential human capital erosion that occurs in case of long-term unemployment, captured by the two shape parameters of the Beta distribution, α_2 and β_2. We vary the two parameters to keep the expected human capital erosion equal to 20% and allow for different degree of uncertainty around it. Figure 7 panel (a) shows the results of an experiment with the expected human capital erosion equal 20% and the standard deviation in the range of 33 – 40%. It turns out that under all distributional assumptions, life cycle profiles are very similar to the benchmark case. This outcome provides a final indication that extremely rare but potentially disastrous labor income shocks may be relevant to understand cautiousness by young workers and their limited risk taking in the stock market.

Figure 7: Uncertainty around the disaster risk

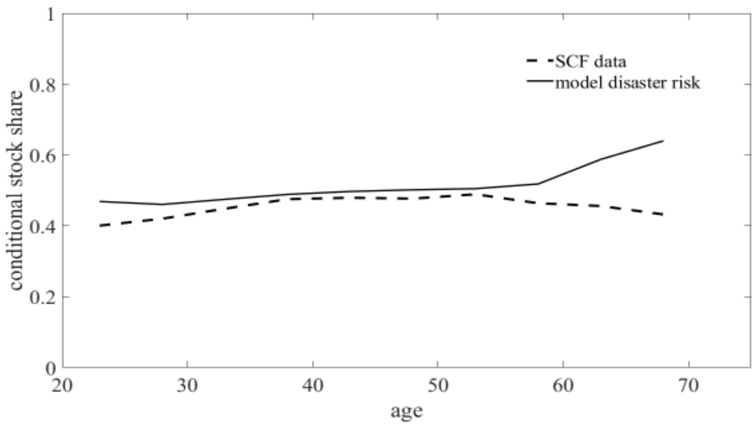


This figure displays the life-cycle profiles of optimal conditional stock holdings (panel a) for different parameters of the distribution that governs human capital erosion. The expected erosion is 20%, the standard deviation is respectively 40% (dashed line), 39% as in the benchmark case (solid line) and 33% (dash-dot line). Panel b reports, for each parametric configuration, the inverse cumulative distribution function of the human capital erosion.

5.4 Household portfolios with personal disaster risk: matching the empirical regularities

The key implication of our model is that optimal investment profiles are almost flat over the life cycle. In this section, we compare our results with conditional stock holdings for U.S. male investors observed in the Survey of Consumer Finances data (waves from 1992 to 2016).

Figure 8: Life-cycle conditional stock share profiles



This figure displays the life-cycle profiles of conditional stock holdings, for age 20 to 100, observed in SCF data (dotted) and obtained from the benchmark model of unemployment plus disaster risk with expected human capital erosion equal to 20% (solid).

Figure 8 compares the stock portfolio shares for stock market participants for different age classes obtained from our model with the corresponding US SCF data. The model is able to closely match the observed life-cycle pattern of equity portfolio shares conditional on participation, yielding an average value over the whole working life of 49%, to be compared with 45.6% in the data.

6 Conclusions

This paper shows that even a small probability of experiencing human capital erosion with an uncertain, but potentially extreme, size generates optimal conditional stock shares in line with those observed in US data, along with a skewed consumption growth distribution. Nonlinear income shocks, which have recently become essential considerations in consumption studies, appear to play a first-order role in choices on risk taking. Because of the remote possibility of a future personal disaster, younger workers face higher uncertainty concerning future income than older workers and optimally invest a higher portfolio share in the risk-free asset. These results are based on a methodological innovation in the way we model human capital erosion conditional on the occurrence of a rare disaster.

Our analysis has implications for the design of pension plans in the United States. The current design tilts the composition of optimal portfolios towards equity investments. Given the scant heterogeneity in optimal life-cycle investments induced by limited and uncertain protection against a personal future disaster, the flatter design should fit workers with different career histories. More generally, the pattern of risk-taking at different ages in Target Date Funds should be related to the share of uninsured disability and long-term unemployment risk.

The model implies that observed variation in the age pattern of stock investing depends on the features of insurance coverage both across countries and across cohorts. We leave the systematic investigation of this insight for future research.

References

[1] Ameriks J., and S. P. Zeldes (2004), How Do Household Portfolio Shares Vary with Age?, Working paper, Columbia University.

[2] Arellano, M., Blundell, R. and Bonhomme, S. (2017), Earnings and consumption dynamics: a nonlinear panel data framework. *Econometrica* 85 (3), 693–734.

[3] Arulampalam W. (2001), Is unemployment really scarring? Effects of unemployment experiences on wages, *Economic Journal* 111(475): F585–606.

[4] Bagliano F. C., C. Fugazza and G. Nicodano (2014), Optimal life-cycle portfolios for heterogeneous workers, *Review of Finance*, 18(6): 2283-2323.

[5] Bagliano F. C., C. Fugazza and G. Nicodano (2019), Life-Cycle Portfolios, Unemployment, and Human Capital Loss, *Journal of Macroeconomics*, vol. 60: 325-340.

[6] Bhattacharjee, M.C., Adaptive Economic Choices under Recurrent Disasters: A Bayesian Perspective, in Gupta A.K. and S. Nadarajah (2004) *Handbook of Beta Distribution and Its Applications*, New York: Marcel Dekker, 457-484.

[7] Barro R.(2006), Rare Disasters and Asset Markets in the Twentieth Century, *Quarterly Journal of Economics*, 121(3): 823-866.

[8] Basten, C., Fagereng, A. and Telle, K. (2016), Saving and Portfolio Allocation Before and After Job Loss, *Journal of Money, Credit and Banking*, 48: 293-324.

[9] Benzoni, L., Collin-Dufresne, P., and Goldstein, R. G. (2007), Portfolio choice over the life-cycle when the stock and labour markets are cointegrated, *Journal of Finance* 62, 2123-2167.

Blundell, R., L. Pistaferri, and I. Preston (2008), Consumption Inequality and Partial Insurance, *American Economic Review*, 98 (5): 1887-1921.

[10] Bodie Z., R. C. Merton and W. Samuelson (1992), Labor Supply Flexibility and Portfolio Choice in a Life Cycle Model, *Journal of Economic Dynamics and Control*, 16: 427-449.

[11] Bonaparte, J., G. Korniotis, and A. Kumar (2014), Income hedging and portfolio decision, *Journal of Financial Economics* 113, 300-324.

[12] Catherine S., P. Sodini and Y. Zhang (2020), Countercyclical Income Risk and Portfolio Choices: Evidence from Sweden, HEC Paris Research Paper No. FIN-2016-1147.

[13] Chang Y., J. H. Hong and M. Karabarbounis (2017), Labor-Market Uncertainty and Portfolio Choice Puzzles, *American Economic Journal: Macroeconomics*, vol.(10): 222-262.

[14] Cocco J. (2004), Portfolio Choice in the Presence of Housing, *Review of Financial Studies*, 18(2): 535-567.

[15] Cocco J., F. J. Gomes and P. J. Maenhout (2005), Consumption and Portfolio Choice over the Life Cycle, *Review of Financial Studies*, 18(2): 491-533.

[16] Cooper E. (2013), The Effect of Unemployment Duration on Future Earnings and Other Outcomes, Federal Reserve Bank of Boston Working Paper No. 13-8.

[17] Couch K. A. and D. W. Placzek (2010), Earnings Losses of Displaced Workers Revisited, *American Economic Review*, 100(1): 572-89.

[18] Currie J. and B. Madrian (1999), Health, Health Insurance and the Labor Market, The Handbook of Labor Economics, vol.3c, David Card and Orley Ashenfelter (eds.), Amsterdam: North Holland, 3309-3407

[19] De Nardi, M., G. Fella, and G. Paz-Pardo (2020), Nonlinear Household Earnings Dynamics, Self-Insurance, and Welfare, *Journal of the European Economic Association*, 18(2), 890–926.

[20] Fagereng A., C. Gottlieb and L. Guiso (2017), Asset Market Participation and Portfolio Choice over the Life-Cycle, *Journal of Finance*, 72(2): 705-750.

[21] Ghayad R. and W. Dickens (2012), What Can We Learn by Disaggregating the Unemployment-Vacancy Relationship?, Federal Reserve Bank of Boston Public Policy Brief 12-3.

[22] Gomes F. J, L.J. Kotlikoff and L.M. Viceira (2008), Optimal Life-Cycle Investing with Flexible Labor Supply: A Welfare Analysis of Life-Cycle Funds, *American Economic Review*, 98(2): 297-303.

[23] Gomes F. J and A. Michaelides (2005), Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence, *Journal of Finance*, 60(2): 869-904.

[24] Gruber J. (2000), Disability insurance benefits and labor supply Journal of Political Economy, 108(6), 1162-1183

[25] Guvenen F., F. Karahan, S. Ozkan and J. Song (2016), What Do Data on Millions of US Workers Reveal About Life-Cycle Earnings Dynamics?, *Federal Reserve Bank of New York Staff Report* n. 710.

[26] Guvenen F., F. Karahan, S. Ozkan and J. Song (2017), Heterogeneous Scarring Effects of Full-Year Nonemployment, *American Economic Review*, 107, 369-73.

[27] Guvenen F., and A. A. Smith (2014), Inferring Labor Income Risk and Partial Insurance from Economic Choices, *Econometrica*, 82(6), 2085-2129

[28] Hubbard G. R., J. Skinner and Zeldes S.F. (1995) Precautionary Saving and Social Insurance. *Journal of Political Economy* 103(2), 360-399.

[29] Hubener A., R. Maurer and O. S. Mitchell (2016), How Family Status and Social Security Claiming Options Shape Optimal Life Cycle Portfolios, *The Review of Financial Studies*, 29, (4): 937-978.

[30] Jacobson L., R. LaLonde and D. Sullivan (1993a), Earnings Losses of Displaced Workers, *American Economic Review*, 83(4): 685-709.

[31] Jacobson L., R. LaLonde and D. Sullivan (1993b), The Costs of Worker Dislocation. Kalamazoo, MI: W. E. Upjohn Institute for Employment Research.

[32] Jacobson L., R. LaLonde and D. Sullivan (2005), Estimating the Returns to Community College Schooling for Displaced Workers, *Journal of Econometrics*, 125(1-2): 271-304.

[33] Jung,P. and M.Kuhn (2019), Earnings Losses and Labor Mobility Over the Life Cycle, *Journal of the European Economic Association*, 17(3): 678-724.

[34] Kraft H. and Munk, C. (2011), Optimal Housing, Consumption, and Investment Decisions over the Life-Cycle, *Management Science*, 57(6): 1025-1041.

[35] Kroft K., F. Lange, M.J., Notowidigdo and L. Katz (2016), Long Term Unemployment and the Great Recession: the Role of Composition, Duration Dependence, and Non Participation, *Journal of Labor Economics*, 34(S1): S7-S54.

[36] Krueger A. B., J. Cramer and D. Cho (2014), Are the Long-Term Unemployed on the Margins of the Labor Market?, *Brookings Papers on Economic Activity*, Spring: 229-280.

[37] Lallemant D. and A Kiremidjian. (2015), A Beta Distribution Model for Characterizing Earthquake Damage State Distribution. *Earthquake Spectra*, 31 (3): 1337-1352.

[38] Low A., C. Meghir and L. Pistaferri (2010), Wage Risk and Employment Risk over the Life Cycle, *American Economic Review,* 100(4): 1432-67.

[39] Low A. and L. Pistaferri (2015), Disability Insurance and the Dynamics of the Incentive Insurance Trade-Off, *American Economic Review*, 105 (10): 2986-3029.

[40] Mayer G. (2014), *The Trend in Long-Term Unemployment and Characteristics of Workers Unemployed for Two Years or More*, Washington, D.C.: Congressional Research Service

[41] Michaelides A. and Y. Zhang (2017), Stock Market Mean Reversion and Portfolio Choice over

the Life Cycle, *Journal of Financial and Quantitative Analysis*, 52(3): 1183-1209.

[42] Munk C. and C. Sorensen (2010), Dynamic asset allocation with stochastic income and interest rates, *Journal of Financial Economics*, 96(3): 433-462.

[43] Schmieder J. F., T. von Wachter, and S. Bender (2016), The Effect of Unemployment Benefits and Nonemployment Durations on Wages, *American Economic Review*, 106(3): 739-77.

[44] Viceira L. M. (2001), Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income, *The Journal of Finance*, 56: 433-470.

[45] Yang, W. (2011), Long-run Risk in Durable Consumption, *Journal of Financial Economics*, 102(1): 45-61.

Latest CeRP Working Papers

N° 205/21	Fabio Bagliano Carolina Fugazza Giovanna Nicodano	Life-Cycle Risk-Taking with Personal Disaster Risk
N° 204/21	Michele Belloni Elena Farina Irene Simonetti Francesca Zantomio	Labour Outcomes Adjustments to Health Shocks over the Long Run: Evidence from Italian Administrative Records
N° 203/20	Yan Alperovych Riccardo Calcagno Martijn Lentz	Entrepreneurs on their financial literacy: evidence from the Netherlands
N° 202/20	Alessandra Colombelli Elena Grinza Valentina Meliciani Mariacristina Rossi	Pulling Effects in Immigrant Entrepreneurship: Does Gender Matter?
N° 201/20	Alessandro Manello Greta Falavigna Eleonora Isaia Mariacristina Rossi	Women in leading corporate positions and credit risk: Evidence from Italian firms
N° 200/20	Francesco Figari Carlo V. Fiorio	Welfare resilience in the immediate aftermath of the COVID-19 outbreak in Italy
N° 199/20	Elsa Fornero Christina Benita Wilke	Pension Policy in Europe and the United States – Towards a new Public-Private Pension Mix
N° 198/20	Ainoa Aparicio Fenoll Flavia Coda-Moscarola Sarah Zaccagni	Mathematics Camps: A Gift for Gifted Students?
N° 197/19	Noemi Oggero	Retirement Expectations in the Aftermath of a Pension Reform
N° 196/19	Francesco Scervini Serena Trucchi	Consumption response to offspring's income risk
N° 195/19	Raffaele Corvino Francesco Ruggiero	The Relative Pricing of Sovereign Credit Risk After the Eurozone Crisis
N° 194/19	Raffaele Corvino	Dynamic Ownership, Private Benefits, and Stock Prices
N° 193/19	Beatrice Magistro	Financial literacy and preferences for economic openness in the U.K.
N° 192/19	Beatrice Magistro	The effects of financial and economic literacy on policy preferences in Italy
N° 191/19	Agata Maida Daniela Sonedda	Getting out of the starting gate on the right foot: employment effects of investment in human capital
N° 190/19	Stefania Basiglio Maria Cristina Rossi Arthur van Soest	Subjective inheritance expectations and economic outcomes
N° 189/19	Stefania Basiglio Paola De Vincentiis Eleonora Isaia Mariacristina Rossi	Credit Access and Approval

The full series is available at: http://www.cerp.carloalberto.org/category/publications/working-papers/